

# Mathematics Education in Iran

## From Ancient to Modern

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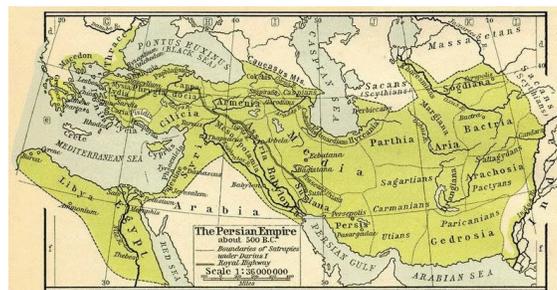
### 1. Introduction

Land of Persia was a cradle of science in ancient times and moved to the modern Iran through historical ups and downs. Iran has been a land of prominent, influential figures in science, arts and literature. It is a country whose impact on the global civilization has permeated centuries. Persian scientists contributed to the understanding of nature, medicine, mathematics, and philosophy, and the unparalleled names of Ferdowsi, Rumi, Rhazes, al-Biruni, al-Khwarizmi and Avicenna attest to the fact that Iran has been perpetually a land of science, knowledge and conscience in which cleverness grows and talent develops. There are considerable advances through education and training in various fields of sciences and mathematics in the ancient and medieval eras to the modern and post-modern periods.

In this survey, we will present advancements of mathematics and math education in Iran from ancient Persia and the Islamic Golden Age toward modern and post-modern eras, we will have some conclusions and remarks in the epilogue.

### 2. Ancient: Rise of the Persian Empire

The Land of Persia is home to one of the world's oldest civilizations, beginning with its formation in 3200–2800 BC and reaching its pinnacle of its power during the Achaemenid Empire. Founded by Cyrus the Great in 550 BC, the Achaemenid Empire at its greatest extent comprised major portions of the ancient world.



Persian Empire

Governing such a vast empire was, for sure in need of financial and administration systems. In the 1930s, researchers from the Oriental Institute of the Chicago University found more than 30,000 clay tablets in Persepolis (capital of the Achaemenid Empire). Tablets provided information about the finance and administration systems, and they utilized weights and enumeration models, arithmetic and accounting systems. Besides arithmetic and numerical systems, geometry also had been used extensively in architecture to construct buildings and cities such as Persepolis. There is no evidence about any educational system or educational institute in the empire, but seems there had been some hands on and in service trainings.

In the ancient period, the only evidence of a structured educational state was in Gondishapur Academy during the Sasanian dynasty (224-650 AC). The Academy of Gondishapur was one of the most important learning institutes in the ancient time. It was the intellectual center of the Sasanian Empire and offered training in medicine, philosophy, theology and science. The faculty was versed in the Zoroastrian and Persian traditions. No evidence of mathematics or math education has been remained but there are some references to a “Royal Astronomical Tables” which indicates that reasonable amounts of math had been developed to provide such astronomical tables.

### **3. Medieval: The Golden Age**

Muslim conquest of Persia led to the end of Sasanian Empire in 651, but after two centuries of the Arab rule, semi-independent and independent Iranian kingdoms began to appear, and by the Samanid era in the 9<sup>th</sup> and 10<sup>th</sup> centuries, the efforts of Iranians to regain their independence had been well solidified.

The blossoming literature, philosophy, medicine, arts and sciences of Iran became major elements in the formation of a new age for the Iranian civilization, during the period known as the *Islamic Golden Age*. The Islamic Golden Age reached its peak by the 10<sup>th</sup> and 11<sup>th</sup> centuries, during which Iran was the main theater of the scientific activities. After the 10<sup>th</sup> century, the Persian language, alongside Arabic, was used for the scientific, philosophical, historical, musical, and medical works, where the important Iranian scientists such as Tusi, Avicenna, and al-Biruni, had major contributions. Mathematics was also in the center of scientific development in the Golden Age and many original math works developed accordingly.

The history of mathematics during the Golden Age of Islam, especially during the 9<sup>th</sup> and 10<sup>th</sup> centuries, built on Greek mathematics and Indian mathematics but saw important development, such as full development of the decimal place-value system to include decimal fractions and the first systemized study of algebra, as well as advances in geometry and trigonometry.

One major contribution came from **Mohammad ibn Musa al-Khwarizmi**, who played a significant role in the development of algebra, algorithms, and Hindu-Arabic numerals in the 9<sup>th</sup> century. Al-Khwarizmi’s contributions to mathematics, geography, astronomy, and cartography established the basis for innovation in algebra and trigonometry. His systematic approach to solving linear and quadratic equations led to algebra, a word derived from title of his 830<sup>th</sup> book on the subject, “*The Compendious Book on Calculation by Completion and Balancing*”.

*On the Calculation with Hindus Numerals* written circa 825, was principally responsible for spreading the Hindu-Arabic numeral system throughout the Middle east and Europe. It was translated into Latin as *Algoritmi de Numero Indorum*. Al-Khwarizmi rendered as (Latin) *Algoritmi*, which led to term “*algorithm*” in computer science.

Al-Khwarizmi provided exhaustive account of solving polynomial equations up to the second degree and discussed the fundamental methods of “reduction” and “balancing”, referring to the transposition of terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of an equation.



A page from al-Khwarizmi’s Algebra

Al-Khwarizmi’s method of solving linear and quadratic equation worked by first reducing the equation to one of the six standard forms and by dividing out the coefficient of the square. By using the two operation *al-jabr* (“restoring” or “completion”) and *al-muqabala* (“balancing”), he developed a methodology for solving equations.

Another famous mathematician of the 11<sup>th</sup> century is **Omar Khayyam**, who wrote the influential book *Treatise on Demonstration of Problems of Algebra* (1070), which laid down the principles of algebra and derived general methods for solving cubic equations and even some higher orders.

In the *Treatise*, he wrote of the triangular arrays of binomial coefficients also known as Pascal’s triangle. In 1077, Khayyam wrote *Sharh ma Ashkala min Musadarat Kitab Uqlidis* (Explanations

of the Difficulties in the Postulates of Euclid) published in English as “On the Difficulties of Euclid’s Definitions”. An important part of the book is concerned with Euclid’s famous parallel postulate, and Khayyam’s advances and criticisms may have contributed to the eventual development of non-Euclidean geometry.

As a mathematician, Khayyam has also made fundamental contributions to the philosophy of mathematics especially in the context of Persian Mathematics and Persian philosophy.

Another significant Persian mathematician and astronomer in the medieval period of the 14<sup>th</sup> century was **Ghiyath al-Din Jamshid Mas’ud al-Kashi**. Al-Kashi developed great contributions in numerical systems and approximation methods. In his famous book *The Treatise on the Chord and Sine*, al-Kashi computed  $\sin 1^\circ$  to nearly as much accuracy as his value for  $\pi$ , which was the most accurate approximation of  $\sin 1^\circ$  in his time and was not surpassed until 16<sup>th</sup> century. In algebra and numerical analysis, he correctly computed  $2\pi$  to 9 sexagesimal digits, and he converted his approximation of  $\pi$  to 17 decimal place of accuracy. Al-Kashi also developed and used both decimal and sexagesimal fractions with great ease in his *Key to Arithmetic*.

Al-Kashi’s work included astronomy as well. He wrote the *Treatise on Astronomical Observational Instruments*, which described a variety of different instruments, and he invented the *Plate of Conjunctions*, an analogue computing instrument used to determine the time of day by using linear interpolation techniques. Al-Kashi also invented a mechanical planetary computer which he called the *Plate of Zones*, which could graphically solve a number of planetary problems, including the prediction of the true positions in longitude of the Sun, Moon, and planets in terms of elliptical orbits, the latitude of the Sun, Moon, and planets, and the ecliptic orbit of the Sun.

In the Golden Age, for scholars training including mathematicians, there existed scientific institutes. In 832, the famous House of Wisdom was founded in Baghdad, there the methods of Gondishapur were emulated and many mathematicians and scientists attracted and joined. Some scholars produced original research in the House of Wisdom for example, al-Khwarizmi worked in House of Wisdom for development of his contribution on algebra. In the northern and central parts of Iran there were also libraries under support of Persian kingdoms where intellectual and scholar training centers and many important scientists and mathematicians were housed and supported. There is no evidence of any educational system or institute other than scholarly training under supervision of scientists and mathematicians.

In the eleventh century **Nezamiyeh Institutes** were founded as a group of the medieval institutions of higher education by Khwaja Nezam al-Mulk (the prime minister in the Sejukian era, the title of *Nezamiyeh* derived also from his name). Nezamiyeh institutes were among the first well organized institutions of higher learning in Iran. The quality of education was among the higher in the Islamic world, and they were even renowned in Europe. The royal establishments of the Seljukian empire

and elite class supported them financially, politically, and spiritually. The Nezamiyeh institutes placed more emphasis on religious studies but science, mathematics and astronomy also were considered.

In 1219, Iran suffered a devastating invasion by the Mongol army of Genghis Khan. The Mongols violence and depredations killed up to three-fourth of the Iran's population of the Iranian, as libraries were burned and a civilization was destroyed. Following the demise of the Mongol Empire in 1256, Hulagu Khan, grand son of Genghis Khan, established the Ilkhanate in Iran. In 1370, yet another conquer, Timur, followed the example of Hulagu, establishing the Timurid Empire that lasted for another century and half. The Ilkhans and Timurids soon came to adopt the customs of the Iranians, choosing surround themselves with a culture that was distinctively the customs of the Iranian. Nevertheless, the scientific establishments never reached to the pre-Mongolian in Iran.

#### **4. Modern Era: Post Mongolian**

In the 1500s the Safavi Empire reunited Iran, and European industrial products touched the Iranian market, but for more than three centuries, no approach for any reform in educational systems could be seen. Theological education dominated the traditional schools, which caused math and science education to hold little value.

Three centuries later, the Russo-Persian war ended in 1828 by the Turkmenchay treaty. Russia completed the conquering of all Caucasian territories from Iran, having previously gained Georgia, Dagestan, and most of contemporary Azerbaijan through the treaty of Gulistan in 1813. After losing the wars, the government officials understood that military losses were due to a lack of modern science and technology, so they dispatched a group of students to Europe to be trained in modern sciences. The next top down approach was performed by Amir Kabir, the reformist prime minister of the mid 19<sup>th</sup> century. He established Dar ul-Funun in 1851 as the first modern higher education institute in Iran. Emphasis was placed on medicine, science and military training, but a modern approach to math education was also in agenda. The public educational system was limited to the traditional elementary schools with emphasis on Arabic and Persian literatures and without much focus on mathematics teaching just arithmetic and a special accounting system called *Siaq*. The next bottom up approach was made in 1887 by Mirza Hassan Roshdiyeh who founded the first modern elementary school in Tabriz in the northwest of Iran. Later in 1897 he established the first modern elementary school in the capital Tehran, but there was still no establishment for the inclusive modern educational system.

In 1906, the constitute revolution caused more new approaches to be made toward reforming the traditional systems. In 1909, the second parliament passed a bill to establish the Ministry of Maearif (culture and science) and to officially establish educational systems including elementary and high schools and also higher education. As a result, elementary and high schools founded

nationwide. Teaching and learning mathematics were core value of the new system, to train more needed teachers including math teachers, the Teachers College was established in 1920.



Teachers College (Tehran)

Teachers College trained professional math teachers; the curriculum included psychological and teaching skills too.

In 1921, the parliament passed another bill to dispatch 100 students annually to Europe for five years to be trained in modern science and technology which included mathematics and math education. Some educated mathematicians and math educators became pioneering leaders in math education reform by developing new curriculums and writing math textbooks at all levels.

In 1934 Tehran University was established as the first modern higher education institute in Iran. Some smaller independent colleges also joined the new university including the Teachers College. Tehran University trained many professional mathematicians and also math teachers.

In 1941 Iran was occupied by the Allies troops during World War II, but after the war ended in 1948 new universities in major cities as Mashhad, Tabriz, Isfahan, and Ahvaz established. Mathematics came to be considered one of the most important majors, and many math teachers taught nationwide. In the 1960s new universities as Aryamehr (Sharif) University of Technology and Pahlavi (Shiraz) University were founded. New universities reformed the educational system, creating new approaches in the math curriculum that impacted the math curriculum in the pre-university level as well.

The pre-university educational system started with six years of elementary and six years of secondary schools. In elementary school, arithmetic and basic concepts of geometry were taught, in the secondary level subjects such as algebra, geometry, pre-calculus and calculus were covered. In algebra and pre-calculus and calculus, courses were primarily based on numerical and symbolic calculation, but in geometry logical thinking and deductive reasoning were in agenda. In the 1960s

the educational system was reformed to comprise five years of elementary, three years of middle and four years of high school. The worldwide Modern Math approach in the 1960s also impacted the math curriculum. Causing subjects such as set theory and logic to appear in textbooks.

The Iranian Mathematical Society was also established in the late 1960s and gathered mathematicians as well as math teachers in annual conferences to discuss future advancements in mathematics.

## 5. 21<sup>st</sup> Century: Post Modern Era

In this 21<sup>st</sup> century the youth majority of the Iranian population is eager for progress and looking for empowerment in the knowledge-based economy. More than 5% of Iran's population of 80 million are college students, and almost half of them are girls. The number of K12 students reached 20 million students in the early 2000, but it has comedown in 2015 to 12 million students, almost 15% of the population. Mathematical literacy is at the heart of the educational system, both in K12 and college level. Some highlights about the educational system and math education follow.

- The Ministry of Science, Research and Technology is responsible for higher education and research as well as technological development. The Ministry of Education is responsible for K12 education.
- There are over 100 public universities and 300 private colleges in Iran. Also, Azad private university and vocational colleges are established in most of major cities. Four and half million students lie at the college level. There are almost 100,000 math major students in undergrad and graduate levels.
- Teachers University and most major universities offering a math teaching degree in the undergrad level. Math education programs in the MSc and PhD levels is also offering.
- PhD programs in mathematical sciences and math education since late 1980s has institutionalized research in math and math education. Researches has been supported widely by the public research institutes. More than 5,000 research papers have been published in the period of 2008 to 2014 in the fields of mathematical sciences.
- The Institute for Research in Fundamental Sciences (previously the Institute for Studies in Theoretical Physics and Mathematics, often shortened to IPM) established in the late 1980s and is a major sponsor of research in mathematical sciences nationwide.
- The Iranian Mathematical Society, as a professional association is supporting math and math education through conferences, workshops and seminars also publications. The annual Math Conference over the last five decades and biyearly Math Education Conference are the most important conferences. The *Bulletin* of the Iranian Mathematical Society, the most important research journal, *Farahang-o-Andisheh Riaz* the expository journal and a monthly newsletter are the most important publications of the society. Iranian Mathematical Society also organizes a yearly student conference and math competition among the undergraduate math students, each university can participate with a team of

five students in the competition and top teams are supported to participate in the International University Students Math Contest.

- The Iranian Statistical Society, also established in 1990. The society is organizing conferences and publishing journals for scientists and professionals.
- The Iranian Math Teachers Association is a nationwide professional association with most math teachers as members in the local chapters. Local chapters organizing seminars and workshops and publishing journals. Association also is co-sponsor of the biyearly math education conference which is the largest gathering of math teachers in Iran.
- There are more than one million teachers in more than 100,000 schools nationwide, of which more than 90% are public schools.
- The national curriculum is in the form of 6 years of elementary and 6 years of high school. Math for all students constitute 3 to 5 hours of each school week up to grade 9. Specific branches are taught in grades 10 to 12: math-physics, natural sciences, humanities and vocational training. 10 to 12 hours of math courses are taught per week for math-physics and 3 to 5 hours for others.
- Number systems, arithmetic and basic geometry are covered in the elementary level, while algebra, pre-calculus, calculus, combinatorics, probability and geometry are covered in high school.
- Standard textbooks at the national level are provided and published by the ministry of education at over 120 million copies per year. The private publishers publish enrichment books.
- Online and digital learning systems are currently progressing rapidly; *maktabkhoneh.org* and *kelasedars.org* are some of the most popular for math courses.
- The National Organization for Development of Exceptional Talents (NODET) was established in 1976 to support gifted students in special schools in the middle school and high school levels nationwide. NODET is supporting math education through advanced enrichment courses.
- The Iranian Mathematical Olympiad is a math contest for high school students organized since 1985 by the Young Scholars Club under the support of the Ministry of Education. Iran's team participated in the International Math Olympiad for the first time in 1987 in Cuba and received just one Bronze medal, but math contest at the high school level quickly became very popular among high school students. Nowadays, with almost 100,000 applicants annually, through three rounds of contests the national team of six students is selected to participate in the International Math Olympiad. In the last three decades, they usually end up in top 10 and ranked first in 1998 with five gold medals and one silver medal. The Math Olympiad as an extra curricular activity has attracted many talented students to mathematical studies which empowered them beyond the standard curriculum.
- Math journals have also had a deep influence in the popularization of mathematics and extra curricular math education. The math journal called *Yekan* started publishing in the 1960s and for two decades it was the most popular journal. Recently *Roshde Amoozesh*

*Riazi*, and *Nashri Riazi* and many other local and nationwide math journals have also attracted many interested students and math teachers.

- Math House is a unique community center for extra-curricular activities in mathematics for students. Empowering and attracting youth to mathematical sciences are primary objectives of Math House. Students are attracted to participate in workshops and seminars and join project groups to develop basic and new concepts in group projects to present in Math Fests quarterly. The first math house was established in Isfahan in the central part of the country in year 2000 in the occasion of the World Mathematical Year-2000. Now, more than 30 math houses are established in major cities.



- The mathematics movement in Iran is flourishing in the 21<sup>st</sup> century and Iranian mathematicians are influencing the world by developing mathematical sciences. These achievements are reflected in various ways, such as the most prestigious Fields medal awarded to Maryam Mirzakhani, the first woman to ever earn the medal. Maryam did her middle school and high school studies in Iran and, as a gold medalist in the International Math Olympiad finished her undergrad in mathematics in the Sharif University of Technology in Iran. She later received her PhD from Harvard University and served as a professional mathematician.

## **6. Epilogue**

In the long journey that is Iranian math education, from the ancient period to the post modern era, there are many lessons that can be learned but to look forward for further advancement of math education, we can look at some obstacles and how to overcome them.

There are two main obstacles in the Iranian educational system as well as math education. First the system is centralized and non-flexible. Second, the system is heavily test oriented and focused on memorization, which can hinder students' creativity.

We may consider a vision of a knowledge-based society and may consider the following issues to reform math education and the educational system as a whole:

- Decentralization, flexibility and diversity
- Devolution of decision power and responsibility to the local level
- Commitment to a culture of trust through professionalism

- Equal educational opportunity
- Look for broad knowledge as new skills are needed
- Emphasizing problem solving in math education
- Developing an experimental learning system in math learning
- Pushing a system that courage learners to be active partners and just not passive receivers
- Developing a project based and inquiry based learning system
- Developing a systematic, flexible and individual support system
- Technology should be used as an enabler for interactive and personalized learning

The motto of the educational system in Iran is a best highlighted by Iranian poet Ferdowsi, who says “knowledge is power”. Let us hope this education gives power for peace and friendship all over the world.



Knowledge is power

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