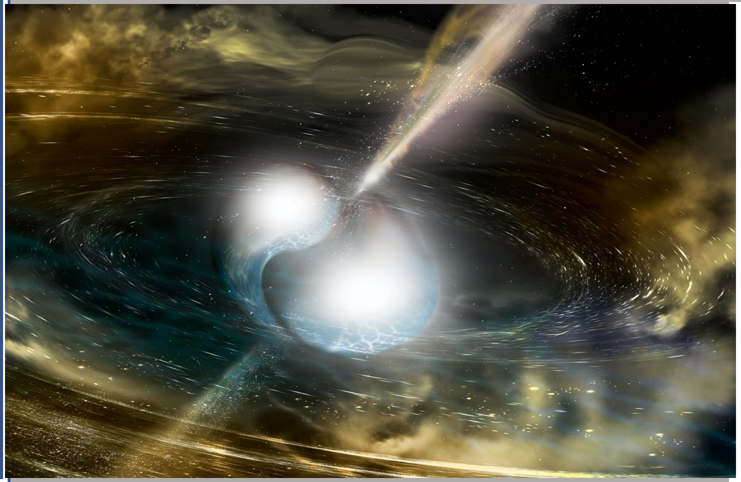
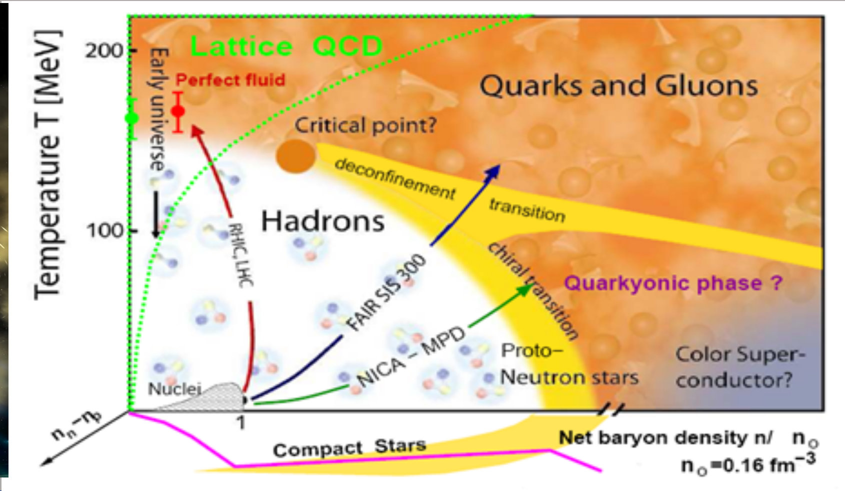
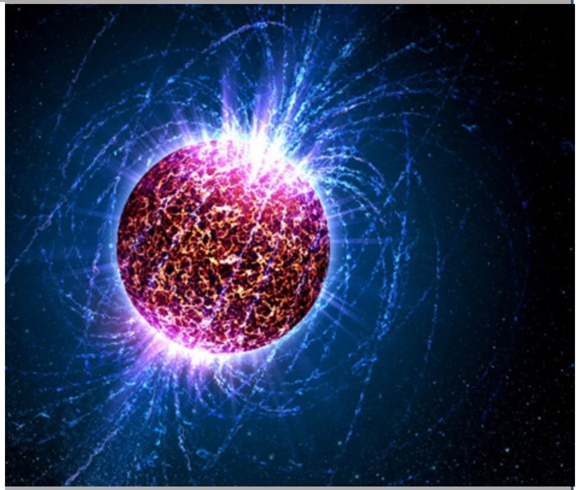
Axion Polariton in Magnetized Dense Quark Matter







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Quark Matter & Relativistic Hydrodynamics
Sharif University of Technology
Nov. 10, 2020

Outline

1. The MDCDW Phase
2. Axion Electrodynamics & Anomalous Quantities
3. Photon-Phonon Interaction & Axion Polariton
4. Primakoff Effect & NS Mass

Neutron Stars

Diameter:

$R \approx 10\ km$

Mass:

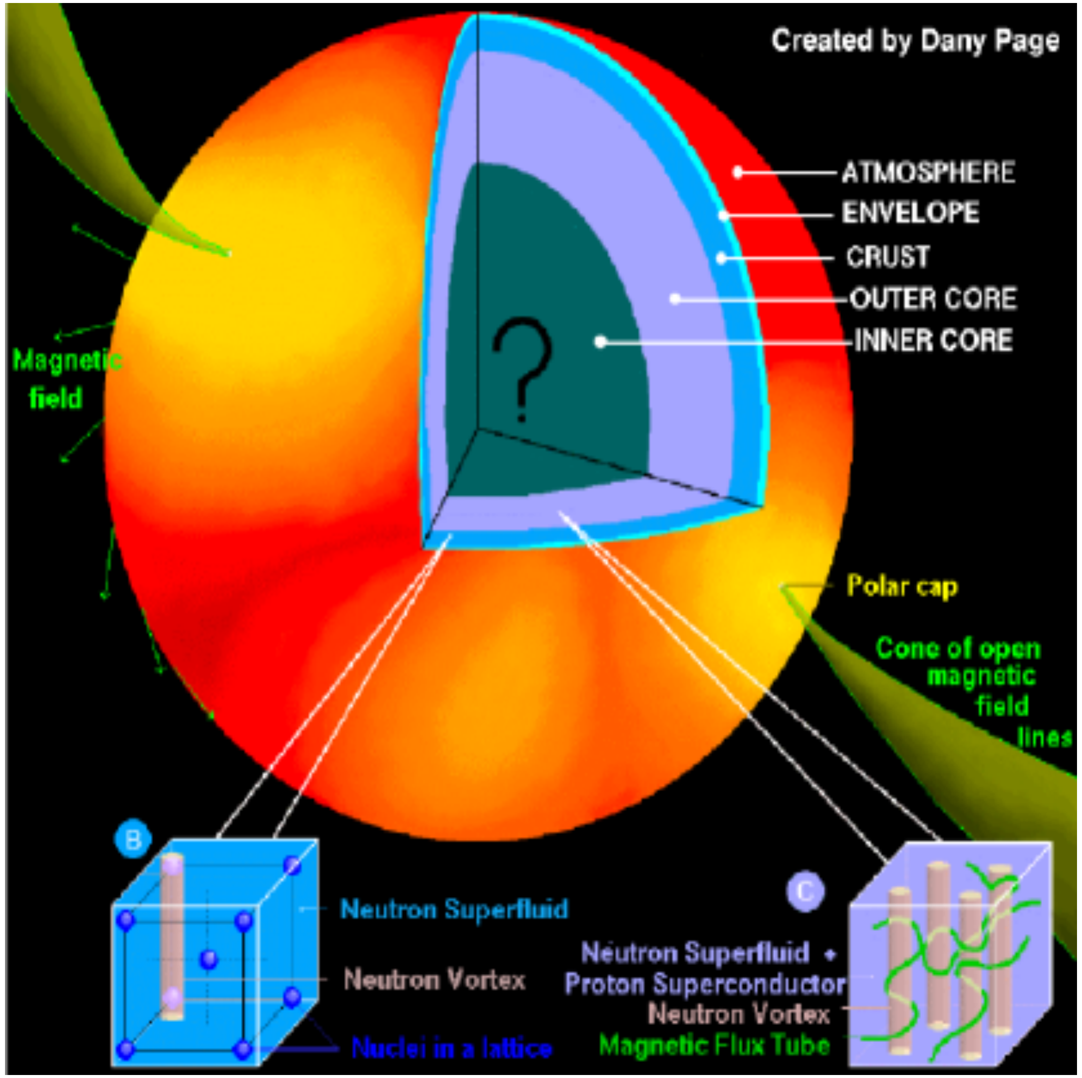
$1.25M_{\odot} \lesssim M \lesssim 2M_{\odot}$

Temperature:

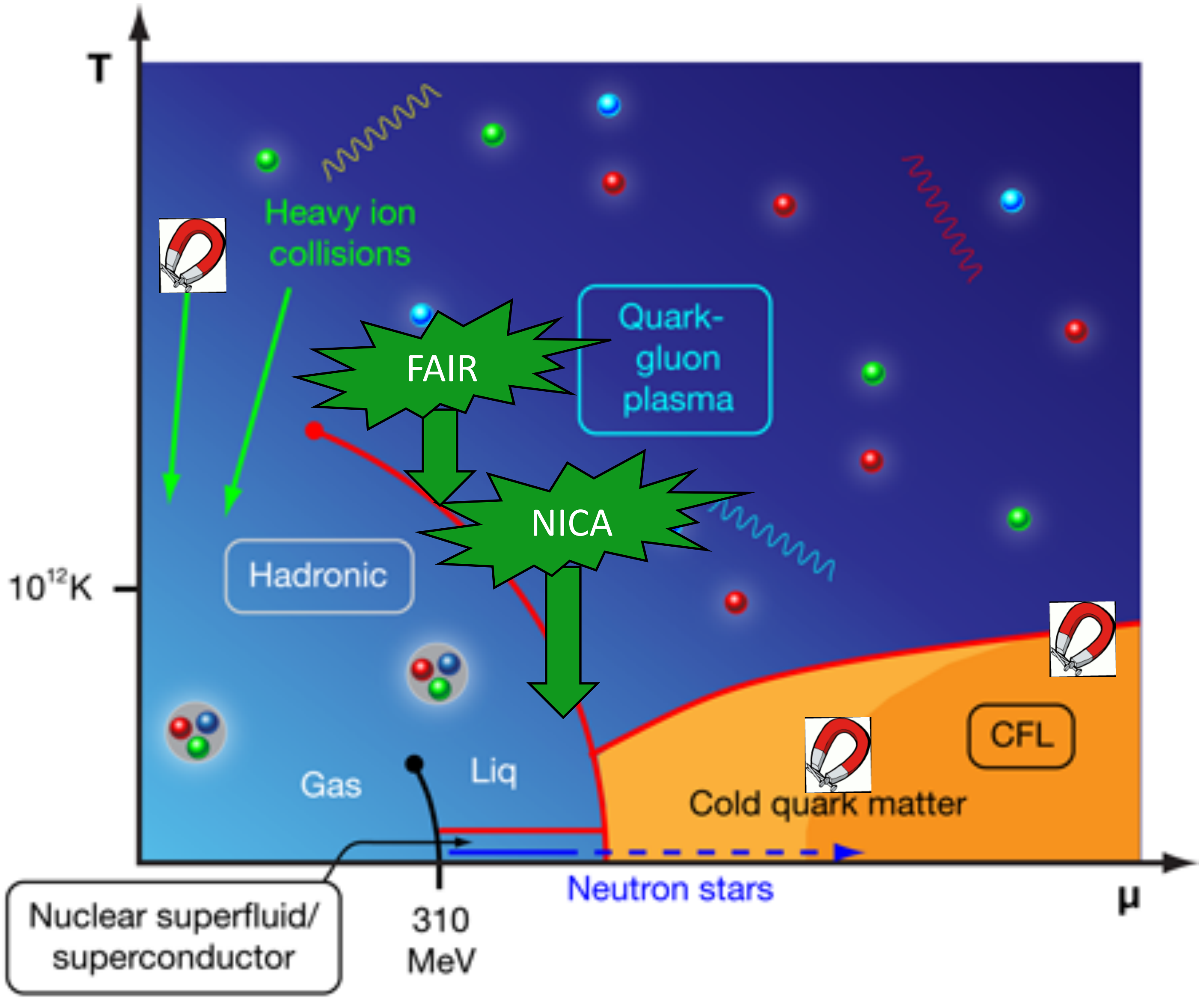
$10\ keV \lesssim T \lesssim 10MeV$

Magnetic fields:

pulsar's surface:
B~ 10^{12}–10^{13}G
magnetar's surface:
B~ 10^{14}–10^{15}G



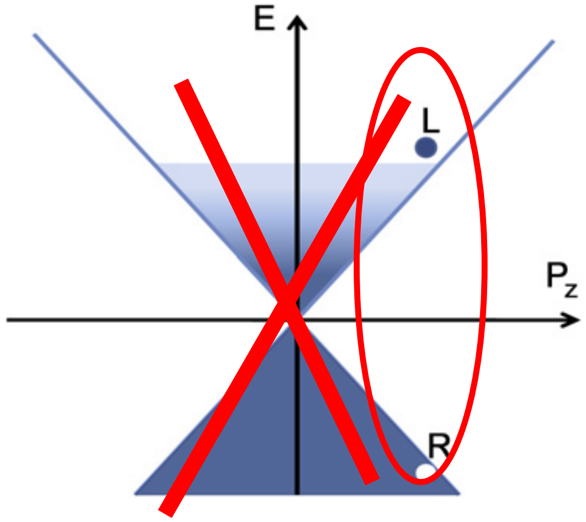
QCD Phase Diagram



Why Inhomogeneous phases at intermediate densities?

Pairing at Intermediate Densities

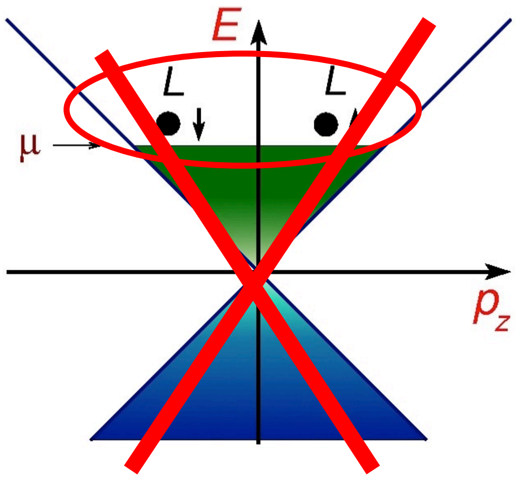
Chiral Condensate



It pairs particle and antiparticle with opposite momentum (homogeneous condensate)

Not favored with increasing density

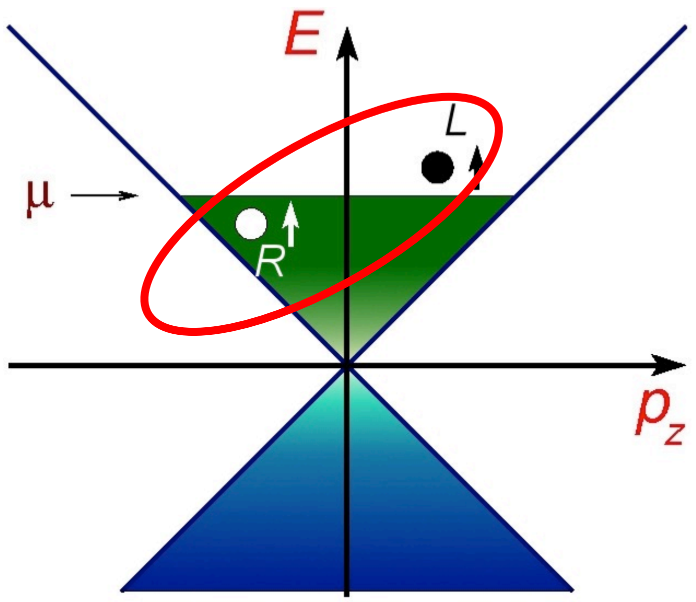
Cooper Pairing



Main channel pairs quarks of different flavors with opposite momenta. Favored at very high densities.

Suffers from Fermi surface mismatch for different flavors leading to chromomagnetic instabilities.

Density Wave Pairing



It pairs particle and hole with opposite spin and parallel momenta (inhomogeneous condensate)

No Fermi surface mismatch

Favored over homogeneous chiral condensate at $B \neq 0$

Favored over CS at large Nc

Basics of the MDCDW Phase

Magnetic Dual Chiral Density Wave Model

2-flavor NJL model + QED at finite baryon density and with magnetic field B$\parallel z$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu) + \gamma_0\mu]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2].$$

It favors the formation of an inhomogeneous chiral condensate

$$\langle\bar{\psi}\psi\rangle = m\cos q_\mu x^\mu, \qquad \langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = m\sin q_\mu x^\mu, \qquad q^\mu = (0,0,0,q)$$

Mean-field Lagrangian

$$\mathcal{L}_{MF} = \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu) + \gamma_0\mu]\psi \boxed{- m\bar{\psi}e^{i\tau_3\gamma_5 q_\mu x^\mu}\psi} - \frac{m^2}{4G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Complex mass term

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

Chiral Transformation & Asymmetric Spectrum

Performing the chiral local transformation

$$\psi \rightarrow U_A\psi = e^{-i\tau_3\gamma_5\frac{qz}{2}}\psi \qquad \bar{\psi} \rightarrow \bar{\psi}\bar{U}_A = \bar{\psi}e^{-i\tau_3\gamma_5\frac{qz}{2}}$$

The MF Lagrangian acquires a constant mass term plus a $\gamma_3\gamma_5$ term

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu 0} + iQA_\mu - i\tau_3\gamma_5\delta_{\mu 3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

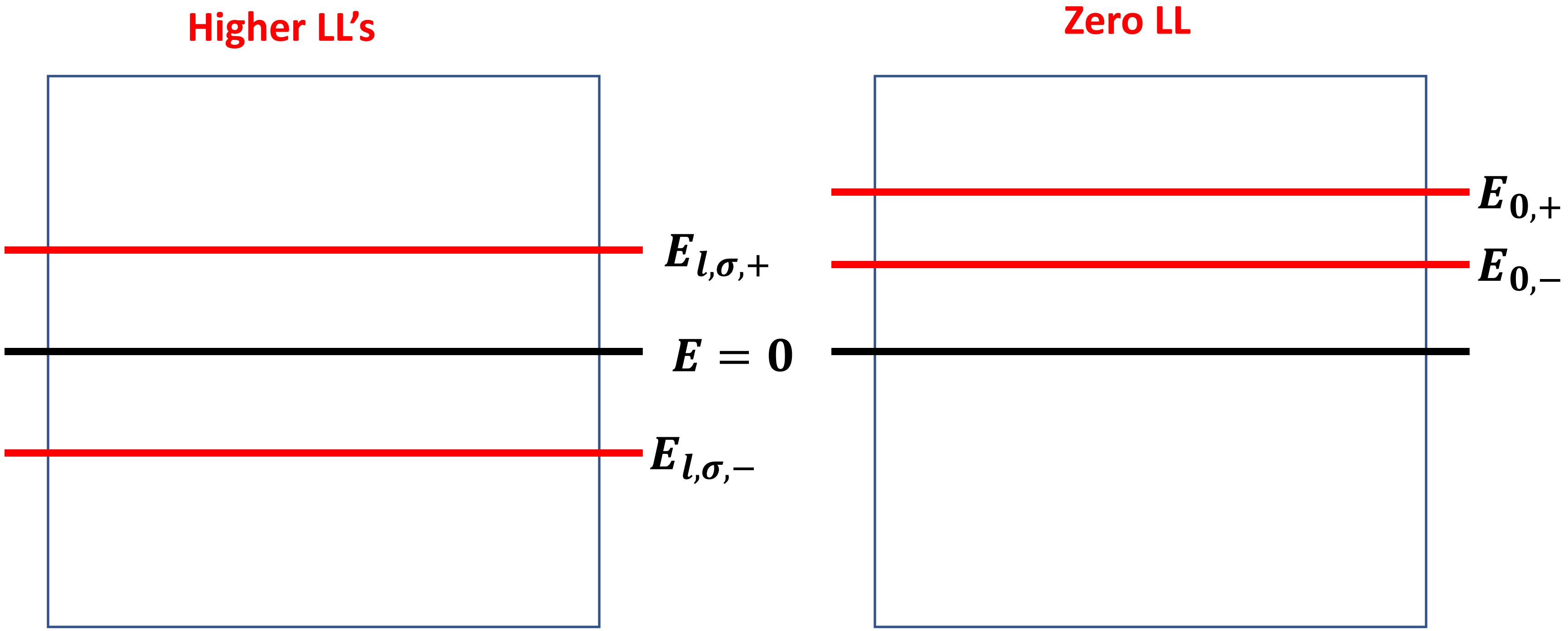
For $A^\mu = (0, 0, Bx, o)$ the corresponding fermion spectrum is

$$\boxed{E_k^{LLL} = \epsilon\sqrt{\Delta^2 + k_3^2} + q/2, \quad \epsilon = \pm}$$

LLL mode is Asymmetric

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1, 2, 3, \ldots$$

Energy Spectrum of the MDCDW Phase



$$E_{l,\sigma,\pm} = \pm\sqrt{E_{0,\sigma}^2 + 2eBl}\,, \quad \sigma = \pm, \quad l = 1, 2, 3, \ldots$$

$$E_{0,\pm} = \pm\sqrt{k_3^2 + m^2} + \mathrm{q}/2$$

A. J. Niemi, Nucl. Phys. B251(1985) 155; A. J. Niemi and G. W.Semenoff, Phys. Reports 135(1986) 99.

Axion Term

EJF & Incera, PLB' 2017; NPB' 2018

Key observation: the fermion measure is not invariant under U_A

$$\boxed{D\bar{\psi}D\psi \rightarrow (\det U_A)^{-2} D\bar{\psi}D\psi} \quad \boxed{(\det U_A)_R^{-2} = e^{i\int d^4x \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}}}$$

The effective MF Lagrangian acquires an axion term:

$$\begin{aligned}\mathcal{L}_{eff} &= \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu - i\tau_3\gamma_5\partial_\mu\theta) + \gamma_0\mu - m]\psi - \frac{m^2}{4G} \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu},}\end{aligned}$$

$\theta = \frac{qz}{2}$

$\kappa = \frac{e^2}{2\pi^2}$

Integrating out the fermions, we find the electromagnetic effective action in the MDCDW model

$$\begin{aligned}\Gamma(A) = \ & V\Omega + \int d^4x \left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}\right] \\ & - \int d^4x A^\mu(x) J_\mu(x) + \cdots,\end{aligned}$$

Nontrivial Topology of the MDCDW Phase

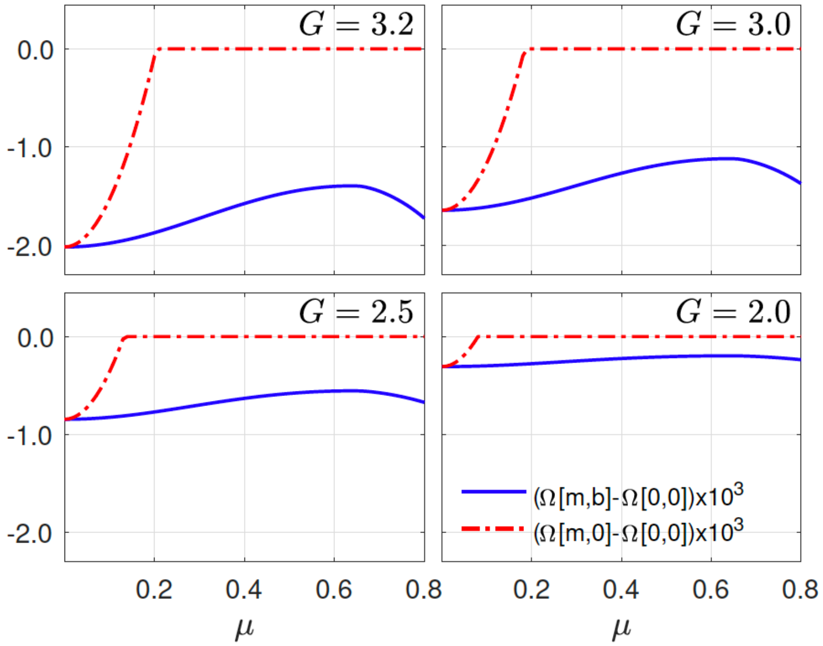
Topology emerges due to the LLL spectral asymmetry & to the axion term.

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_T(B,\mu,T) + \frac{m^2}{4G}.$$

$$\boxed{\Omega^f_{anom} = -\frac{N_c|e_f B|}{(2\pi)^2} q\mu}$$

$$\rho^A_B = 3\frac{|e|}{4\pi^2} qB$$

Anomalous baryon number density

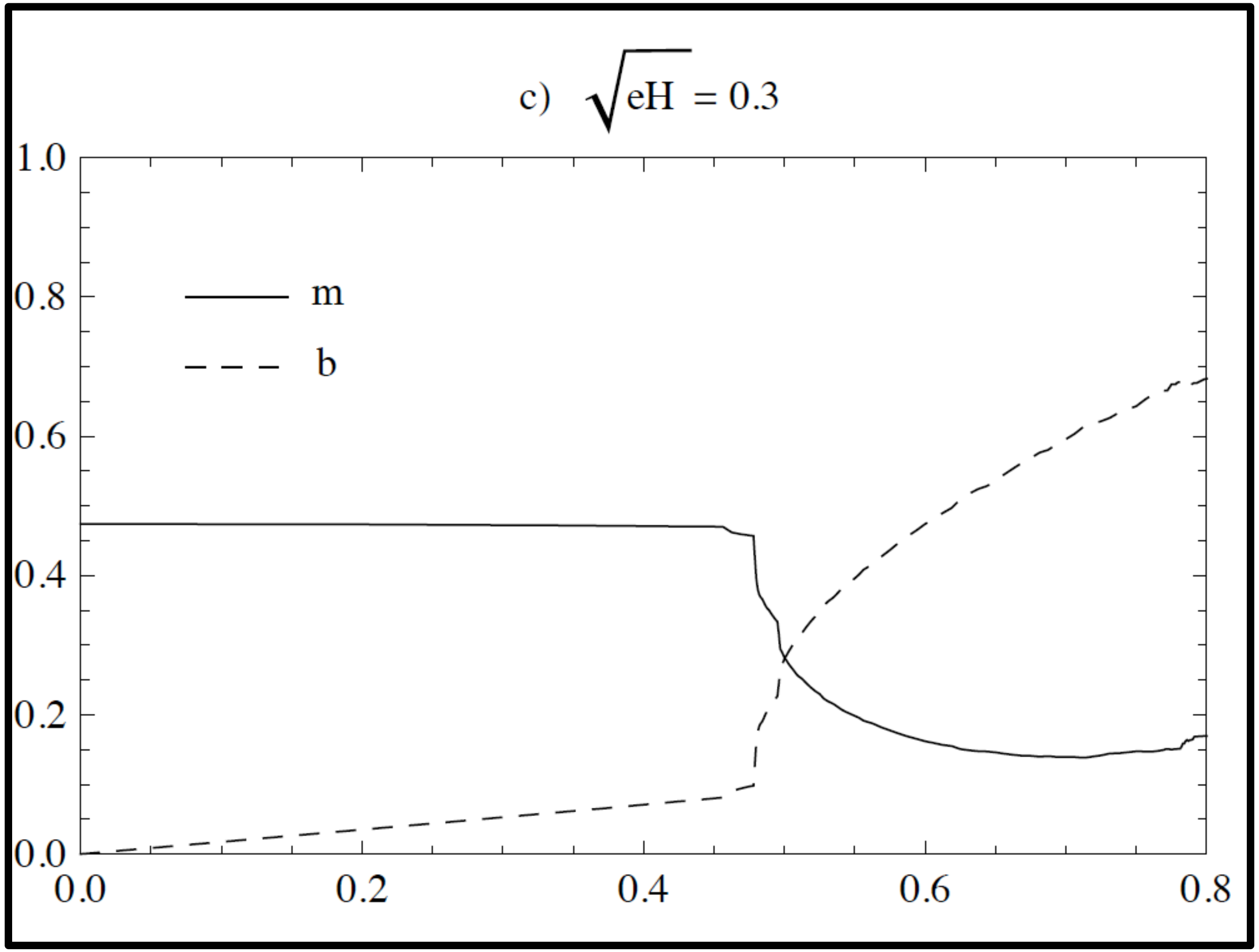


The anomaly makes the MDCDW solution energetically favored over the homogeneous condensate

Feng/EJF/Portillo *PRD* 101 (2020) 056012

Adding a Magnetic Field: MDCDW Phase

I. E. Frolov et al, PRD 82 (2010) 076002



QED in MDCDW is Axion QED

$$\nabla \cdot \mathbf{E} = J^0 + \frac{e^2}{4\pi^2} qB,$$
$$\nabla \times \mathbf{B} - \partial \mathbf{E}/\partial t = \mathbf{J} - \frac{e^2}{4\pi^2} \mathbf{q} \times \mathbf{E},$$
$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \partial \mathbf{B}/\partial t = 0,$$

Anomalous charge

Dissipationless Hall current
⊥ to both B and E

Anomalous Hall conductivity

$$\boxed{\sigma_{xy}^{anom} = e^2 q / 4\pi^2}$$

EJF & Incera, Phys.Lett. B769 (2017) 208; Nucl.Phys. B931 (2018) 192

Magnetoelectricity

$$\nabla \cdot \mathrm{D} = J_0 \quad \nabla \times \mathrm{H} - \frac{\partial \mathrm{D}}{\partial t} = \mathrm{J}_V$$

$$\nabla \cdot \mathrm{B} = 0 \quad \nabla \times \mathrm{E} + \frac{\partial \mathrm{B}}{\partial t} = 0$$

$$\mathrm{H} = \mathrm{B} - \kappa\theta\mathrm{E} \qquad \mathrm{D} = \mathrm{E} + \kappa\theta\mathrm{B}$$

Anomalous magnetization

Anomalous polarization

MDCDW Symmetry-Breaking Pattern

Explicit Symmetry Breaking by the Magnetic field

$$SU_V(2)\times SU_A(2)\times SO(3)\times R^3 \to U_V(1)\times U_A(1)\times SO(2)\times R^3$$

MDCDW Single-Modulated Density Wave Ansatz

$$M(z) = me^{iqz}$$

Spontaneous Symmetry Breaking by the Inhomogeneous Condensate

$$U_V(1)\times U_A(1)\times SO(2)\times R^3 \to U_V(1)\times SO(2)\times R^2$$

GL Expansion

EJF & Incera, PRD`2020

Low Energy GL Expansion of the MDCDW Free Energy

$$\mathcal{F} = a_{2,0}|M|^2 - i\frac{b_{3,1}}{2}[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] + a_{4,0}|M|^4 + a_{4,2}^{(0)}|\nabla M|^2$$
$$+ a_{4,2}^{(1)}(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) - i\frac{b_{5,1}}{2}|M|^2[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M]$$
$$+ \frac{ib_{5,3}}{2}[(\nabla^2 M^*)\hat{B}\cdot\nabla M - \hat{B}\cdot\nabla M^*(\nabla^2 M)] + a_{6,0}|M|^6 + a_{6,2}^{(0)}|M|^2|\nabla M|^2$$
$$+ a_{6,2}^{(1)}|M|^2(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) + a_{6,4}|\nabla^2 M|^2 + \cdots.$$

MDCDW ansatz

$$M(z) = me^{iqz} \Rightarrow$$

$$\boxed{\begin{aligned}\mathcal{F} = {} & a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4 \\ & + b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2,\end{aligned}}$$

The b coefficients are a consequence of the asymmetry of the LLL spectrum

The $a_{x,y}^{(1)}$ coefficients are a consequence of having an external vector

GL Expansion

Ferrer & VI, PRD'2020

Low Energy GL Expansion of the MDCDW Free Energy

$$\mathcal{F} = a_{2,0}|M|^2 - i\boxed{\frac{b_{3,1}}{2}[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M]} + a_{4,0}|M|^4 + a_{4,2}^{(0)}|\nabla M|^2$$
$$+ a_{4,2}^{(1)}(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) - i\boxed{\frac{b_{5,1}}{2}|M|^2[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M]}$$
$$\boxed{+\frac{ib_{5,3}}{2}[(\nabla^2 M^*)\hat{B}\cdot\nabla M - \hat{B}\cdot\nabla M^*(\nabla^2 M)]} + a_{6,0}|M|^6 + a_{6,2}^{(0)}|M|^2|\nabla M|^2$$
$$+ a_{6,2}^{(1)}|M|^2(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) + a_{6,4}|\nabla^2 M|^2 + \cdots.$$

MDCDW ansatz

$$M(z) = me^{iqz}$$

$$\boxed{\begin{aligned}\mathcal{F} = a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4 \\ + b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2,\end{aligned}}$$

The b coefficients are a consequence of the asymmetry of the LLL spectrum

The $a_{x,y}^{(1)}$ coefficients are a consequence of having an external vector

Spontaneous Breaking of Chiral & Translational Symmetries

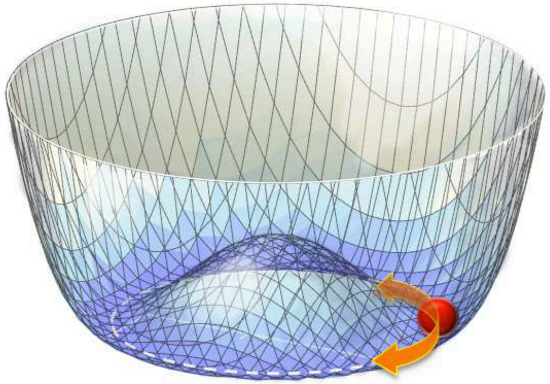
$\bar{M}(z) = m e^{iqz}$ with m and q solutions of the stationary equations:

$$\partial \mathcal{F}/\partial m = 2m\{a_{2,0} + 2a_{4,0}m^2 + 3a_{6,0}m^4 + q^2[a_{4,2} + 2a_{6,2}m^2 + a_{6,4}q^2] + q[b_{3,1} + 2b_{5,1}m^2 + b_{5,3}q^2]\} = 0,$$

$$\partial \mathcal{F}/\partial q = m^2\{2q[a_{4,2} + a_{6,2}m^2 + 2a_{6,4}q^2] + b_{3,1} + b_{5,1}m^2 + 3b_{5,3}q^2\} = 0.$$

$$\boxed{a_{4,2} = a_{4,2}^{(0)} + a_{4,2}^{(1)},\ a_{6,2} = a_{6,2}^{(0)} + a_{6,2}^{(1)}}$$

Symmetry is reduced to $U_V(1) \times SO(2) \times R^2$



Fluctuations of the condensate come from two Goldstone Bosons: A pion and a phonon.

Phonon Low Energy Theory

Chiral and translation transformations are locked

$$M(z) \to e^{i\tau} M\big(z + u(x)\big) = e^{i(\tau + qu(x))} M(z)$$

Phonon Fluctuation Field $u(x)$

$$M(x) = M\big(z + u(x)\big) \approx M_0(z) + M_0'(z)u(x) + \frac{1}{2} M_0''(x)u^2(x)$$

Low-Energy Theory:

$$\mathcal{L}_1 = \frac{1}{2}[(\partial_0\theta)^2 - v_z^2(\partial_z\theta)^2 - v_\perp^2(\partial_\perp\theta)^2],$$

$$\theta = qmu(x)$$

$$v_z^2 = a_{4,2} + m^2 a_{6,2} + 6q^2 a_{6,4} + 3qb_{5,3}, \; v_\perp^2 = a_{4,2} + m^2 a_{6,2} + 2q^2 a_{6,4} + qb_{5,3} - a_{4,2}^{(1)} - m^2 a_{6,2}^{(1)}.$$

The low-energy theory is described by a generalized GL expansion of the thermodynamic potential in powers of the order parameter and its derivatives.

Photon-Phonon Axion Electrodynamic at $B \neq 0$

Taking now into account the contribution of the anomalous photon-phonon interaction $\frac{\kappa}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$, the axion-electrodynamic/phonon equations are:

$$\nabla \cdot \mathbf{E} = J^0 + \frac{\kappa}{2}\nabla\theta_0 \cdot \mathbf{B} + \frac{\kappa}{2}\nabla\theta \cdot \mathbf{B},$$

$$\nabla \times \mathbf{B} - \partial \mathbf{E}/\partial t = \mathbf{J} - \frac{\kappa}{2}\left(\frac{\partial\theta}{\partial t}\mathbf{B} + \nabla\theta \times \mathbf{E}\right),$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \partial \mathbf{B}/\partial t = 0$$

$$\partial_0^2\theta - v_z^2\partial_z^2\theta - v_\perp^2\partial_\perp^2\theta + \frac{\kappa}{2}\mathbf{B}\cdot\mathbf{E} = 0$$

Here we assume that a linearly polarized electromagnetic wave with electric field parallel to the background magnetic field B_0 propagates in the MDCDW medium

Linearized Field Equations

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \nabla^2 \mathbf{E} + \frac{\kappa}{2}\frac{\partial^2 \theta}{\partial t^2}\mathbf{B}_0$$

$$\frac{\partial^2 \theta}{\partial t^2} - v_z^2 \frac{\partial^2 \theta}{\partial z^2} - v_\perp^2 \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + \frac{\kappa}{2}\mathbf{B}_0 \cdot \mathbf{E} = 0.$$

For applications to Neutron Stars, we consider that the medium is neutral. So, we assume that there exist a background of electrons with electric charge J_0 that ensures overall neutrality. Moreover, in the presence of a static and uniform background magnetic field, the coupling between the phonon and the photon is linear.

Eigenmodes Energy Spectrum

The dispersion relations of the hybrid modes are

$$\omega_\gamma^2 = A - B, \quad \omega_{AP}^2 = A + B$$

with

$$A = \frac{1}{2}[p^2 + q^2 + (\frac{\kappa}{2}B_0)^2],$$

$$B = \frac{1}{2}\sqrt{[p^2 + q^2 + (\frac{\kappa}{2}B_0)^2]^2 - 4p^2q^2},$$

$$q^2 = v_z^2 p_z^2 + v_\perp^2 p_\perp^2$$

Axion-Polariton Mass

$$m_{AP} = \alpha B_0 / \pi m$$

Stability against the Fluctuations

EJF & Incera, PRD'2020

$$\langle M \rangle = m e^{iqz} \langle \cos qu \rangle \qquad \langle \cos qu \rangle = e^{-\langle (qu)^2 \rangle / 2}$$

$$\boxed{\begin{aligned}\langle q^2 u^2 \rangle &= \frac{1}{(2\pi)^2} \int_0^\infty dk_\perp k_\perp \\ &\times \int_{-\infty}^{\infty} dk_z \frac{T}{m^2 (v_z^2 k_z^2 + v_\perp^2 k_\perp^2 + \zeta^2 k^4)} \\ &\simeq \frac{\pi T}{m \sqrt{v_z^2 v_\perp^2}},\end{aligned}}$$

Finite! Thanks to B there are no soft transverse modes, hence no Landau-Peierls instability. The MDCDW phase is stable against thermal fluctuations.

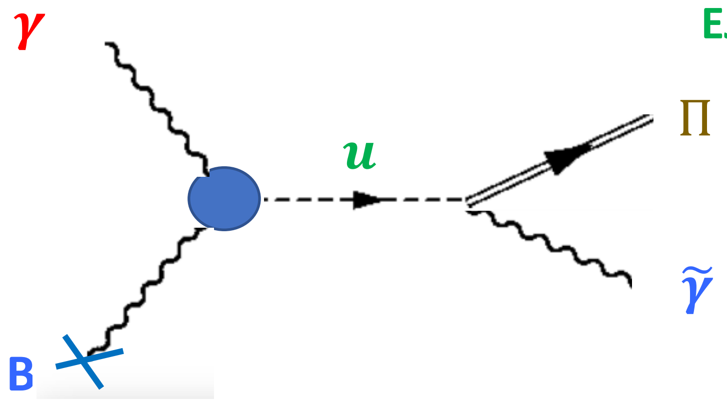
In contrast at B=0

$$\langle u^2 q^2 \rangle \simeq \frac{T}{4\pi v_z \zeta} \ln\left(\frac{v_z l_\perp}{\zeta} \right)$$

Infrared divergent. Any finite T no matter how small destroy the long-range order

Primakoff Effect as a Mechanism to Increase the Star Mass

H. Primakoff, Phys. Rev. 81 (1951) 899;
EJF & Incera, arXiv: 2010.02314 [hep-ph]



$$\Pi = \cos\theta \;\; u + \sin\theta \;\; A_3$$

$$\tilde{A}_3 = -\sin\theta\, u + \cos\theta\, A_3$$

$$\cos\theta = \frac{1}{\sqrt{2}}\left[1 + \frac{\sqrt{(X_1)^2 - (X_2)^2}}{X_1}\right]^{1/2} \qquad \sin\theta = \frac{1}{\sqrt{2}}\left[1 - \frac{\sqrt{(X_1)^2 - (X_2)^2}}{X_1}\right]^{1/2}$$

$$X_1 = (v_z^2 - 1)p_z^2 + (v_\perp^2 - 1)p_\perp^2 \qquad X_2 = 2\kappa B_0 p_4$$

Missing Pulsar Problem in Galactic Center & Axion Polaritons

EJF & Incera, arXiv: 2010.02314 [hep-ph]

Chandrasekhar limit

$$N^{Ch}_{AP} = \left(\frac{M_{pl}}{m_{AP}}\right)^2 = 1.5 \times 10^{46} \left(\frac{10MeV}{m_{AP}}\right)^3$$

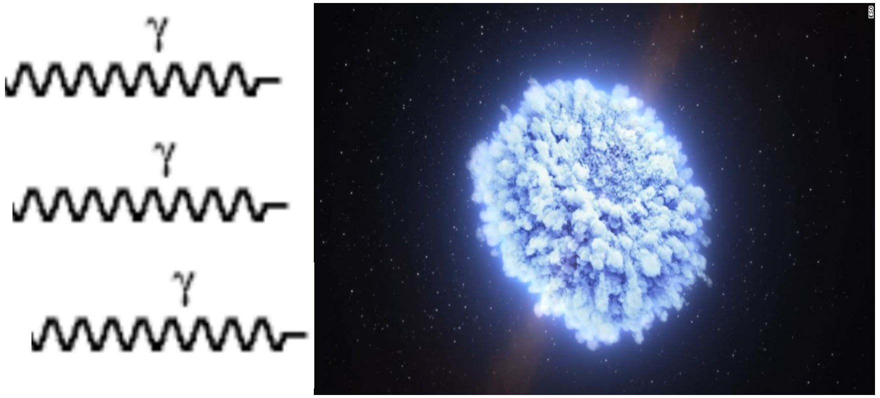
$$m_{AP} = \alpha B_0/\pi m = 0.8\, MeV, \quad for\ \mu = 350\, MeV, \quad B_0 = 5{\times}10^{18}\, G, \quad m = 89\, MeV$$

We find that $\dot{N}^{Ch}_{AP} = 2.9 \times 10^{49}$

Each GRB energy output: $\sim 10^{56}$ MeV

Photons' energy range: 0.1 – 1 MeV

Photons produced in each event: $N_\gamma \geq 10^{56}$



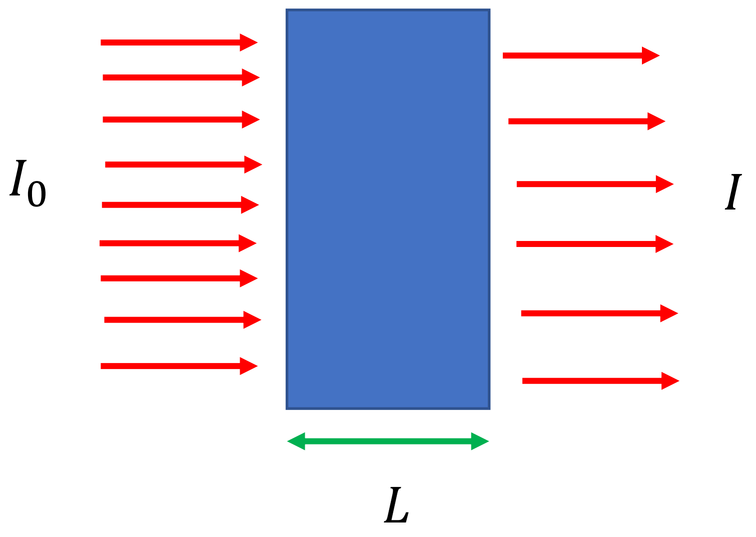
This means that if just 10^{-4} % of the photons reaching the core has energy ≥ 0.8 MeV, they will generate enough number of axion polaritons to produce the NS collapse into a black hole.

γ-ray Attenuation

- Photoelectric Effect
- Compton Effect
    - Thomson Scattering Approximation ($E_\gamma \leq 2m_e$)
- Pair Creation

$$I = I_0 e^{-\sigma n L}$$

σ *Crosss Section*
n *Number Particle Density*
L *Medium Thickness*



Thomson Scattering Cross Section:

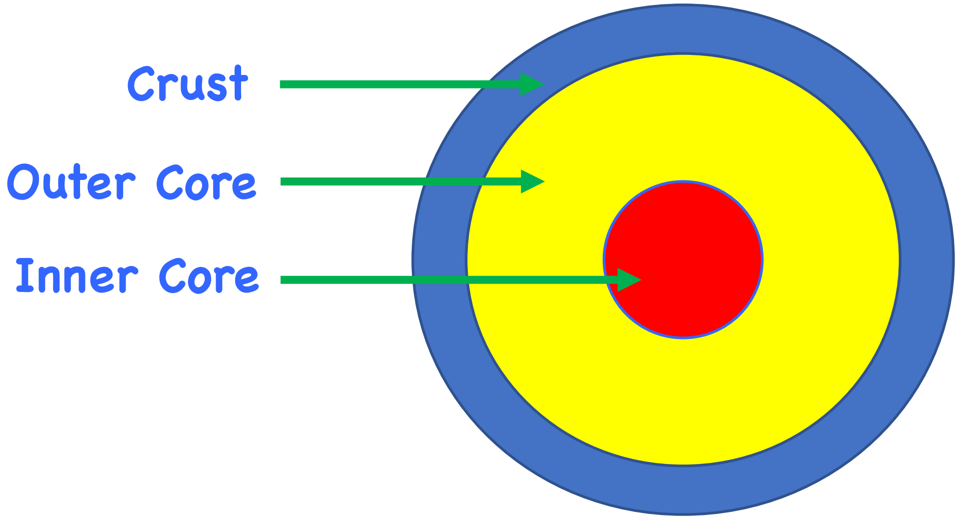
$$\sigma_T = 6.653{\times}10^{-25}\ cm^2$$

Hybrid Star

Crust: Formed by nuclei, mainly Fe and relativistic degenerate electrons, $\rho \sim 10^6 - 10^{11}\ g/cm^3$

Outer Core: Formed by n, p, e, mesons. Density $0.5\,\rho_S\ \leq\ \rho \leq 2\rho_S$, with ρ_S the saturation density $\rho_S = 2.8\times 10^{14} g/cm^3$. Electron to neutron fraction 1:8.

Inner Core: Possibly formed by quarks in the MDCDW phase.



Attenuation of γ-rays in the outer core:

$$L = -(\sigma_T n_e)^{-1} ln\left(\frac{I}{I_0}\right) = -\left(\sigma_T\left[\frac{m_n^{-1}2\rho_S}{8}\right]\right)^{-1} ln\left(\frac{I}{I_0}\right) = -0.15 ln\left(\frac{I}{I_0}\right)\ \mathrm{fm}$$

L	I/I_0
6.45 fm	10^{-20}
13.5 fm	10^{-40}

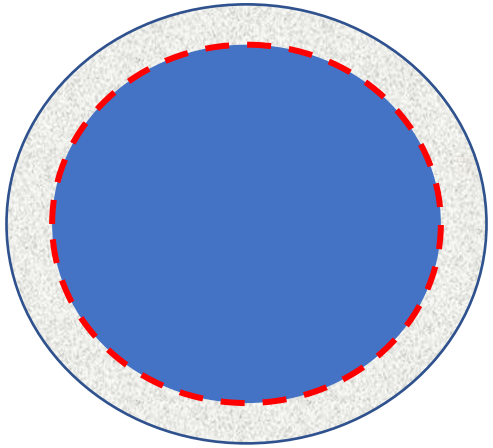
Strange Star

Alcock/Farhi/Olinto, The Astrop. Jour. 310 (1986) 261; Xu/Zhang/Qiao, Astropart. Phys. 12 (2001) 101.

- Strange matter is absolutely stable at all pressures. More stable that Fe (Witten 1984). Formed by roughly equal number of u, d and s quarks plus a smaller number of electrons.
- The number density of electrons beyond the quark surface, which extends to a thickness of several hundred fermis is

$$n_e = \frac{9.49\times10^{35}}{(1.2[z/(10^{-11}cm)] + 4)^3} cm^{-3}$$

z is the heigh above the quark surface.



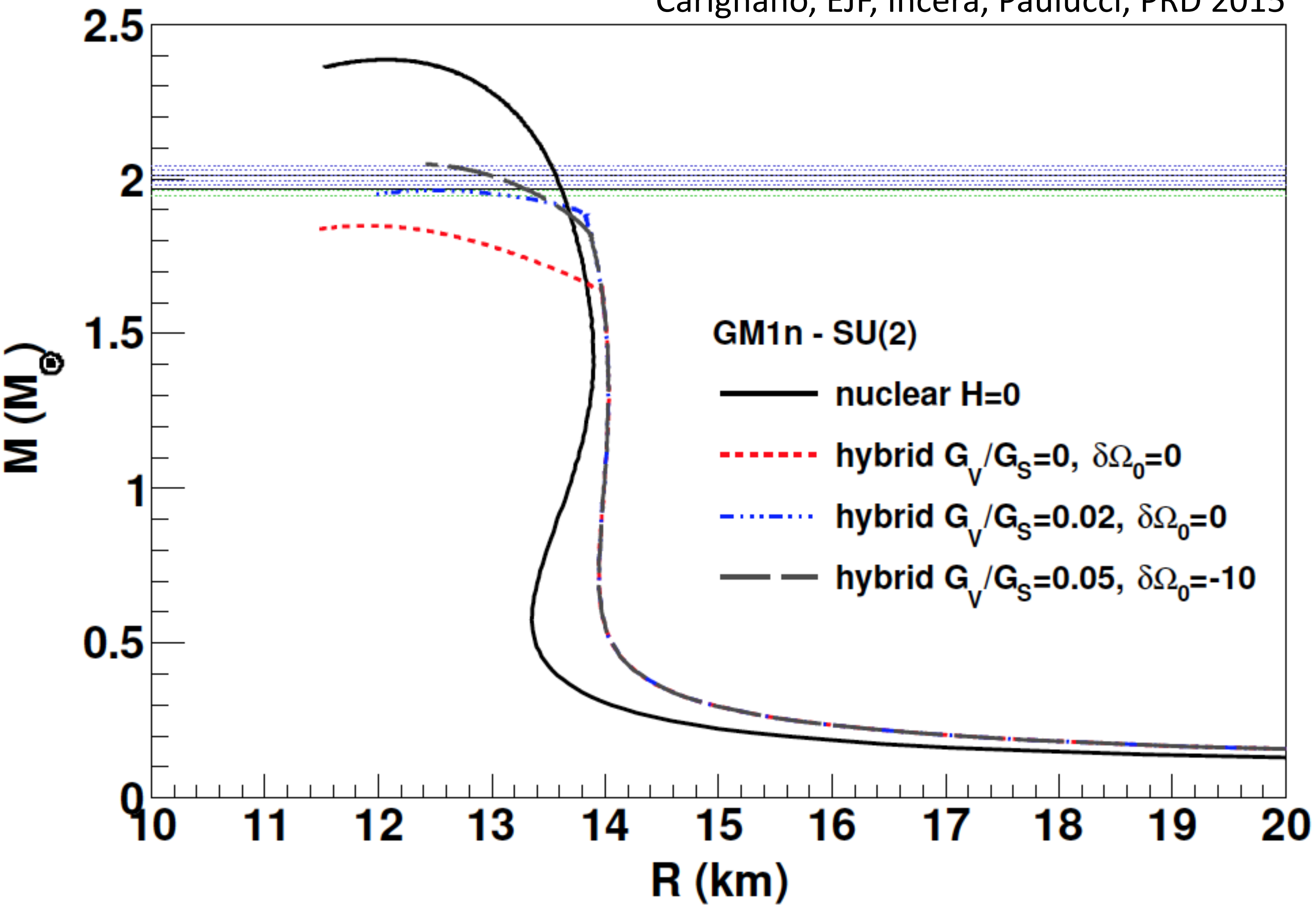
Calculating the attenuation for $z = 100\ fm$ and $L = 100\ fm$

$$ln(I/I_0) = -\sigma_T n_e L = -(6.65\times10^{-25}cm^{-2})(0.07\times10^{35}cm^{-3})(10^{-11}cm)$$

$$I = 0.95 I_0$$

Mass-Radius Relationship $B_c \approx 10^{18}$G

Carignano, EJF, Incera, Paulucci, PRD 2015



Summary:

- Due to an anomalous effect, the electromagnetism is modified in the MDCDW phase of quark matter at intermediate densities giving rise to a rotated photon and an axion polariton
- The anomalous two-photon/phonon interaction present in the MDCDW phase, produces a new mechanism to increase the strange star mass.

Outlook:

- Needed more measurable NS observables that can discriminate between intermediate density candidates: MDCDW, Quarkyonic, CS Phases.