Fluid classification by response to shear stress

- Newtonian
- Ideal Fluid
- Ideal plastic

\[ \tau = \mu \frac{du}{dy} \]

Rate of deformation

Fluid Viscosity

- Examples of highly viscous fluids
  - molasses, tar, 20w-50 oil
- Fundamental mechanisms
  - Gases - transfer of molecular momentum
    - Viscosity **increases** as temperature increases.
    - Viscosity **increases** as pressure increases.
  - Liquids - cohesion and momentum transfer
    - Viscosity **decreases** as temperature increases.
    - Relatively independent of pressure (incompressible)

Example: Measure the viscosity of water

The inner cylinder is 10 cm in diameter and rotates at 10 rpm. The fluid layer is 2 mm thick and 10 cm high. The power required to turn the inner cylinder is 50x10^{-6} watts. What is the dynamic viscosity of the fluid?

Solution Scheme

- Restate the goal
- Identify the given parameters and represent the parameters using symbols
- Outline your solution including the equations describing the physical constraints and any simplifying assumptions
- Solve for the unknown symbolically
- Substitute numerical values with units and do the arithmetic
  - Check your units!
  - Check the reasonableness of your answer
Viscosity Measurement: Solution

\[ F = \mu \frac{AU}{t} \quad U = \omega r \quad A = 2\pi rh \]
\[ F = \mu \frac{2\pi \omega^2 r^3 h}{t} \]
\[ P = \frac{F \omega}{r} \]
\[ P = \frac{2\pi \omega^2 r^3 h}{t} \]
\[ \mu = \frac{P t}{2\pi \omega^2 r^3 h} \]
\[ \omega = 10 \text{ rpm} \]
\[ = 10(2\pi)/60 \text{ } 1/\text{s} = 1.047/\text{s} \]

Outer cylinder

Inner cylinder

Thin layer of water

- Statics
  - Fluids at rest have no relative motion between layers of fluid and thus \( \frac{du}{dy} = 0 \)
  - Therefore the shear stress is \( \text{zero} \) and is independent of the fluid viscosity

- Flows
  - Fluid viscosity is very important when the fluid is moving

Dynamic and Kinematic Viscosity

- Kinematic viscosity \( (\nu) \) is a fluid property obtained by dividing the dynamic viscosity \( (\mu) \) by the fluid density

\[ \nu = \frac{\mu}{\rho} \]
\[ \mu \Rightarrow \left[ \frac{\text{N} \cdot \text{s}}{\text{m}^2} \right] \]
\[ [\text{N}] = \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right] \]
\[ [\mu] = \left[ \frac{\text{kg} \cdot \text{s}}{\text{m}^2} \right] \]

Elasticity (Compressibility)

- Deformation per unit of pressure change

\[ E_v = -\frac{dp}{dV/V} = \frac{dp}{d\rho/\rho} \]

- Bulk Modulus of Elasticity

- For water \( E_v = 2.2 \text{ GPa} \),
  - 1 MPa pressure change = 0.05% volume change
  - Water is relatively incompressible
Example

- **Given:** Pressure of 2 MPa is applied to a mass of water that initially filled 1000-cm³ volume.
- **Find:** Volume after the pressure is applied.
- **Solution:**
  
  \[
  E_v = -\frac{\Delta p}{\Delta V/V}
  \]
  
  \[
  \Delta V = -\frac{\Delta p}{E_v} V
  \]
  
  \[
  = -\frac{2 \times 10^6 \text{ Pa}}{2.2 \times 10^9 \text{ Pa}} (-1000 \text{ cm}^3)
  \]
  
  \[
  = -0.909 \text{ cm}^3
  \]
  
  \[
  V_{\text{final}} = V + \Delta V
  \]
  
  \[
  = 1000 - 0.909
  \]
  
  \[
  V_{\text{final}} = 999.01 \text{ cm}^3
  \]

Vapor Pressure

- Pressure at which a liquid will boil for given temp.
- Vapor pressure increases with temperature
- If you reduce the pressure in water at this temperature, boiling will occur (cavitation)

What is vapor pressure of water at 100°C? **101 kPa**

Cavitation Damage

- Below surface, forces act equally in all directions
- At surface, some forces are missing, pulls molecules down and together, like membrane exerting **tension** on the surface
- If interface is curved, higher pressure will exist on concave side
- Pressure increase is balanced by surface tension, \( \sigma \)
  
  \[
  \sigma = 0.073 \text{ N/m (at 20°C)}
  \]
Capillary Rise

- **Given:** Water @ 20°C, \( d = 1.6 \text{ mm} \)
- **Find:** Height of water
- **Solution:** Sum forces in vertical
  Assume \( \theta \text{ small}, \cos \theta \approx 1 \)
  \[
  F_{\sigma,z} - W = 0
  \]
  \[
  \sigma d \cos \theta - \gamma (\Delta h) \left( \frac{\pi}{4} d^2 \right) = 0
  \]
  \[
  \Delta h = \frac{4 \sigma}{\rho d}
  \]
  \[
  = \frac{4 \times 0.073}{9790 \times 1.6 \times 10^{-3}}
  \]
  \[
  \Delta h = 18.6 \text{ mm}
  \]

Example

- **Find:** Capillary rise between two vertical glass plates 1 mm apart.
  \( \sigma = 7.3 \times 10^{-2} \text{ N/m} \)
  \( t \) is into the page
- **Solution:**
  \[
  \sum F_{\text{vertical}} = 0
  \]
  \[
  2 \sigma t - \kappa t y = 0
  \]
  \[
  h = \frac{2 \sigma}{\gamma}
  \]
  \[
  h = \frac{2 \times 7.3 \times 10^{-2}}{0.001 \times 9810}
  \]
  \[
  h = 0.0149 \text{ m}
  \]
  \[
  h = 14.9 \text{ mm}
  \]

Examples of Surface Tension

- **Pressure increase in a spherical droplet**
  \[
  \Delta p \pi R^2 = 2 \pi R \sigma
  \]
  \[
  \Delta p = \frac{2 \sigma}{R}
  \]
  \[
  \Delta p \pi R^2 = 2 \pi R \sigma
  \]

Surface Tension

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  \[
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  \]
Example: Surface Tension

- Estimate the difference in pressure (in Pa) between the inside and outside of a bubble of air in 20°C water. The air bubble is 0.3 mm in diameter.

\[ p = \frac{2\sigma}{R} \]

\[ R = 0.15 \times 10^{-3} \text{ m} \]

\[ \sigma = 0.073 \text{ N/m} \]

\[ p = 970 \text{ Pa} \]

Statics! \[ p = \gamma h \]

\[ h = \frac{p}{\gamma} = \frac{974 \text{ Pa}}{9806 \text{ N/m}^3} = 0.1 \text{ m water} \]

What is the difference between pressure in a water droplet and in an air bubble?

Example

- Find: The formula for the gage pressure within a spherical droplet of water?
- Solution: Surface tension force is resisted by the force due to pressure on the cut section of the drop

\[ p(\pi r^2) = 2\pi \sigma \]

\[ p = \frac{2\sigma}{r} \]

Dimensions and Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>ML(^{-3})</td>
</tr>
<tr>
<td>Specific Weight</td>
<td>( \gamma )</td>
<td>ML(^{-2})T(^{-2})</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>( \mu )</td>
<td>ML(^{-1})T(^{-1})</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>( \nu )</td>
<td>L(^2)T(^{-1})</td>
</tr>
<tr>
<td>Surface tension</td>
<td>( \sigma )</td>
<td>MT(^{-1})</td>
</tr>
<tr>
<td>Bulk mod of elasticity</td>
<td>( E )</td>
<td>ML(^{-1})T(^{-2})</td>
</tr>
</tbody>
</table>

These are **fluid** properties!

How many independent properties? \(\_4\_\)
Exercise:

1.10, 1.13, 1.54, 1.64, 1.97