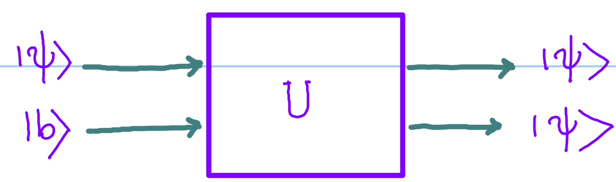
شبیه‌سازی حالت‌های کوانتومی

[1] W. K. Wootters and W. H. Zurek, Nature 299, 802 (1982).

• قضیه No-cloning

• استدلال بر پایه خطی بودن QM



$$U(|0\rangle|b\rangle) = |0\rangle|0\rangle \qquad U(|1\rangle|b\rangle) = |1\rangle|1\rangle$$

$$U((|0\rangle+|1\rangle)|b\rangle) = (|0\rangle+|1\rangle)(|0\rangle+|1\rangle) = |0\rangle|0\rangle+|1\rangle|1\rangle+|0\rangle|1\rangle+|1\rangle|0\rangle$$

$$|\psi\rangle|b\rangle|M\rangle \xrightarrow{U} |\psi\rangle|\psi\rangle|M_\psi\rangle$$

$$U(|0,b,M\rangle) = |0,0,M_0\rangle \qquad U(|1,b,M\rangle) = |1,1,M_1\rangle \qquad (1)$$

$$(1) \rightarrow \quad U((|0\rangle+|1\rangle)|b\rangle|M\rangle) = |0,0\rangle|M_0\rangle + |1,1\rangle|M_1\rangle \qquad (2)$$

$$\text{cloning} \rightarrow \quad = (|0,0\rangle+|0,1\rangle+|1,0\rangle+|1,1\rangle)|N\rangle \qquad (3)$$

$$2 \neq 3.$$

• استدلال برای کپی نشدن QM.

$$|\psi\rangle|b\rangle|M\rangle \xrightarrow{\ U\ } |\psi\rangle|\psi\rangle|M_\psi\rangle$$

$$|\phi\rangle|b\rangle|M\rangle \xrightarrow{\ U\ } |\phi\rangle|\phi\rangle|M_\phi\rangle$$

$$\rightarrow \quad \langle\psi|\phi\rangle = \langle\psi|\phi\rangle^2 \langle M_\psi|M_\phi\rangle$$

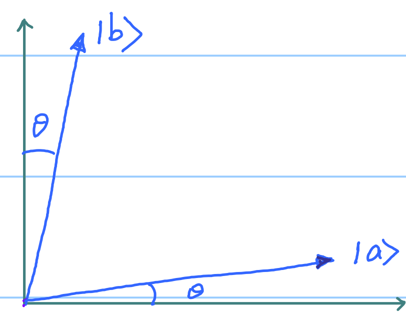
$$\langle\psi|\phi\rangle = \frac{1}{\langle M_\psi|M_\phi\rangle}$$

$\longrightarrow$ ① $\langle\psi|\phi\rangle = 0$

or

$\longrightarrow$ ② $\langle\psi|\phi\rangle = 1$

State Dependent Cloning



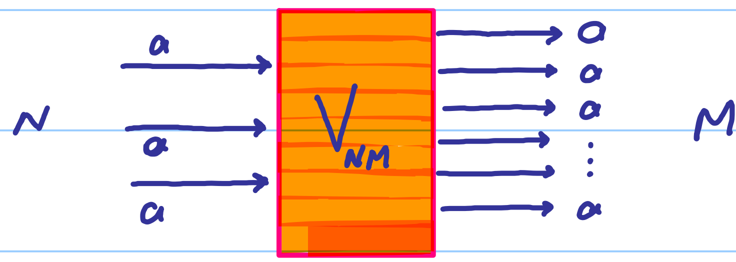
$$|a\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$

$$|b\rangle = \sin\theta\,|0\rangle + \cos\theta\,|1\rangle$$

$$\langle a|b\rangle = \sin 2\theta =: S$$

$$N \longrightarrow M$$

یک حد پایینی برای فیدلیتی پیدا کنیم



[6] D. Bruß, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A **57**, 2368 (1998).

[7] C. Macchiavello, J. Optics B **2**, 144 (2000).

We will consider a unitary operator V_{NM} acting on the Hilbert space of M qubits and define the final states $|\alpha_{NM}\rangle$ and $|\beta_{NM}\rangle$ as

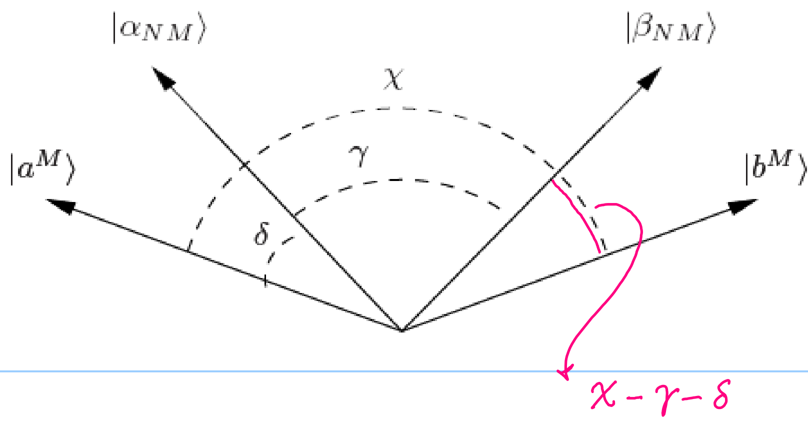
$$|\alpha_{NM}\rangle = V_{NM}\left(|a\rangle^{\otimes N} \otimes |0\rangle^{\otimes M-N}\right), \quad (4.5)$$
$$|\beta_{NM}\rangle = V_{NM}\left(|b\rangle^{\otimes N} \otimes |0\rangle^{\otimes M-N}\right). \quad (4.6)$$

Unitarity gives the following constraint on the scalar product of the final states:

$$\langle\alpha_{NM}|\beta_{NM}\rangle = (\langle a|b\rangle)^N = S^N. \quad (4.7)$$

As a convenient criterion for optimality of the cloning transformation, we maximize the average global fidelity $F_g(N,M)$ of both final states $|\alpha_{NM}\rangle$ and $|\beta_{NM}\rangle$ with respect to the perfectly cloned states $|a^M\rangle \equiv |a\rangle^{\otimes M}$ and $|b^M\rangle \equiv |b\rangle^{\otimes M}$. The average global fidelity is defined formally as

$$F_g(N,M) = \frac{1}{2}\left(|\langle\alpha_{NM}|a^M\rangle|^2 + |\langle\beta_{NM}|b^M\rangle|^2\right). \quad (4.8)$$



$$F_g(N,M) = \frac{1}{2}\left[\cos^2\delta + \cos^2(\chi-\gamma-\delta)\right]$$

Symmetry $\longrightarrow$ $\chi-\gamma-\delta = \delta$ $\longrightarrow$ $\delta = \dfrac{\chi-\gamma}{2}$

$$F_g(N,M) = \frac{1}{2}\left[\cos^2\frac{\chi-\gamma}{2} + \cos^2\frac{\chi-\gamma}{2}\right]$$

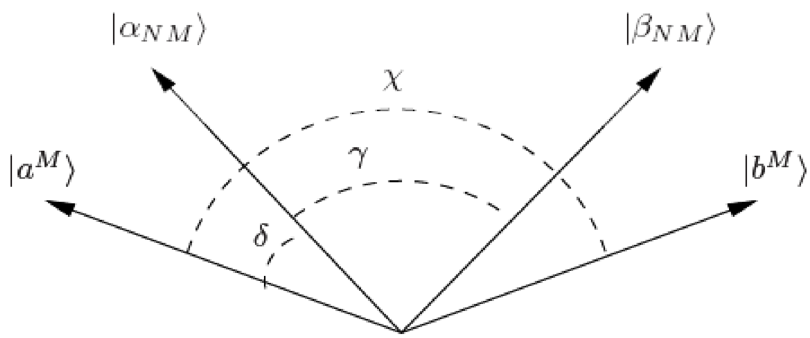
$$F_g(N,M) = \cos^2\frac{\chi-\gamma}{2} = \frac{1}{2}\left(1+\cos(\chi-\gamma)\right)$$

$$F_g(N,M) = \frac{1}{2}\left\{1 + \cos\chi\cos\gamma + \sin\chi\sin\gamma\right\}$$

$$\langle a^M | b^M \rangle = \langle a|b \rangle^M = S^M \longrightarrow \quad \cos x = S^M$$

$$\cos \gamma = \langle \alpha_{MN} | \beta_{MN} \rangle = S^N$$

$$F_g(M,N) = \frac{1}{2}\left[1 + S^{M+N} + \sqrt{1-S^M}\sqrt{1-S^N} \right]$$



$$|\alpha_{NM}\rangle = x \; |\alpha^M\rangle + y \; |\beta^M\rangle$$

$$|\beta_{NM}\rangle = y \; |\alpha^M\rangle + x \; |\beta^M\rangle$$

$$x^2 + y^2 + 2xy\, S^M = 1$$

$$S^N \equiv \langle \alpha_{NM} | \beta_{NM} \rangle = 2xy + (x^2+y^2)\, S^M$$

$$x = A+B \quad , \quad y = A-B \longrightarrow \begin{cases} x^2+y^2 = 2(A^2+B^2). \\ 2xy = 2(A^2-B^2). \end{cases}$$

قبلا داشتیم

$$\begin{cases} x^2+y^2+2S^M xy = 1 \\ S^N \equiv \langle \alpha_{NM} | \beta_{NM} \rangle = 2xy + (x^2+y^2) S^M \end{cases}$$

$$
\begin{aligned}
|\alpha_{NM}\rangle &= (A+B)|a^M\rangle + (A-B)|b^M\rangle \\
|\beta_{NM}\rangle &= (A-B)|a^M\rangle + (A+B)|b^M\rangle,
\end{aligned}
$$

where

$$A = \frac{1}{2}\sqrt{\frac{1+S^N}{1+S^M}}, \quad B = \frac{1}{2}\sqrt{\frac{1-S^N}{1-S^M}}.$$

$$2(A^2+B^2) + 2(A^2-B^2)S^M = 1$$

$$2(A^2-B^2) + 2(A^2+B^2)S^M = S^N$$

$$\rho_\alpha = (A+B)^2|a\rangle\langle a| + (A-B)^2|b\rangle\langle b| + (A^2-B^2)S^{M-1}(|a\rangle\langle b| + |b\rangle\langle a|$$

idelity is then calculated as

$$
\begin{aligned}
F_{\mathrm{sd}}(N,M) &= \langle a|\rho_\alpha|a\rangle \\
&= A^2(1+S^2+2S^M) + B^2(1+S^2-2S^M) + 2AB(1-S^2).
\end{aligned}
$$

$$F_{\mathrm{sd}}(1,2) = \frac{1}{2}\left[1 + \frac{1-S^2}{\sqrt{1+S^2}} + \frac{S^2(1+S)}{1+S^2}\right].$$

$$F_{\mathrm{sd}}(1,3) = \frac{1}{4}\left[2S^3\left(\frac{1+S}{1+S^3} - \frac{1-S}{1-S^3}\right) + (1+S^2)\left(\frac{1+S}{1+S^3} + \frac{1-S}{1-S^3}\right) + 2(1-S^2)\sqrt{\frac{1-S^2}{1-S^6}}\right].$$

In Fig. 4.2 we show the fidelities for the $1 \to 2$ and $1 \to 3$ cloners as functions of the parameter θ. The dashed curve corresponds to $F_{\mathrm{sd}}(1,2)$, the full curve to $F_{\mathrm{sd}}(1,3)$. As expected, the values of the fidelity are always much higher than $F_{\mathrm{pc}}^{\mathrm{opt}}(1,2) = (\sqrt{2}+1)/(2\sqrt{2}) \approx 0.854$ and $F_{\mathrm{u}}^{\mathrm{opt}}(1,2) = 5/6 \approx 0.833$ for the $1 \to 2$ optimal phase covariant and universal cloners, respectively, and than $F_{\mathrm{pc}}^{\mathrm{opt}}(1,3) = (7+2\sqrt{3})/[2(2\sqrt{3}+3)] \approx 0.809$ and $F_{\mathrm{u}}^{\mathrm{opt}}(1,3) = 7/9 \approx 0.778$, see Sections 4.4 and 4.5.

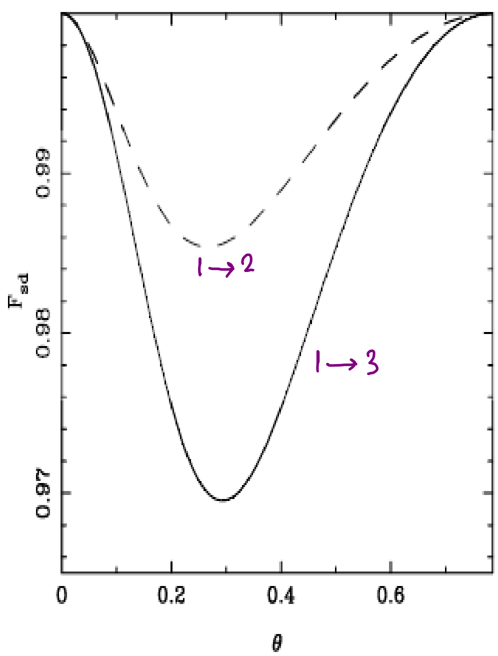
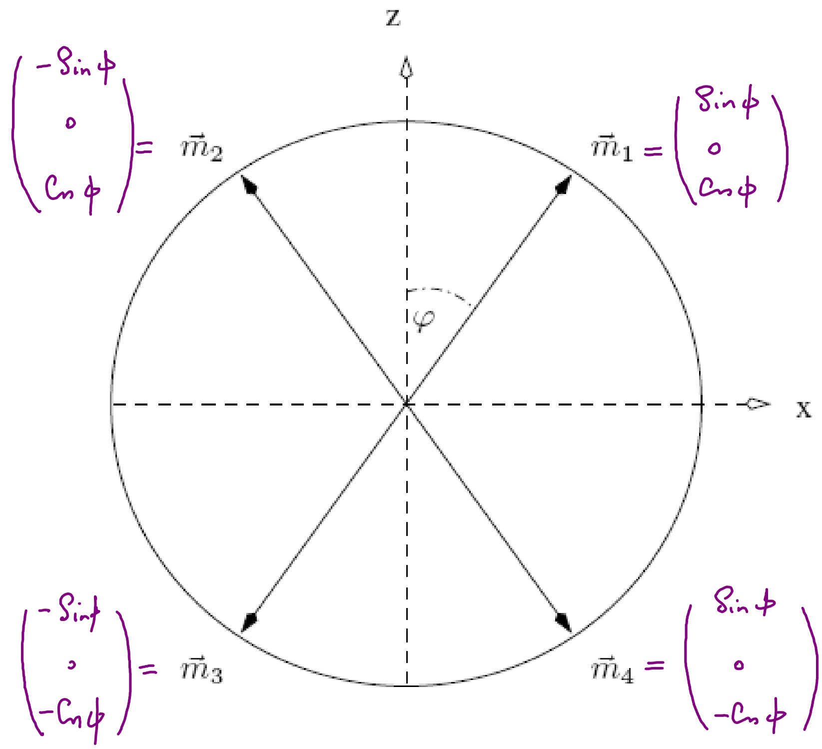
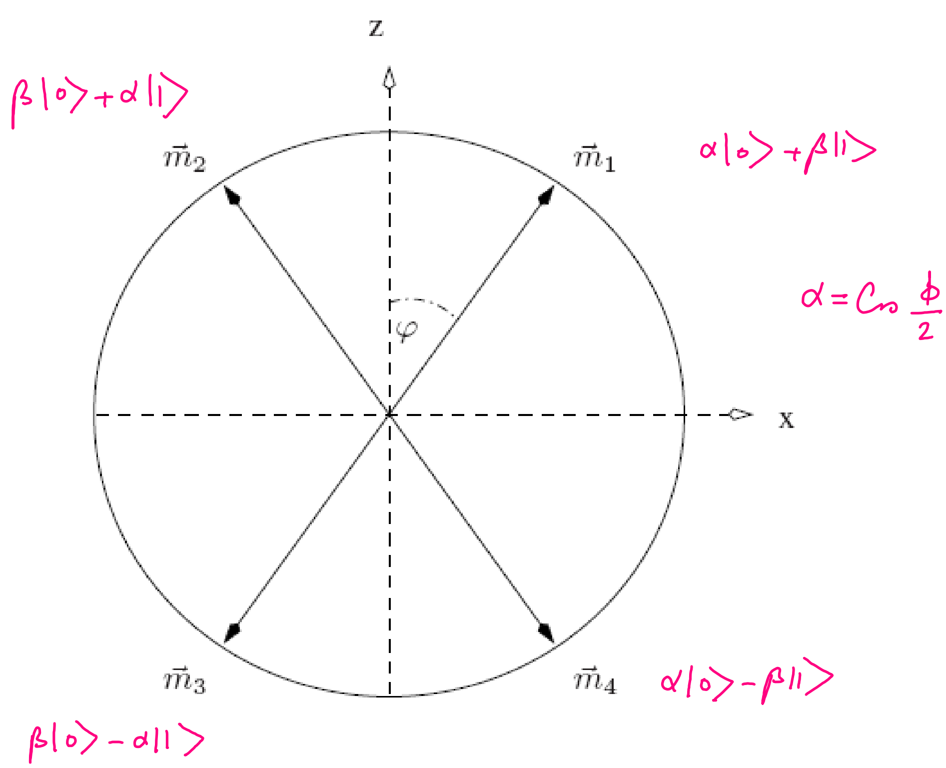


Figure 4.2. Fidelity for each output copy of the state-dependent cloner as a function of the parameter θ. The dashed curve refers to the $1 \to 2$ cloner (Eq. (4.15)), while the full curve corresponds to the $1 \to 3$ cloner (Eq. (4.16)).

• دستگاهی برای کپی کردن چهار حالت مختلف


z
x
φ
$\begin{pmatrix} -\sin\phi \\ 0 \\ \cos\phi \end{pmatrix} = \vec{m}_2$
$\vec{m}_1 = \begin{pmatrix} \sin\phi \\ 0 \\ \cos\phi \end{pmatrix}$
$\begin{pmatrix} -\sin\phi \\ 0 \\ -\cos\phi \end{pmatrix} = \vec{m}_3$
$\vec{m}_4 = \begin{pmatrix} \sin\phi \\ 0 \\ -\cos\phi \end{pmatrix}$


z
x
φ
$\beta|0\rangle + \alpha|1\rangle$
$\vec{m}_2$
$\vec{m}_1$
$\alpha|0\rangle + \beta|1\rangle$
$\alpha = \cos\frac{\phi}{2}$
$\vec{m}_3$
$\vec{m}_4$
$\alpha|0\rangle - \beta|1\rangle$
$\beta|0\rangle - \alpha|1\rangle$

$$
\begin{aligned}
U\,|0\rangle|0\rangle|X\rangle &= a|00\rangle|A\rangle + b(|01\rangle + |10\rangle)|B\rangle + c|11\rangle|C\rangle, \\
U\,|1\rangle|0\rangle|X\rangle &= \tilde{a}|11\rangle|\tilde{A}\rangle + \tilde{b}(|10\rangle + |01\rangle)|\tilde{B}\rangle + \tilde{c}|00\rangle|\tilde{C}\rangle,
\end{aligned}
$$

if $|0\rangle \longleftrightarrow |1\rangle \implies |\psi_1\rangle \longleftrightarrow |\psi_3\rangle, \quad |\psi_2\rangle \longleftrightarrow |\psi_3\rangle$

$\Rightarrow a = \tilde{a}, \quad b = \tilde{b}, \quad c = \tilde{c}.$

Normalization: $a^2 + 2b^2 + c^2 = 1$

orthogonality: $ac(\langle A|C\rangle + \langle C|A\rangle) + 2b^2\, \mathrm{Re}(\langle B|\tilde{B}\rangle) = 0$

$$
\begin{aligned}
U\,|0\rangle|0\rangle|X\rangle &= a|00\rangle|A\rangle + b(|01\rangle + |10\rangle)|B\rangle + c|11\rangle|C\rangle, \\
U\,|1\rangle|0\rangle|X\rangle &= \tilde{a}|11\rangle|\tilde{A}\rangle + \tilde{b}(|10\rangle + |01\rangle)|\tilde{B}\rangle + \tilde{c}|00\rangle|\tilde{C}\rangle,
\end{aligned}
$$

$$
|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha\left(a|00\rangle|A\rangle + b(|01\rangle + |10\rangle)|B\rangle + c|11\rangle|C\rangle\right)
$$
$$
+ \beta\left(a|11\rangle|\tilde{A}\rangle + b(|01\rangle + |10\rangle)|\tilde{B}\rangle + c|00\rangle|\tilde{C}\rangle\right)
$$

$$
F_1 = \langle\psi_1|\rho^{out}|\psi_1\rangle.
$$

$$F = a^2(\alpha^4 + \beta^4) + 2c^2\alpha^2\beta^2 + b^2 + \alpha^2\beta^2\Big[ab \cdot 2\,\mathrm{Re}(\langle A|\tilde{B}\rangle + \langle B|\tilde{A}\rangle) + bc \cdot 2\,\mathrm{Re}(\langle B|\tilde{C}\rangle + \langle C|\tilde{B}\rangle)\Big] \,.$$

Independently of the coefficients a, b, c the fidelity will be maximal for the following choice of scalar products between the auxiliary states:

$$\begin{aligned} \langle A|\tilde{B}\rangle &= 1 = \langle B|\tilde{A}\rangle \,, \\ \langle B|\tilde{C}\rangle &= 1 = \langle C|\tilde{B}\rangle \,, \end{aligned} \tag{4.26}$$

which can be reached with a two-dimensional ancilla and, e.g., the choice

$$\begin{aligned} |A\rangle &= |0\rangle \,,\ |B\rangle = |1\rangle \,,\ |C\rangle = |0\rangle \,, \\ |\tilde{A}\rangle &= |1\rangle \,,\ |\tilde{B}\rangle = |0\rangle \,,\ |\tilde{C}\rangle = |1\rangle \,. \end{aligned} \tag{4.27}$$

Inserting this into Eq. (4.25) we arrive at

$$F = \tfrac{1}{2} + \tfrac{1}{2}(a^2 - c^2)\cos^2\varphi + b(a+c)\sin^2\varphi \,.$$

$$\begin{aligned} a\cos^2\varphi + b\sin^2\varphi &= 2a\lambda \,, \\ (a+c)\sin^2\varphi &= 4b\lambda \,, \\ -c\cos^2\varphi + b\sin^2\varphi &= 2c\lambda \,, \\ a^2 + 2b^2 + c^2 &= 1 \,, \end{aligned}$$

where λ is the Lagrange multiplier. The solution for the coeffic

$$\begin{aligned} a &= \tfrac{1}{2}\Big(1 + \cos^2\varphi\sqrt{\frac{1}{\sin^4\varphi + \cos^4\varphi}}\Big) \,, \\ b &= \tfrac{1}{2}\sin^2\varphi\sqrt{\frac{1}{\sin^4\varphi + \cos^4\varphi}} \,, \\ c &= \tfrac{1}{2}\Big(1 - \cos^2\varphi\sqrt{\frac{1}{\sin^4\varphi + \cos^4\varphi}}\Big) \,. \end{aligned}$$

$$F^{\rm opt} = \frac{1}{2}(1 + \sqrt{\sin^4\varphi + \cos^4\varphi}) \, .$$

[5] D. Bruß, G. M. D'Ariano, C. Macchiavello, M. F. Sacchi, Phys. Rev. A **62**, 62302 (2000).

[6] D. Bruß, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A **57**, 2368 (1998).

[7] C. Macchiavello, J. Optics B **2**, 144 (2000).

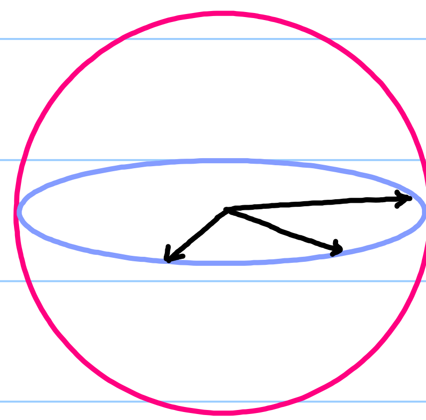
[8] D. Bruß and C. Macchiavello, J. Phys. A: Math. Gen. **34**, 1 (2001).

[9] D. Bruß, M. Cinchetti, G. M. D'Ariano, and C. Macchiavello, Phys. Rev. A **62**, 12302 (2000).

phase Covariant cloning

$$|\psi_\phi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + e^{i\phi}|1\rangle\right] \; ,$$

$\theta = \frac{\pi}{2}$

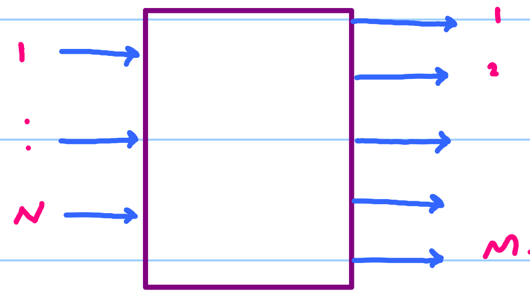


phase Covariance:

$$U_\phi = e^{i\phi|1\rangle\langle 1|} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$$

if $\rho_{in} \longrightarrow U_\phi \, \rho_{in} \, U_\phi^\dagger$

then $\rho_{out} \longrightarrow U_\phi \, \rho_{out} \, U_\phi^\dagger$



$$U_{\varphi}\rho^{out}U_{\varphi}^{\dagger} = Tr_{M-1}[C_{NM}(U_{\varphi}^{N}|\psi_{\phi}\rangle\langle\psi_{\phi}|^{\otimes N}U_{\varphi}^{\dagger N})]$$

[10] C. A. Fuchs, N. Gisin, R. B. Griffiths, C.-S. Niu, and A. Peres, Phys. Rev. A **56**, 1163 (1997).

[11] H. Fan et al., Phys. Rev. A **65**, 012304 (2002).

[12] G.M. D'Ariano and C. Macchiavello, Phys. Rev. A **67**, 042306 (2003).

[13] M. Keyl and R. Werner, J. Math. Phys. **40**, 3283 (1999).

For $1 \longrightarrow M$ cloning

$$F_{pcc} = \frac{1}{2}\left(1 + \frac{M+1}{2M}\right) \quad \text{for odd } M\,,$$

$$F_{pcc} = \frac{1}{2}\left(1 + \frac{\sqrt{M(M+2)}}{2M}\right) \quad \text{for even } M\,.$$

For $N \longrightarrow M$ cloning

$$F_{\mathrm{pcc}} = \frac{1}{2} + \frac{1}{M2^N} \sum_{j=0}^{N-1} \sqrt{C(N,j)C(N,j+1)} \sqrt{\left[\frac{(M+N)}{2} - j\right]\left[\frac{(M-N)}{2} + j + 1\right]},$$

$\binom{N}{j}$

Phase covariant cloning in d-dimensions:

$$|\psi(\{\phi_j\})\rangle = \frac{1}{\sqrt{d}}(|0\rangle + e^{i\phi_1}|1\rangle + e^{i\phi_2}|2\rangle + \cdots + e^{i\phi_{d-1}}|d-1\rangle),$$

[14] H. Fan, H. Imai, K. Matsumoto, and X.-B. Wang, Phys. Rev. A **67**, 022317 (2003).

optimal Fidelity for $1 \longrightarrow 2$

$$F^{\mathrm{opt}}_{\mathrm{d,pcc}} = \frac{1}{d} + \frac{1}{4d}\left(d - 2 + \sqrt{d^2 + 4d - 4}\right).$$

[15] F. Buscemi, G.M. D'Ariano, and C. Macchiavello, Phys. Rev. A **71**, 042327 (2005).

Optimal Fidelity for $N \longrightarrow M$

$$F^{\rm opt}_{\rm d,pcc} = \frac{1}{d} + \frac{1}{M d^{N+1}} \sum_{\{n_j\}} \sum_{i\neq j} \frac{N!}{n_0!\ldots n_i!\ldots n_j!\ldots} \sqrt{\frac{(n_i+k+1)(n_j+k+1)}{(n_i+1)(n_j+1)}}.$$

$$\sum_{j=0}^{d-1} n_j = N-1$$

Universal Cloning

Quantum copying: Beyond the no-cloning theorem

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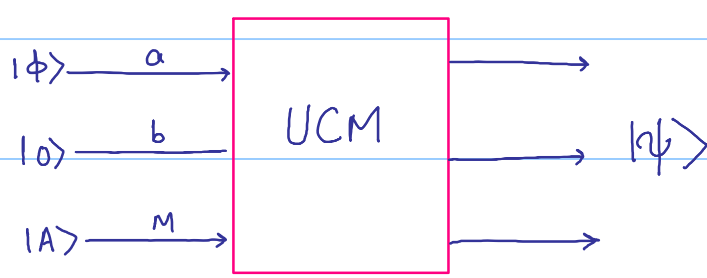
[1] *Department of Physics and Astronomy, Hunter College of the City University of New York, 695 Park Avenue, New York, NY 10021, USA*
[2] *Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Slovakia*
(February 1, 2008)

We analyze a possibility of copying ($\equiv$ cloning) of arbitrary states of quantum-mechanical spin-1/2 system. We show that there exists a "universal quantum-copying machine" (i.e. transformation) which approximately copies quantum-mechanical states such that the quality of its output does not depend on the input. We also examine a machine which combines a unitary transformation and a selective measurement to produce good copies of states in the neighborhood of a particular state. We discuss the problem of measurement of the output states.

03.65.Bz

PRA (1996)



Wootters, Zurek:

$$|0\rangle|0\rangle|Q\rangle \xrightarrow{U} |0\rangle|0\rangle|Q_0\rangle$$

$$|1\rangle|0\rangle|Q\rangle \xrightarrow{U} |1\rangle|1\rangle|Q_1\rangle$$

Norm $\longrightarrow$ $\langle Q_0|Q_0\rangle = \langle Q_1|Q_1\rangle = 1 \qquad \langle Q_0|Q_1\rangle = 0$

$$|s\rangle = a|0\rangle + b|1\rangle. \qquad \rho_{in}^{id} = |s\rangle\langle s|$$

$$|s\rangle|0\rangle|Q\rangle \xrightarrow{U} \underbrace{a\,|0\rangle|0\rangle|Q_0\rangle + b\,|1\rangle|1\rangle|Q_1\rangle}_{\psi_{out} = \psi}$$

$$\rho_a^{out} = \mathrm{tr}_{bM}(|\psi\rangle\langle\psi|) = a^2|0\rangle\langle 0| + b^2|1\rangle\langle 1|$$

$$\rho_b^{out} = \mathrm{tr}_{aM}(|\psi\rangle\langle\psi|) = a^2|0\rangle\langle 0| + b^2|1\rangle\langle 1|$$

$$\rho_a = \rho_b \longrightarrow \text{Symmetric cloner.}$$

$$D_a := \| \rho_{in}^{id} - \rho_a^{out} \|$$

$$\| A \| = \mathrm{tr}\, A^2 \qquad \rho_a^{id} = |s\rangle\langle s|$$

مطالب ما ← ① D_a به حالت ورودی بستگی دارد.

② متوسط D_a روی تمام ورودی‌ها برابر است با $\frac{1}{3}$.

Buzek, Hillery

$$|00\rangle|Q\rangle \longrightarrow |00\rangle|Q_0\rangle + (|01\rangle + |10\rangle)|Y_0\rangle$$

$$|10\rangle|Q\rangle \longrightarrow |11\rangle|Q_1\rangle + (|01\rangle + |10\rangle)|Y_1\rangle$$

Constraints:

$$\langle Q_0|Q_0\rangle + 2\langle Y_0|Y_0\rangle = 1$$

$$\langle Q_1|Q_1\rangle + 2\langle Y_1|Y_1\rangle = 1$$

$$\langle Y_0 | Y_1 \rangle = 0$$

فرض در راستای $\quad \langle Q_0 | Q_1 \rangle = 0 \quad , \quad \langle Q_0 | Y_0 \rangle = \langle Q_1 | Y_1 \rangle = 0$

مراحل مختلف مقاله Buzek, Hillery:

(1) $|S\rangle = a|0\rangle + b|1\rangle \longrightarrow a\left(|00\rangle|Q_0\rangle + \sqrt{2}\,|+\rangle|Y_0\rangle\right) + b\left(|11\rangle|Q_1\rangle + \sqrt{2}\,|+\rangle|Y_1\rangle\right)$

(2) $\rho_a^{out},\ \rho_b^{out} \longleftarrow \rho_{ab}^{out} \longleftarrow |\psi_{out}\rangle$

(3) تقاضا می‌کنیم که $D_a := tr\left(\left(\rho_a^{out} - \rho_{in}^{id}\right)^2\right)$ مستقل از ورودی باشد.

(4) تقاضا می‌کنیم که فاصله ρ_{ab}^{out} و $\rho_a^{out} \otimes \rho_b^{out}$ کمترین مقدار ممکن را داشته باشد.

$$
\begin{aligned}
U|0\rangle|0\rangle|A\rangle &= \sqrt{\frac{2}{3}}|00\rangle|0\rangle + \sqrt{\frac{1}{6}}(|01\rangle + |10\rangle)|1\rangle, \\
U|1\rangle|0\rangle|A\rangle &= \sqrt{\frac{2}{3}}|11\rangle|1\rangle + \sqrt{\frac{1}{6}}(|01\rangle + |10\rangle)|0\rangle.
\end{aligned}
$$

$$F = \langle \psi_a | \rho_a^{out} | \psi_a \rangle = \frac{5}{6}$$

[6] D. Bruß, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A **57**, 2368 (1998).

BH Cloning machine is optimal.

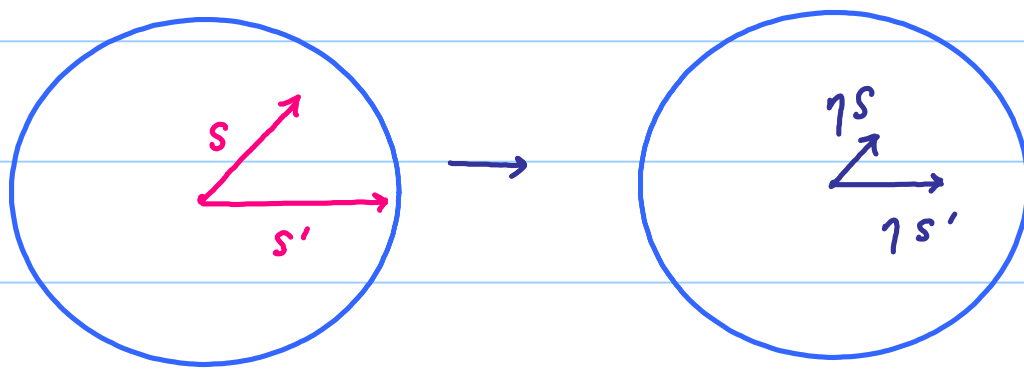
یک روش دیگر :

[6] D. Bruß, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A **57**, 2368 (1998).

هیچ ماشینی وجود ندارد که همه بردارهای $\vec{r}$ در کره بلوخ را به یک اندازه بچرخاند.

[16] D. Bruß, A. Ekert, and C. Macchiavello, Phys. Rev. Lett. **81**, 2598 (1998).

$\rho_{in} = \frac{1}{2}\left(I + \vec{s}.\vec{\sigma} \right) \longrightarrow \rho_{\text{out}} = \frac{1}{2}(\mathbb{1} + \eta_{\text{u}}(N,M)\vec{s}\cdot\vec{\sigma})$,



η = shrinking Factor

$$F(\rho_{in}, \rho_{out}) = \frac{1+\eta}{2}$$ Fidelity

$$F(\rho, \sigma) = \left\{ tr \sqrt{\rho^{1/2} \sigma \rho^{1/2}} \right\}^2$$

if $\rho = |\psi\rangle\langle\psi| \longrightarrow \rho^{1/2} = |\psi\rangle\langle\psi| \longrightarrow$

$$\rho^{1/2}\sigma\rho^{1/2} = |\psi\rangle\langle\psi|\sigma|\psi\rangle\langle\psi| = |\psi\rangle\langle\psi| \langle\psi|\sigma|\psi\rangle$$

$$\sqrt{\rho^{1/2}\sigma\rho^{1/2}} = |\psi\rangle\langle\psi| \sqrt{\langle\psi|\sigma|\psi\rangle}$$

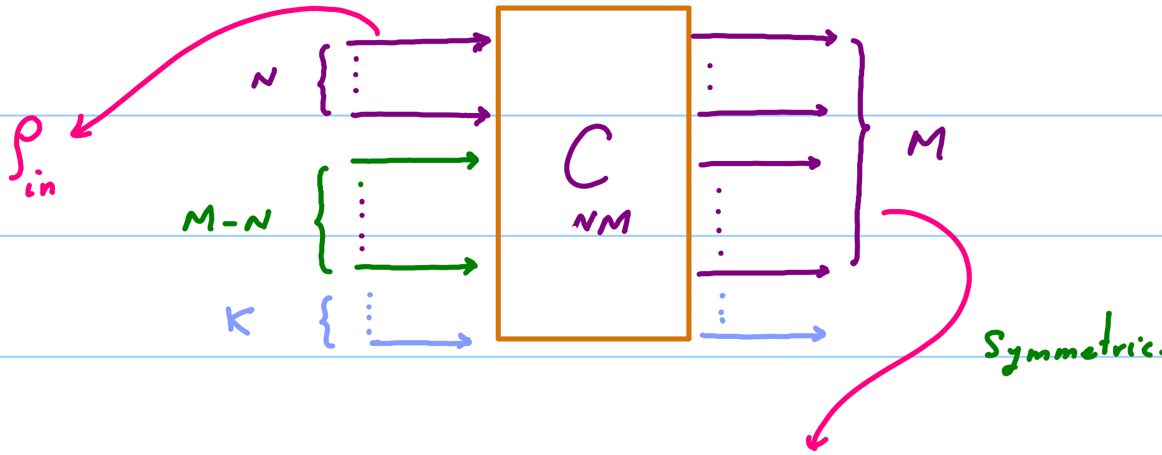
$$\rightarrow \quad F(\rho,\sigma) = \langle\psi|\sigma|\psi\rangle = \mathrm{tr}(\rho\sigma)$$

$$\rho_{in} = \frac{1}{2}(1 + s\cdot\sigma) \qquad \rho_{out} = \frac{1}{2}(1 + \eta\, s\cdot\sigma)$$

$$\rightarrow \quad F = \mathrm{tr}(\rho_{in}\rho_{out}) = \frac{1}{2}(1+\eta).$$

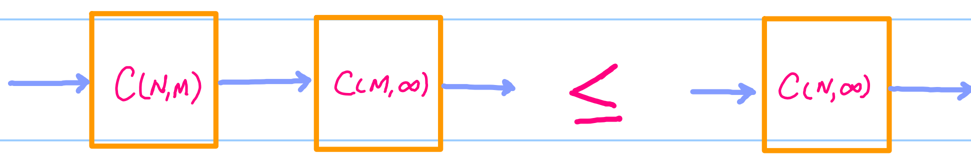
[16] D. Bruß, A. Ekert, and C. Macchiavello, Phys. Rev. Lett. **81**, 2598 (1998).

$$\rho \longrightarrow \eta\rho + \frac{1}{2}(1-\eta) I.$$



$$\rho' = \rho_{out} = \mathrm{Tr}_{M-1}\left(\rho_{out}^{1,\dots,M}\right)$$

حال شبکه‌ی زیر را در نظر بگیرید:



$$\eta_{op}(N,M)\,\eta_{op}(M,\infty) \leq \eta_{op}(N,\infty) \qquad -1$$

$$\eta_{op}(N,M) \leq \frac{\eta_{op}(N,\infty)}{\eta_{op}(M,\infty)}$$

$$\eta_{op}(M,\infty) = \eta_{meas.}(M) \qquad (2$$

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Optimal Extraction of Information from Finite Quantum Ensembles

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Given only a finite ensemble of identically prepared particles, how precisely can one determine their states? We describe optimal measurement procedures in the case of spin 1/2 particles. Furthermore, we prove that optimal measurement procedures must necessarily view the ensemble as a single composite system rather than as the sum of its components, i.e., optimal measurements cannot be realized by

$$\eta_{meas}(M) = \frac{M}{M+2}$$

$$\rightarrow \eta_{op}(N,M) = \frac{\left(\frac{N}{N+2}\right)}{\left(\frac{M}{M+2}\right)} = \frac{N}{M}\,\frac{M+2}{N+2}$$

$$F_{N,M} = \frac{1}{2}(1+\eta) = \frac{M+N+MN}{M(N+2)}$$

$$\boxed{F_{1,2} = \frac{2+1+2}{2(1+2)} = \frac{5}{6}.}$$

Buzek-Hillery

[18] R. Werner, Phys. Rev. A **58**, 1827 (1998).

$$F_{\mathrm{u}}^{\mathrm{opt}}(N, M) = \frac{M - N + N(M+d)}{M(N+d)},$$

[21] S. Albeverio and S.M. Fei, Eur. Phys. J. B **14**, 669 (2000).

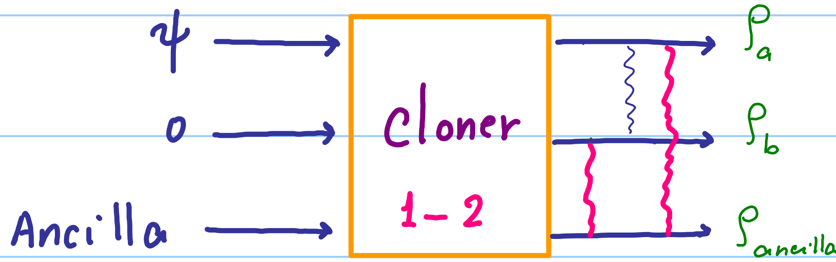
Explicit Construction of the unitary map.

if we now let $M \to \infty$ and use

$$F_{op}(N, \infty) = F_{meas}(N)$$

$$\rightarrow \quad F_{meas}(N) = \lim_{M \to \infty} \frac{M - N + N(M+d)}{M(N+d)} = \frac{N+1}{N+d}.$$

Entanglement Structure.



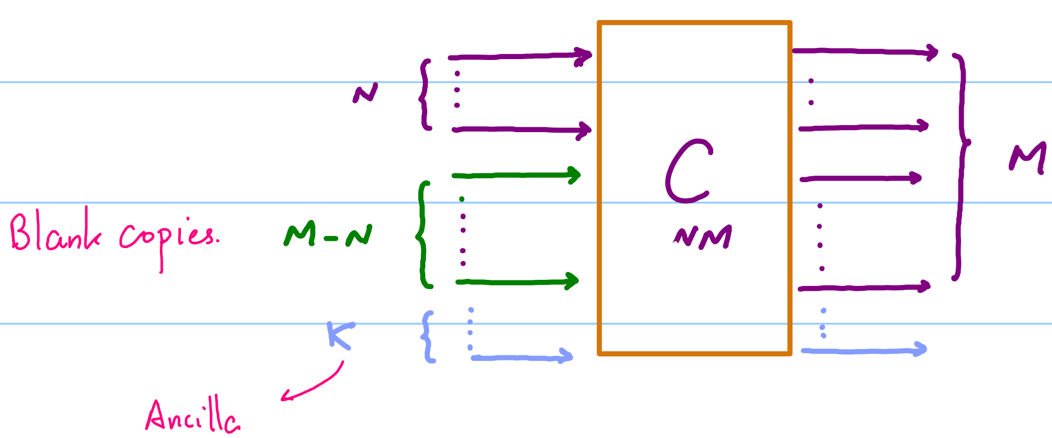
[23] D. Bruß and C. Macchiavello, Found. Phys. **33**, 1617 (2003).

① حالت خروجی در کلاس W است

$$|W\rangle = \frac{1}{\sqrt{3}}\left(|100\rangle + |010\rangle + |001\rangle\right)$$

② درهم تنیدگی بین کپی‌ها خروجی با هم کمتر از درهم تنیدگی بین آنها و Ancilla است.

[24] K. O'Connor and W. Wootters, Phys. Rev. A **63**, 052302 (2001).



$$C_{cc}(1, M) = 2 \max\left(\frac{1}{6} - \frac{\sqrt{(3M+2)(M-2)}}{6M}, 0\right). \quad (1)$$

$M = 2 \longrightarrow C_{cc}(1,2) = 2 \max\left(\frac{1}{6}, 0\right) = \frac{1}{3}$

$M = 3 \longrightarrow C_{cc}(1,3) = 2 \max\left(\frac{1}{6} - \frac{1}{3}\sqrt{\frac{11}{2}}, 0\right) = 0$

$C_{cc} = 0 \quad \text{if} \quad M > 3$

$$C_{ca}(1, M) = \frac{1}{3}\left(\frac{M+2}{M} - \sqrt{\frac{M-2}{M}}\right). \quad (2)$$

But $C_{ca}(1, M) \geq 0$ & $C_{ca}(1, M) \xrightarrow{M \to \infty} 0$

- Asymmetric cloning
- Probabilistic cloning
- Experimental cloning

آنچه که گفته نشد.
