Chapter 12
Equilibrium and Elasticity

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12-1 What is Physics?

► Human buildings should be stable in spite of the gravitational force and wind forces on it, and a bridge should be stable in spite of the gravitational force pulling it downward and the repeated jolting it receives from cars and trucks.

► In this chapter we examine the two main aspects of stability: the equilibrium of the forces and torques acting on rigid objects and the elasticity of non-rigid objects, a property that governs how such objects can deform.
Equilibrium:

- **Equilibrium**: (Force and torque are zero),
  
  \[ \overrightarrow{P} = \text{a constant} \quad \text{and} \quad \overrightarrow{L} = \text{a constant}. \]

- **Static equilibrium**: (constant = 0)
12-3 The Requirements of Equilibrium

1. The net external force on the object must equal zero:
\[ \sum \vec{F}_{\text{ext}} = 0 \]

2. The net external torque on the object about any axis must be zero:
\[ \sum \vec{r}_{\text{ext}} = 0 \]

(that is, \( v_{\text{CM}} = 0 \) and \( \omega = 0 \)).

- Examples: with zero net external force and zero net external torque
Coplanar problems:

1. The net external force on the object must equal zero:
   \[ \sum \overrightarrow{F}_{\text{ext}} = 0 \]

2. The net external torque on the object about any axis must be zero:
   \[ \sum \overrightarrow{\tau}_{\text{ext}} = 0 \]

where the location of the axis of the torque equation is arbitrary.
12-4 The Center of Gravity

Center of mass:

\[ x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \]
Center of gravity:

\[(m_1 + m_2 + m_3 + \cdots)g_{CG} x_{CG} = m_1 g_1 x_1 + m_2 g_2 x_2 + m_3 g_3 x_3 + \cdots\]
Equating the torque resulting from $Mg_{CG}$ acting at the center of gravity to the sum of the torques acting on the individual particles gives

$$(m_1 + m_2 + m_3 + \cdots)g_{CG} x_{CG} = m_1 g_1 x_1 + m_2 g_2 x_2 + m_3 g_3 x_3 + \cdots$$

This expression accounts for the possibility that the value of $g$ can in general vary over the object. If we assume uniform $g$ over the object (as is usually the case), the $g$ factors cancel and we obtain

$$x_{CG} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$
Example:

A seesaw consisting of a uniform board of mass $M$ and length $\ell$ supports at rest a father and daughter with masses $m_f$ and $m_d$, respectively, as shown in Figure 12.7. The support (called the fulcrum) is under the center of gravity of the board, the father is a distance $d$ from the center, and the daughter is a distance $\ell/2$ from the center.

(A) Determine the magnitude of the upward force $\vec{n}$ exerted by the support on the board.

(B) Determine where the father should sit to balance the system at rest.

Figure 12.7 (Example 12.1) A balanced system.
\[ n - m_fg - m_dg - Mg = 0 \]

\[ n = m_fg + m_dg + Mg = (m_f + m_d + M)g \]

\[ (m_fg)(d) - (m_dg)\frac{\ell}{2} = 0 \]

\[ d = \left(\frac{m_d}{m_f}\right)\frac{\ell}{2} \]
Example:

A uniform horizontal beam with a length of \( \ell = 8.00 \) m and a weight of \( W_b = 200 \) N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of \( \phi = 53.0^\circ \) with the beam (Fig. 12.8a). A person of weight \( W_p = 600 \) N stands a distance \( d = 2.00 \) m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.
(1) \[ \sum F_x = R \cos \theta - T \cos \phi = 0 \]

(2) \[ \sum F_y = R \sin \theta + T \sin \phi - W_p - W_b = 0 \]

\[ \sum \tau_z = (T \sin \phi)(\ell) - W_p d - W_b \left( \frac{\ell}{2} \right) = 0 \]

\[ T = \frac{W_p d + W_b (\ell/2)}{\ell \sin \phi} = \frac{(600 \text{ N})(2.00 \text{ m}) + (200 \text{ N})(4.00 \text{ m})}{(8.00 \text{ m}) \sin 53.0^\circ} = 313 \text{ N} \]

\[ \frac{R \sin \theta}{R \cos \theta} = \tan \theta = \frac{W_p + W_b - T \sin \phi}{T \cos \phi} \]

\[ \theta = \tan^{-1} \left( \frac{W_p + W_b - T \sin \phi}{T \cos \phi} \right) \]

\[ = \tan^{-1} \left[ \frac{600 \text{ N} + 200 \text{ N} - (313 \text{ N}) \sin 53.0^\circ}{(313 \text{ N}) \cos 53.0^\circ} \right] = 71.1^\circ \]

\[ R = \frac{T \cos \phi}{\cos \theta} = \frac{(313 \text{ N}) \cos 53.0^\circ}{\cos 71.1^\circ} = 581 \text{ N} \]
Example:

A uniform ladder of length $\ell$ rests against a smooth, vertical wall (Fig. 12.9a). The mass of the ladder is $m$, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle $\theta_{\text{min}}$ at which the ladder does not slip.
(1) \( \sum F_x = f_s - P = 0 \)

(2) \( \sum F_y = n - mg = 0 \)

(3) \( P = f_s \)

(4) \( n = mg \)

(5) \( P = f_{s,\text{max}} = \mu_s n = \mu_s mg \)

\[ \sum \tau = P \ell \sin \theta_{\text{min}} - mg \frac{\ell}{2} \cos \theta_{\text{min}} = 0 \]

\[ \frac{\sin \theta_{\text{min}}}{\cos \theta_{\text{min}}} = \tan \theta_{\text{min}} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s} \]

\[ \theta_{\text{min}} = \tan^{-1} \left( \frac{1}{2\mu_s} \right) = \tan^{-1} \left[ \frac{1}{2(0.40)} \right] = 51^\circ \]
(A) Estimate the magnitude of the force \( \vec{F} \) a person must apply to a wheelchair’s main wheel to roll up over a sidewalk curb (Fig. 12.10a). This main wheel that comes in contact with the curb has a radius \( r \), and the height of the curb is \( h \).

(B) Determine the magnitude and direction of \( \vec{R} \).
(1) \[ d = \sqrt{r^2 - (r - h)^2} = \sqrt{2rh - h^2} \]

(2) \[ \sum \tau_A = mgd - F(2r - h) = 0 \]

\[ mg\sqrt{2rh - h^2} - F(2r - h) = 0 \]

\[ F = \frac{mg\sqrt{2rh - h^2}}{2r - h} \]

\[ F = \frac{(700 \text{ N}) \sqrt{2(0.3 \text{ m})(0.1 \text{ m})} - (0.1 \text{ m})^2}{2(0.3 \text{ m}) - 0.1 \text{ m}} \]

\[ = 3 \times 10^2 \text{ N} \]
(B) Determine the magnitude and direction of $\vec{R}$.

(3) $\sum F_x = F - R \cos \theta = 0$

(4) $\sum F_y = R \sin \theta - mg = 0$

\[ \frac{R \sin \theta}{R \cos \theta} = \tan \theta = \frac{mg}{F} \]

\[ \theta = \tan^{-1} \left( \frac{mg}{F} \right) = \tan^{-1} \left( \frac{700 \text{ N}}{300 \text{ N}} \right) = 70^\circ \]

\[ R = \frac{mg}{\sin \theta} = \frac{700 \text{ N}}{\sin 70^\circ} = 8 \times 10^2 \text{ N} \]
Example:

In Fig. 12-5a, a uniform beam, of length $L$ and mass $m = 1.8$ kg, is at rest on two scales. A uniform block, with mass $M = 2.7$ kg, is at rest on the beam, with its center a distance $L/4$ from the beam’s left end. What do the scales read?
\[ F_l + F_r - Mg - mg = 0. \]

\[
(0)(F_l) - (L/4)(Mg) - (L/2)(mg) + (L)(F_r) = 0,
\]

\[
F_r = \frac{1}{4}Mg + \frac{1}{2}mg
\]
\[
= \frac{1}{4}(2.7 \text{ kg})(9.8 \text{ m/s}^2) + \frac{1}{2}(1.8 \text{ kg})(9.8 \text{ m/s}^2)
\]
\[
= 15.44 \text{ N} \approx 15 \text{ N}. \quad \text{(Answer)}
\]

\[ F_l = (M + m)g - F_r \]
\[
S_{mg} = (2.7 \text{ kg} + 1.8 \text{ kg})(9.8 \text{ m/s}^2) - 15.44 \text{ N}
\]
\[
= 28.66 \text{ N} \approx 29 \text{ N}. \quad \text{(Answer)}
\]
In Fig. 12-6a, a ladder of length \( L = 12 \) m and mass \( m = 45 \) kg leans against a slick (frictionless) wall. The ladder’s upper end is at height \( h = 9.3 \) m above the pavement on which the lower end rests (the pavement is not frictionless). The ladder’s center of mass is \( L/3 \) from the lower end. A firefighter of mass \( M = 72 \) kg climbs the ladder until her center of mass is \( L/2 \) from the lower end. What then are the magnitudes of the forces on the ladder from the wall and the pavement?
\[-(h)(F_w) + (a/2)(Mg) + (a/3)(mg) + (0)(F_{px}) + (0)(F_{py}) = 0.\]

Using the Pythagorean theorem, we find that
\[a = \sqrt{L^2 - h^2} = 7.58 \text{ m}.\]

Then Eq. 12-19 gives us
\[F_w = \frac{ga(M/2 + m/3)}{h}\]
\[= \frac{(9.8 \text{ m/s}^2)(7.58 \text{ m})(72/2 \text{ kg} + 45/3 \text{ kg})}{9.3 \text{ m}}\]
\[= 407 \text{ N} \approx 410 \text{ N}. \quad \text{(Answer)}\]

Now we need to use the force balancing equations. The equation \(F_{\text{net},x} = 0\) gives us
\[F_w - F_{px} = 0,\]
so
\[F_{px} = F_w = 410 \text{ N}. \quad \text{(Answer)}\]

The equation \(F_{\text{net},y} = 0\) gives us
\[F_{py} - Mg - mg = 0,\]
so
\[F_{py} = (M + m)g = (72 \text{ kg} + 45 \text{ kg})(9.8 \text{ m/s}^2)\]
\[= 1146.6 \text{ N} \approx 1100 \text{ N}. \quad \text{(Answer)}\]
Figure 12-7a shows a safe, of mass $M = 430$ kg, hanging by a rope from a boom with dimensions $a = 1.9$ m and $b = 2.5$ m. The boom consists of a hinged beam and a horizontal cable. The uniform beam has a mass $m$ of 85 kg; the masses of the cable and rope are negligible.

(a) What is the tension $T_c$ in the cable? In other words, what is the magnitude of the force $\vec{T_c}$ on the beam from the cable?

(b) Find the magnitude $F$ of the net force on the beam from the hinge.
Writing torques in the form of \( r \perp F \) and using our rule about signs for torques, the balancing equation \( \tau_{\text{net},z} = 0 \) becomes

\[
(a)(T_c) - (b)(T_r) - \left(\frac{1}{2}b\right)(mg) = 0.
\]

Substituting \( Mg \) for \( T_r \) and solving for \( T_c \), we find that

\[
T_c = \frac{gb(M + \frac{1}{2}m)}{a} = \frac{(9.8 \text{ m/s}^2)(2.5 \text{ m})(430 \text{ kg} + 85/2 \text{ kg})}{1.9 \text{ m}} = 6093 \text{ N} \approx 6100 \text{ N}. \quad \text{(Answer)}
\]
(b) Find the magnitude $F$ of the net force on the beam from the hinge.

**Calculations:** For the horizontal balance, we write $F_{\text{net},x} = 0$ as

$$F_h - T_c = 0,$$

and so

$$F_h = T_c = 6093 \text{ N}.$$

For the vertical balance, we write $F_{\text{net},y} = 0$ as

$$F_v - mg - T_r = 0.$$

Substituting $Mg$ for $T_r$ and solving for $F_v$, we find that

$$F_v = (m + M)g = (85 \text{ kg} + 430 \text{ kg})(9.8 \text{ m/s}^2)$$

$$= 5047 \text{ N}.$$

From the Pythagorean theorem, we now have

$$F = \sqrt{F_h^2 + F_v^2}$$

$$= \sqrt{(6093 \text{ N})^2 + (5047 \text{ N})^2} \approx 7900 \text{ N.} \quad \text{(Answer)}$$
For the problems of this chapter, we have only three independent equations at our disposal, usually two balance of forces equations and one balance of torques equation about a given rotation axis. Thus, if a problem has more than three unknowns, we cannot solve it.
Consider an unsymmetrically loaded car. What are the forces—all different on the four tires?

We can solve an equilibrium problem for a table with three legs but not for one with four legs.

Problems like these, in which there are more unknowns than equations, are called indeterminate.

To solve such indeterminate equilibrium problems, we must supplement equilibrium equations with some knowledge of elasticity.
The atoms of a metallic solid are distributed on a repetitive three-dimensional lattice. The springs represent interatomic forces.
stress = modulus × strain.

\[ \frac{F}{A} = E \frac{\Delta L}{L} \]
Stress-Strain curve

Stress-versus-strain curve for an elastic solid.
Elastic modulus

\[ \text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}} \]

1. Young’s modulus measures the resistance of a solid to a change in its length.

2. Shear modulus measures the resistance to motion of the planes within a solid parallel to each other.

3. Bulk modulus measures the resistance of solids or liquids to changes in their volume.
Young’s Modulus: Elasticity in Length (E or Y)

The amount by which the length of the bar changes due to the applied force is $\Delta L$.

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$
<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Young’s Modulus $E$ ($10^9$ N/m$^2$)</th>
<th>Ultimate Strength $S_{\text{u}}$ ($10^6$ N/m$^2$)</th>
<th>Yield Strength $S_{\text{y}}$ ($10^6$ N/m$^2$)</th>
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</thead>
<tbody>
<tr>
<td>Steel$^a$</td>
<td>7860</td>
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<td>400</td>
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<td>$50^b$</td>
<td>—</td>
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<tr>
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<td>$40^b$</td>
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<td>Wood$^d$</td>
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<td>13</td>
<td>$50^b$</td>
<td>—</td>
</tr>
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<td>Bone</td>
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<td>$9^b$</td>
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</tr>
<tr>
<td>Polystyrene</td>
<td>1050</td>
<td>3</td>
<td>48</td>
<td>—</td>
</tr>
</tbody>
</table>
Shear Modulus: Elasticity of Shape

\[ S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h} \]

The shear stress causes the top face of the block to move to the right relative to the bottom.
Bulk Modulus: Volume Elasticity

Bulk modulus $B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$

The cube undergoes a change in volume but no change in shape.
Example:

One end of a steel rod of radius $R = 9.5 \text{ mm}$ and length $L = 81 \text{ cm}$ is held in a vise. A force of magnitude $F = 62 \text{ kN}$ is then applied perpendicularly to the end face (uniformly across the area) at the other end. What are the stress on the rod and the elongation $\Delta L$ and strain of the rod?
stress = \frac{F}{A} = \frac{F}{\pi R^2} = \frac{6.2 \times 10^4 \text{ N}}{(\pi)(9.5 \times 10^{-3} \text{ m})^2}
= 2.2 \times 10^8 \text{ N/m}^2. \quad \text{(Answer)}

The yield strength for structural steel is $2.5 \times 10^8 \text{ N/m}^2$, so this rod is dangerously close to its yield strength.

We find the value of Young’s modulus for steel in Table 12-1. Then from Eq. 12-23 we find the elongation:

$$
\Delta L = \frac{(F/A)L}{E} = \frac{(2.2 \times 10^8 \text{ N/m}^2)(0.81 \text{ m})}{2.0 \times 10^{11} \text{ N/m}^2}
= 8.9 \times 10^{-4} \text{ m} = 0.89 \text{ mm}. \quad \text{(Answer)}
$$

For the strain, we have

$$
\frac{\Delta L}{L} = \frac{8.9 \times 10^{-4} \text{ m}}{0.81 \text{ m}}
= 1.1 \times 10^{-3} = 0.11\%. \quad \text{(Answer)}
$$
Example:

A table has three legs that are 1.00 m in length and a fourth leg that is longer by $d = 0.50 \text{ mm}$, so that the table wobbles slightly. A steel cylinder with mass $M = 290 \text{ kg}$ is placed on the table (which has a mass much less than $M$) so that all four legs are compressed but unbuckled and the table is level but no longer wobbles. The legs are wooden cylinders with cross-sectional area $A = 1.0 \text{ cm}^2$; Young’s modulus is $E = 1.3 \times 10^{10} \text{ N/m}^2$. What are the magnitudes of the forces on the legs from the floor?
We take the table plus steel cylinder as our system. The situation is like that in Fig. 12-9, except now we have a steel cylinder on the table. If the tabletop remains level, the legs must be compressed in the following ways: Each of the short legs must be compressed by the same amount (call it $\Delta L_3$) and thus by the same force of magnitude $F_3$. The single long leg must be compressed by a larger amount $\Delta L_4$ and thus by a force with a larger magnitude $F_4$. In other words, for a level tabletop, we must have

$$\Delta L_4 = \Delta L_3 + d.$$  \hspace{1cm} (12-26)

From Eq. 12-23, we can relate a change in length to the force causing the change with $\Delta L = FL/AE$, where $L$ is the original length of a leg. We can use this relation to replace $\Delta L_4$ and $\Delta L_3$ in Eq. 12-26. However, note that
we can approximate the original length $L$ as being the same for all four legs.

**Calculations:** Making those replacements and that approximation gives us

$$\frac{F_4 L}{AE} = \frac{F_3 L}{AE} + d. \quad (12-27)$$

We cannot solve this equation because it has two unknowns, $F_4$ and $F_3$.

To get a second equation containing $F_4$ and $F_3$, we can use a vertical $y$ axis and then write the balance of vertical forces ($F_{\text{net},y} = 0$) as

$$3F_3 + F_4 - Mg = 0, \quad (12-28)$$

where $Mg$ is equal to the magnitude of the gravitational force on the system. *(Three legs have force $F_3$ on them.)*

To solve the simultaneous equations 12-27 and 12-28 for, say, $F_3$, we first use Eq. 12-28 to find that $F_4 = Mg - 3F_3$. Substituting that into Eq. 12-27 then yields, after some algebra,

$$F_3 = \frac{Mg}{4} - \frac{dAE}{4L}$$

$$= \frac{(290 \text{ kg})(9.8 \text{ m/s}^2)}{4} - \frac{(5.0 \times 10^{-4} \text{ m})(10^{-4} \text{ m}^2)(1.3 \times 10^{10} \text{ N/m}^2)}{(4)(1.00 \text{ m})}$$

$$= 548 \text{ N} \approx 5.5 \times 10^2 \text{ N.} \quad \text{(Answer)}$$
From Eq. 12-28 we then find

\[ F_4 = Mg - 3F_3 = (290 \text{ kg})(9.8 \text{ m/s}^2) - 3(548 \text{ N}) \]
\[ \approx 1.2 \text{ kN}. \]

(Answer)

You can show that to reach their equilibrium configuration, the three short legs are each compressed by 0.42 mm and the single long leg by 0.92 mm.
17. A flexible chain weighing 40.0 N hangs between two hooks located at the same height (Fig. P12.17). At each hook, the tangent to the chain makes an angle $\theta = 42.0^\circ$ with the horizontal. Find (a) the magnitude of the force each hook exerts on the chain and (b) the tension in the chain at its midpoint. *Suggestion:* For part (b), make a force diagram for half of the chain.
A uniform beam of length $L$ and mass $m$ shown in Figure P12.16 is inclined at an angle $\theta$ to the horizontal. Its upper end is connected to a wall by a rope, and its lower end rests on a rough, horizontal surface. The coefficient of static friction between the beam and surface is $\mu_s$. Assume the angle $\theta$ is such that the static friction force is at its maximum value. (a) Draw a force diagram for the beam. (b) Using the condition of rotational equilibrium, find an expression for the tension $T$ in the rope in terms of $m$, $g$, and $\theta$. (c) Using the condition of translational equilibrium, find a second expression for $T$ in terms of $\mu_s$, $m$, and $g$. (d) Using the results from parts (a) through (c), obtain an expression for $\mu_s$ involving only the angle $\theta$. (e) What happens if the ladder is lifted upward and its base is placed back on the ground slightly to the left of its position in Figure P12.16? Explain.
23. One end of a uniform 4.00-m-long rod of weight $F_g$ is supported by a cable at an angle of $\theta = 37^\circ$ with the rod. The other end rests against the wall, where it is held by friction as shown in Figure P12.23. The coefficient of static friction between the wall and the rod is $\mu_s = 0.500$. Determine the minimum distance $x$ from point $A$ at which an additional object, also with the same weight $F_g$, can be hung without causing the rod to slip at point $A$. 

Figure P12.23
Example:

24. A 10.0-kg monkey climbs a uniform ladder with weight \(1.20 \times 10^2\) N and length \(L = 3.00\) m as shown in Figure P12.24. The ladder rests against the wall and makes an angle of \(\theta = 60.0^\circ\) with the ground. The upper and lower ends of the ladder rest on frictionless surfaces. The lower end is connected to the wall by a horizontal rope that is frayed and can support a maximum tension of only 80.0 N. (a) Draw a force diagram for the ladder. (b) Find the normal force exerted on the bottom of the ladder. (c) Find the tension in the rope when the monkey is two-thirds of the way up the ladder. (d) Find the maximum distance \(d\) that the monkey can climb up the ladder before the rope breaks. (e) If the horizontal surface were rough and the rope were removed, how would your analysis of the problem change? What other information would you need to answer parts (c) and (d)?
A uniform sign of weight $F_g$ and width $2L$ hangs from a light, horizontal beam hinged at the wall and supported by a cable (Fig. P12.45). Determine (a) the tension in the cable and (b) the components of the reaction force exerted by the wall on the beam in terms of $F_g$, $d$, $L$, and $\theta$. 

![Figure P12.45](image_url)
Example:

When a gymnast performing on the rings executes the iron cross, he maintains the position at rest shown in Figure P12.53a. In this maneuver, the gymnast’s feet (not shown) are off the floor. The primary muscles involved in supporting this position are the latissimus dorsi (“lats”) and the pectoralis major (“pecs”). One of the rings exerts an upward force $\vec{F}_h$ on a hand as shown in Figure P12.53b.
A uniform beam of mass $m$ is inclined at an angle $\theta$ to the horizontal. Its upper end (point $P$) produces a $90^\circ$ bend in a very rough rope tied to a wall, and its lower end rests on a rough floor (Fig. P12.51). Let $\mu_s$ represent the coefficient of static friction between beam and floor. Assume $\mu_s$ is less than the cotangent of $\theta$. (a) Find an expression for the maximum mass $M$ that can be suspended from the top before the beam slips. Determine (b) the magnitude of the reaction force at the floor and (c) the magnitude of the force exerted by the beam on the rope at $P$ in terms of $m$, $M$, and $\mu_s$. 

![Diagram of the beam and rope system with labels $P$, $M$, and $\theta$.]