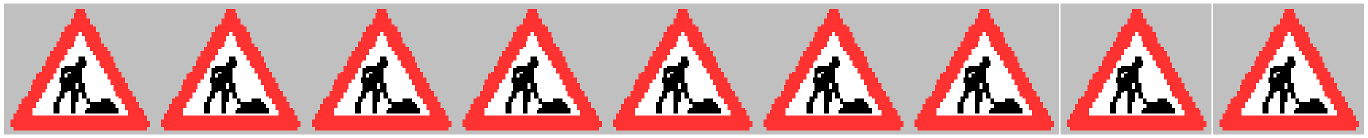
A non-perturbative definition of fluctuating hydrodynamics, based on Zubarev hydrodynamics and Crooks theorem

G.Torrieri

Ongoing work with Francesco Becattini



This is an unpublished stuff me and Francesco are still discussing.

Dont take any of my answers too seriously, for they could be wrong.

But think about the issues I am rasing, for they are important!

- The necessity to redefine hydro
- A possible answer: Zubarev and Crooks!
- Discussion

Some experimental data warmup

(2004) Matter in heavy ion collisions seems to behave as a perfect fluid, characterized by a very rapid thermalization

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

April 18, 2005

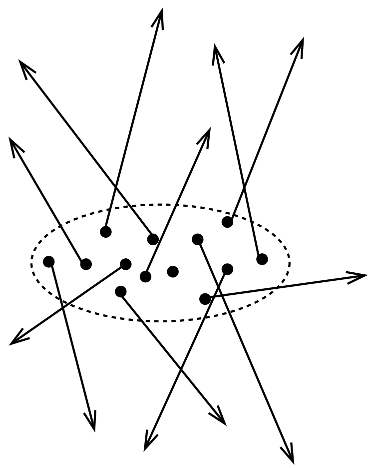
TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

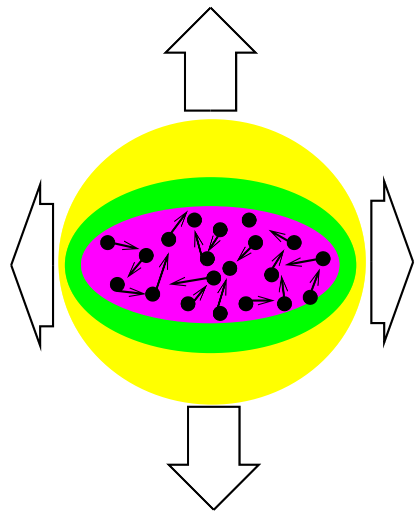
"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

The technical details

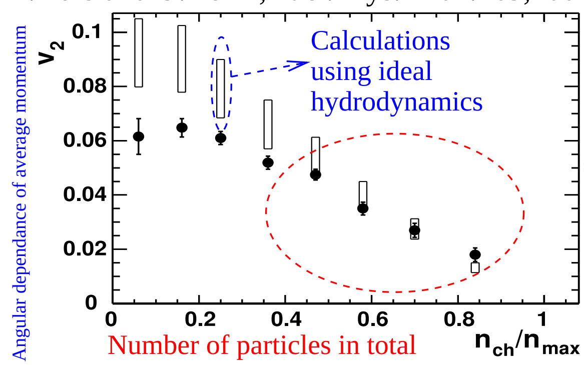
A "dust"
Particles ignore each other, their path is independent of initial shape



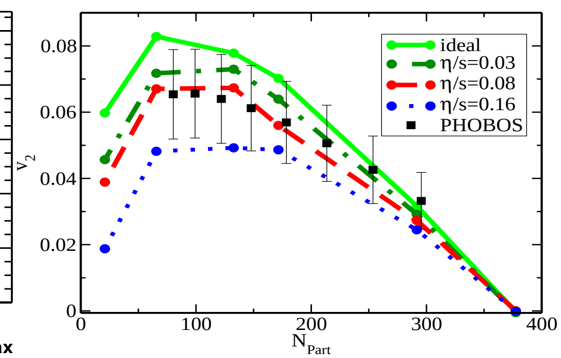


A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape)

P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.



P.Romatschke,PRL99:172301,2007



The conventional widsom

Hydrodynamics is an "effective theory", built around coarse-graining and "fast thermalization". Fast w.r.t. Gradients of coarse-grained variables
If thermalization instantaneus, then isotropy,EoS enough to close evolution

$$T_{\mu\nu} = (e + P(e))u_\mu u_\nu + P(e) g_{\mu\nu}$$

In rest-frame at rest w.r.t. u^μ

$$T_{\mu\nu} = \mathrm{Diag}\,(e(p), p, p, p)$$

(NB: For simplicity we assume no conserved charges, $\mu_B = 0$)

If thermalization not instantaneous,

$$T_{\mu\nu} = T_{\mu\nu}^{eq} + \Pi_{\mu\nu} \quad , \quad u_\mu \Pi^{\mu\nu} = 0$$

$$\sum_n \tau_{n\Pi} \partial_\tau^n \Pi_{\mu\nu} = -\Pi_{\mu\nu} + \mathcal{O}\left(\partial u\right) + \mathcal{O}\left((\partial u)^2\right) + ...$$

A series whose "small parameter" (Barring phase transitions/critical points/... all of these these same order):

$$K \sim \frac{l_{micro}}{l_{macro}} \sim \frac{\eta}{sT} \nabla u \sim \frac{\mathrm{Det}\Pi_{\mu\nu}}{\mathrm{Det}T_{\mu\nu}} \sim ...$$

and the transport coefficients calculable from asymptotic correlators of microscopic theory

Navier-Stokes $\sim K$, Israel-Stewart $\sim K^2$ etc.

So hydrodynamics is an EFT in terms of K and correlators

$$\eta = \lim_{k\to 0}\frac{1}{k}\int dx \left\langle \hat{T}_{xy}(x)\hat{T}_{xy}(y)\right\rangle \exp\left[ik(x-y)\right] \quad , \quad \tau_\pi \sim \int e^{ikx}\left\langle TTT\right\rangle , ...$$

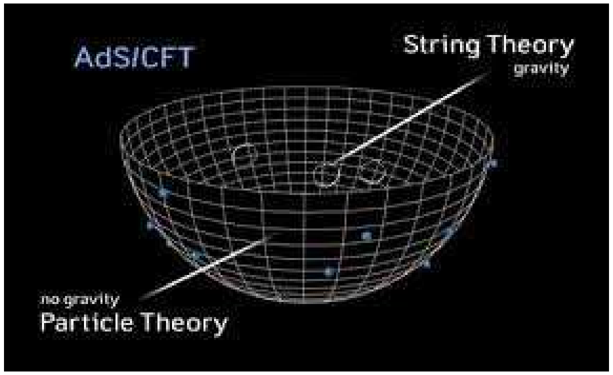
This is a classical theory , $\hat{T}_{\mu\nu} \rightarrow \langle T_{\mu\nu}\rangle$ Higher order correlators $\langle T_{\mu\nu}(x)...T_{\mu\nu}\rangle$ play role in transport coefficients, not in EoM (if you know equation and initial conditions, you know the whole evolution!)

As is the case with 99% of physics we know how to calculate rigorously mostly in perturbative limit. But 2nd law of thermodynamics tells us that A Knudsen number of some sort can be defined in any limit as a thermalization timescale can always be defined Strong coupling $\rightarrow$ lots of interaction $\rightarrow$ "fast" thermalization $\rightarrow$ "low" K

e.g. "Lower limits" on viscosity

Danielewicz and Gyulassy used the uncertainity principle and Boltzmann equation

$$\eta \sim \frac{1}{5} \langle p \rangle n l_{mfp} \quad , \quad l_{mfp} \sim \langle p \rangle^{-1} \rightarrow \frac{\eta}{s} = \frac{1}{15}$$



KSS and extensions from AdS/CFT (actually any Gauge/gravity): Viscosity$\equiv$ Black hole graviton scattering $\rightarrow \frac{\eta}{s} = \frac{1}{4\pi}$

but both theories not realistic

Danielewitz+Gyulassy In strongly coupled system the Boltzmann equation is inappropriate

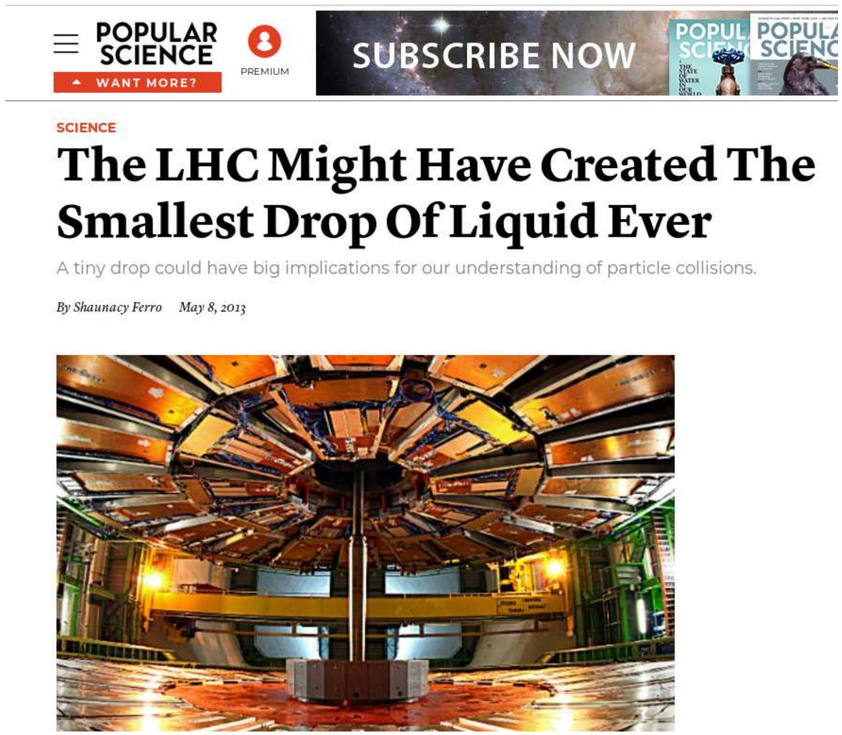
KSS UV-completion is conformal,planar, strong

Is there a general and intuitive way of thinking about these things? e.g. minimal viscosity calculable just from hydrodynamics and, e.g., Lorentz symmetry and Quantum mechanics? after all,

$$\chi(w) = \int dx \left\langle \hat{T}_{xy}(x)\hat{T}_{xy}(y)\right\rangle \exp\left[ik(x-y)\right] \quad , \quad \left\{ \begin{array}{c} c_s^2 \\ \eta \end{array} \right\} \sim \lim_{w\to 0} w^{-1} \left[\begin{array}{c} Re[\chi \\ Im[\chi \end{array} \right.$$

and Kramers-Konig $\left\{ \begin{array}{c} Re \\ Im \end{array} \right\} \chi = \frac{1}{\pi} \int \frac{dw'}{w-w'} \left\{ \begin{array}{c} Im \\ -Re \end{array} \right\} \chi$

2011-2013 FLuid-like behavior has been observed down to very small sizes, $p-p$ collisions of 50 particles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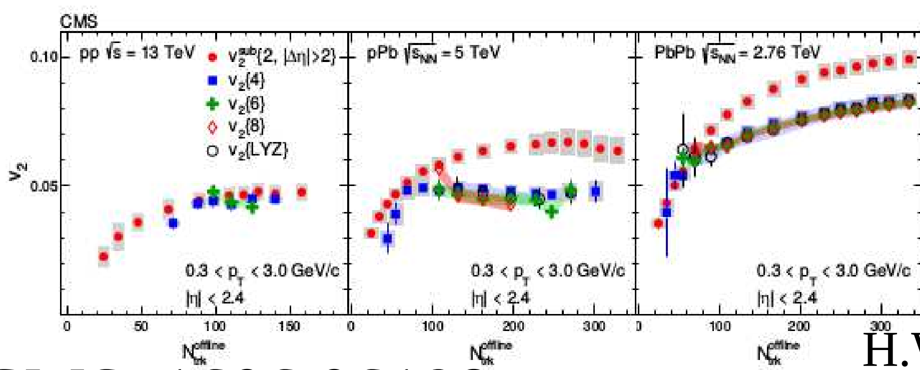
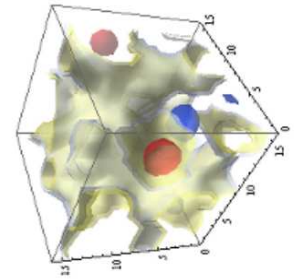
The LHC Might Have Created The Smallest Drop Of Liquid Ever

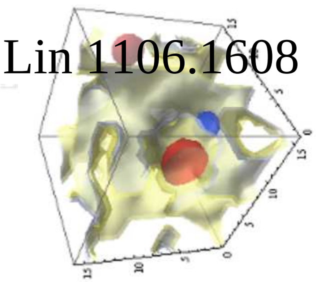
A tiny drop could have big implications for our understanding of particle collisions.

By Shaunacy Ferro May 8, 2013

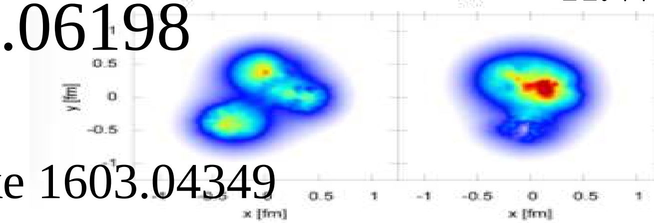


CMS 1606.06198





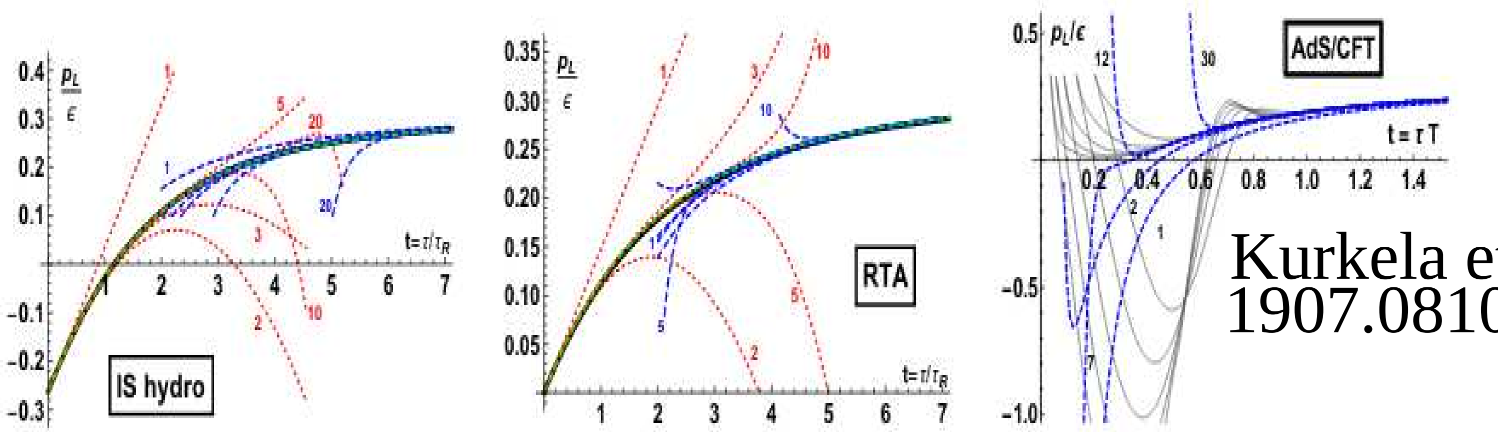
H.W.Lin 1106.1608



BSchenke 1603.04349

1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles <u>"just as collective"</u> as for 1000.

Hydrodynamics in small systems: “hydrodynamization” /”fake equilibrium”
A lot more work in both AdS/CFT and transport theory about ”hydrodynamization” /”Hydrodynamic attractors”



Fluid-like systems far from equilibrium (large gradients)! Usually from 1D solution of Boltzmann AdS/CFT equations!

But I have a basic question: ensemble averaging!

- What is hydrodynamics if $N \sim 50$ ...
  - Ensemble averaging , $\langle F(\{x_i\}, t)\rangle \neq F(\{\langle x_i\rangle\}, t)$ suspect for any non-linear theory. molecular chaos in Boltzmann, Large N_c in AdS/CFT, all assumed . But for $\mathcal{O}(50)$ particles?!?!
  - For water, a cube of length $\eta/(sT)$ has $\mathcal{O}\left(10^9\right)$ molecules,

$$P(N \neq \langle N\rangle) \sim \exp\left[-\langle N\rangle^{-1}(N-\langle N\rangle)^2\right] \ll 1$$

.

- How do microscopic, macroscopic and quantum corrections talk to eac other? EoS is given by $p = T\ln Z$ but $\partial^2 \ln Z/\partial T^2, dP/dV$??

NB: nothing to do with equilibration timescale . Even "things born in equilibrium" locally via Eigenstate thermalization have fluctuations!

And there is more... How does dissipation work in such a "semi-microscopic system"?

- What does local and global equilibrium mean there?
- If $T_{\mu\nu} \rightarrow \hat{T}_{\mu\nu}$ what is $\hat{\Pi}_{\mu\nu}$ Second law fluctuations? Sometimes because of a fluctuation entropy <u>decreases!</u>

Bottom line: Either hydrodynamics is not the right explanation for these observables (possible!) or we are not understanding something basic about what fluctuations do!

Landau and Lifshitz (also D.Rishke,B Betz et al): Hydrodynamics has <u>three</u> length scales

$$\underbrace{l_{micro}}_{\sim s^{-1/3},n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

Weakly coupled: Ensemble averaging in Boltzmann equation good up to $\mathcal{O}\left((1/\rho)^{1/3}\partial_\mu f(...)\right)$
Strongly coupled: classical supergravity requires $\lambda \gg 1$ but $\lambda N_c^{-1} = g_{YM} \ll 1$ so

$$\frac{1}{TN_c^{2/3}} \ll \frac{\eta}{sT} \quad \left(\quad or \quad \frac{1}{\sqrt{\lambda}T}\right) \ll L_{macro}$$

QGP: $N_c = 3 \ll \infty$,so $l_{micro} \sim \frac{\eta}{sT}$. Cold atoms: $l_{micro} \sim n^{-1/3} > \frac{\eta}{sT}$?

Why is $l_{micro} \ll l_{mfp}$ necessary? Without it, microscopic fluctuations (which come from the finite number of DoFs and have nothing to do with viscosity) will drive fluid evolution.

$\Delta\rho/\rho \sim C_V^{-1} \sim N_c^{-2}$, thermal fluctuations "too small" to be important!

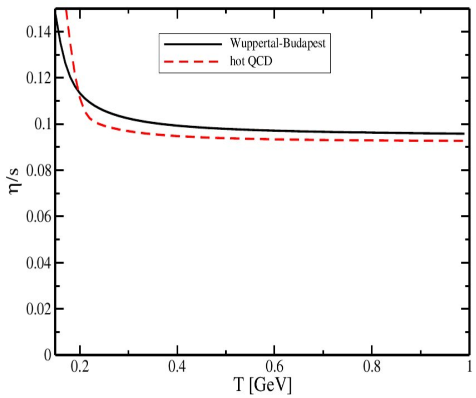
Kovtun, Moore, Romatschke, 1104.1586 As $\eta \to 0$ "infinite propagation of soundwaves" inpacts "IR limit of Kubo formula"

$$\lim_{\eta,k\to 0} \int d^3x e^{ikx} \langle T^{xyxy}(x) T^{xyxy}(0) \rangle \simeq -i\omega \frac{7Tp_{max}}{60\pi^2\gamma_\eta} + (i+1)\omega^{\frac{3}{2}} \frac{7T}{240\pi\gamma_\eta^{\frac{3}{2}}}$$

where p_{max} is the maximum momentum scale and $\gamma_\eta = \eta/(e+p)$

Kovtun,Moore and Romatschke plug in p_{max} into viscosity

$$\eta^{-1} \sim \eta_{bare}^{-1} + \frac{p_{max}}{T} \geq \frac{T}{p_{max}} \geq \frac{T}{s^{1/3}}$$

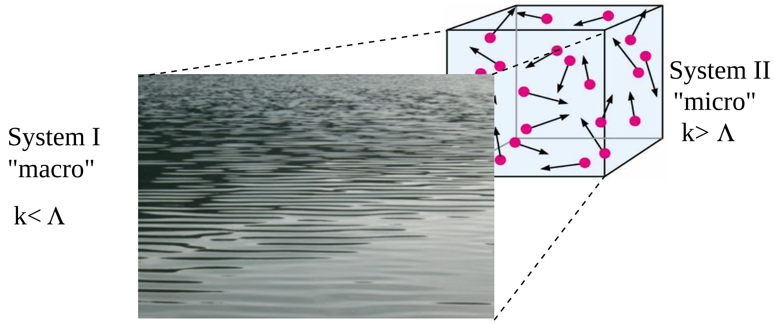


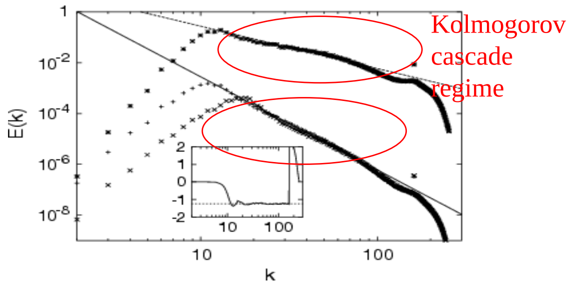
G.Moore,P.Romatschke

Phys.Rev.D84:025006,2011 arXiv:1104.1586

Nc=3, η/ s=KSS

This however, "assumes what you are trying to prove": If there is a "microscopic length", you will eventually get a viscosity. What happens when macro and micro talk to each other in a strongly coupled/turbulent regime?



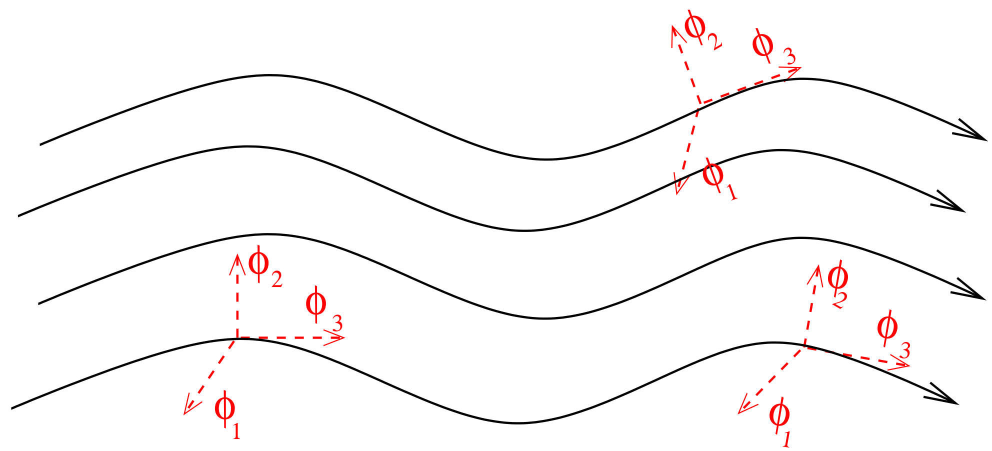


A classical low-viscosity fluid is turbulent. Typically, low-k modes cascade into higher and higher k modes via sound and vortex emission (phase space looks more "fractal"). Classically

$$\eta/(sT) \ll L_{eddy} \ll L_{boundary} \quad , \quad E(k) \sim \left(\frac{dE}{dt}\right)^{2/3} k^{-5/3}$$

For a classical ideal fluid, no limit! since $\lim_{\delta\rho\rightarrow 0, k\rightarrow\infty} \delta E(k) \sim \delta\rho k c_s \rightarrow 0$ but for quantum perturbations, $E \geq k$ so conservation of energy has to cap cascade. A quantum viscosity!

My previous attempt Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I=1...3$ of the position of a fluid cell originally at $\phi_I(t=0,x^i), I=1...3$.



The system is a Fluid if it's Lagrangian obeys some symmetries (Ideal hydrodynamics $\leftrightarrow$ Isotropy in comoving frame)

$$L \to \ln \mathcal{Z} \quad \mathcal{Z} = \int \mathcal{D}\phi_i \exp\left[-T_0^4 \int F(B(\phi_I)) d^4x\right], \langle \mathcal{O} \rangle \sim \frac{\partial \ln \mathcal{Z}}{\partial ...}$$

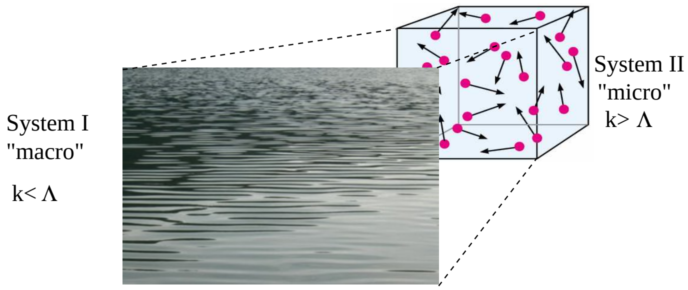
A lot of work on this 1903.08729

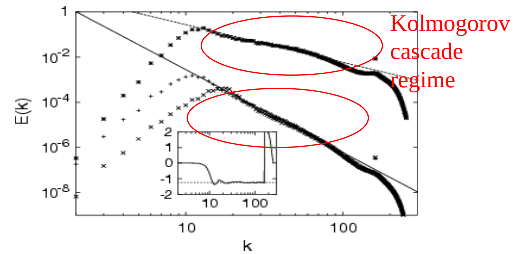
Some accomplishments EFT techniques, insights from Ostrogradski's theorem

Some limitations no clear way to incorporate microscopic fluctuations, functional integral hard , lattice regularization possible but limited to hydrostatic case (1502.05421)

Using a volume cell as a DoF makes it hard to understand fluctuations within it!

More fundamentally: Let us take a stationary slab of fluid at local equilibrium.

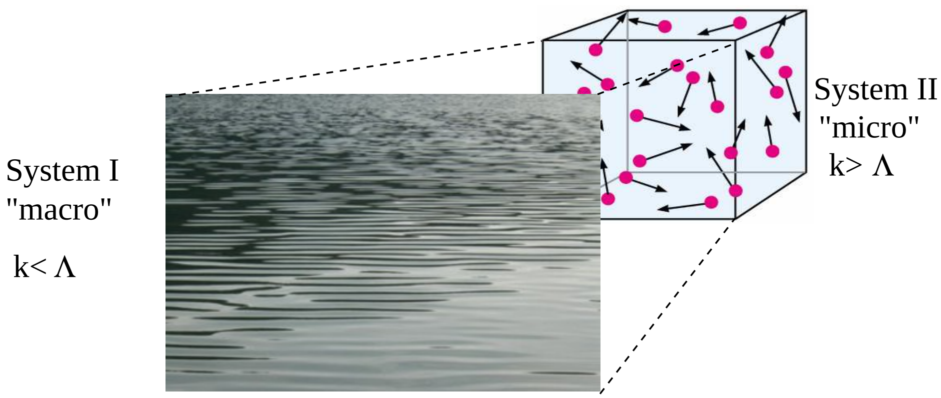


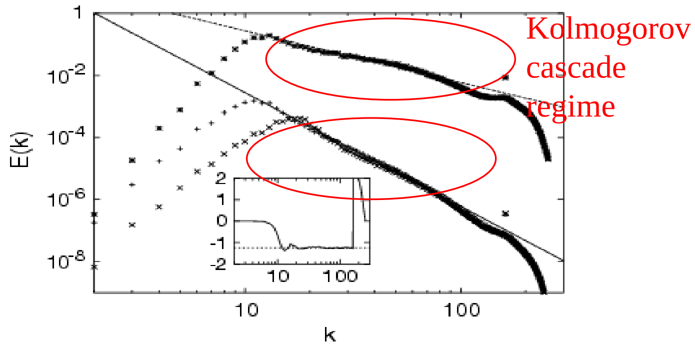


Statistical mechanics: This is a system in global equilibrium, described by a partition function $Z(T, V, \mu)$, whose derivatives give expectation values $\langle E \rangle$,fluctuations $\langle (\Delta E)^2 \rangle$ etc. in terms of parameters representing conserved charges

Fluid dynamics: This is the state of a field in local equilibrium which can be perturbed in an infinity of ways. The perturbations will then interact and dissipate according to the Euler/N-S equations

More fundamentally: Let us take a stationary slab of fluid at local equilibrium.





To what extent are these two pictures the same?

- Global equilibrium is also local equilibrium, if you forget fluctuations
- Dissipation scale in local equilibrium $\eta/(Ts)$, global equilibration timescale $(Ts)/\eta$

Some insight from maths

Millenium problem: existence and smoothness of the Navier-Stokes equations

Important tool are "weak solutions" , similar to what we call "coarse-graining".

$$F\left(\frac{d}{dx}, f(x)\right) = 0 \Rightarrow F\left(\int \frac{d}{dx}\phi(x)..., f(x)\right) = 0$$

$\phi(x)$ "test function", similar to coarse-graining!

Existance of Wild/Nightmare solutions and non-uniqueness of weak solutions shows this tension is non-trivial, coarse-graining “dangerous”

I am a physicist so I care little about the ”existence of ethernal solutions” to an approximate equation, Turbulent regime and microscopic local equilibria need to be consistent

Thermal fluctuations could both ”stabilize” hydrodynamics and ”accellerate” local thermalization

Our proposal

Every statistical theory needs a "state space" and an "evolution dynamics"
The ingredients

State space:Zubarev hydrodynamics Mixes micro and macro DoFs

Dynamics: Crooks fluctuation theorem provides the dynamics via a definition of $\Pi_{\mu\nu}$ from fluctuations

$\hat{T}^{\mu\nu}$ is an operator, so any decomposition, such as $\hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$ must be too!

Zubarev: A partition function for local equilibrium

Let us generalize the GC ensemble to a co-moving frame $E/T \to \beta_\mu T^\mu_\nu$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

Z is a partition function with a field of Lagrange multiples β_μ , with microscopic and quantum fluctuations included.

Effective action from $\ln Tr[Z]$. Correction to Lagrangian picture?

But....

- Dynamics is not clear. Gradient expansion in β_μ but...
  - 2nd order Gradient expansion (Navier stokes) non-causal perhaps...
  - Use Israel-Stewart, $\Pi_{\mu\nu}$ arbitrary perhaps...
  - Foliation $d\Sigma_\mu$ arbitrary but not clear how to link to Arbitrary $\Pi_{\mu\nu}$
- What about fluctuations? Coarse-graining and fluctuations mix? How does one truncate?

An operator formulation

$$\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}_{\mu\nu}$$

and $\hat{T}_0^{\mu\nu}$ truly in equilibrium!

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

describes all cumulants and probabilities

$$\langle T_0^{\mu\nu}(x_1) T_0^{\mu\nu}(x_2) ... T_0^{\mu\nu}(x_n) \rangle = \prod_i \frac{\delta^n}{\delta \beta_\mu(x_i)} \ln Z$$

and also the full energy-momentum tensor

$$\langle T^{\mu\nu}(x_1)T^{\mu\nu}(x_2)...T^{\mu\nu}(x_n)\rangle = \prod_i \frac{\delta^n}{\delta g_{\mu\nu}(x_i)} \ln Z$$

What this means

- Equilibrium at "probabilistic" level

$$\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$$

- KMS Condition obeyed by "part of density matrix" in equilibrium, "expand" around that! An operator constrained by KMS condition is still an operator! $\equiv$ time dependence in interaction picture

Does this make sense

$$\hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu} \qquad , \qquad \hat{\rho}_{T_{\mu\nu}} = \frac{\hat{\rho}_{T_0} + \hat{\rho}_{\Pi_0}}{\mathrm{Tr}\left(\hat{\rho}_{T_0} + \hat{\rho}_{\Pi_0}\right)} \simeq \hat{\rho}_{T_0}\left(1 + \delta\hat{\rho}\right)$$

For <u>any</u> flow field β_μ and <u>lagrangian</u> we can <u>define</u>

$$Z_{T_0}(J(y)) = \int \mathcal{D}\phi \exp\left[-\int_0^{T^{-1}(x_i^\mu)} d\tau' \int d^3x \left(L(\phi) + J(y)\phi\right)\right] \propto$$

$$\propto \exp\left[-\beta^0 \hat{T}_{00}\right]\Big|_{\beta_\mu = (T^{-1}(x,t),\vec{0})}$$

E.g. Nishioka, 1801.10352 $\langle x|\,\rho\,|x'\rangle =$

$$= \frac{1}{Z}\int_{\tau=-\infty}^{\tau=\infty}\int \left[\mathcal{D}\phi, \mathcal{D}y(\tau)\,\mathcal{D}y'(\tau)\right]\, e^{-iS(\phi y, y')} \cdot \underbrace{\delta\left[y(0^+) - x'\right]\,\delta\left[y'(0^-) - x\right]}_{\frac{\delta J_i(y(0^+))}{\delta J_i(x')}\frac{\delta J_j(y(0^-))}{\delta J_j(x)}}$$

$$\Rightarrow \frac{\delta^2}{\delta J_i(x)\delta J_j(x')} \ln\left[Z_{T_0}(T^{\mu\nu}, J) \times Z_\Pi(J)\right]_{J=J_1(x)+J_2(x')}$$

$J_1(x) + J_2(x')$ chosen to respect Matsubara conditions!

Any ρ can be separated like this for any β_μ . The question is, is this a good approximation? Is dynamics given by Crooks theorem?

The source J related to the smearing in "weak solutions". Pure maths angle?

Entropy/Deviations from equilibrium

- In quantum mechanics Entropy function of density matrix

$$s = Tr(\hat{\rho}\ln\hat{\rho}) = \frac{d}{dT}\left(T\ln Z\right)$$

Conserved in quantum evolution, not coarse-graining/gradient expansion

- In IS entropy function of the dissipative part of E-M tensor

$$n^{\nu}\partial_{\nu}\left(su^{\mu}\right) = n^{\mu}\frac{\Pi^{\alpha\beta}}{T}\partial_{\alpha}\beta_{\beta} \quad , \qquad \geq 0$$

$n_{\mu} = d\Sigma_{\mu}/|d\Sigma_{\mu}|, \Pi_{\mu\nu}$ arbitrary. How to combine coarse-graining? if vorticity non-zero $n_{\mu}u^{\mu} \neq 0$

What about fluctuations

$$n^\nu \partial_\nu \left(s u^\mu\right) = n^\mu \frac{\Pi^{\alpha\beta}}{T} \partial_\alpha \beta_\beta \qquad , \qquad \geq 0$$

- If n_μ arbitrary cannot be true for "any" choice
- 2nd law is true for "averages" anyways, <u>sometimes</u> entropy can decrease

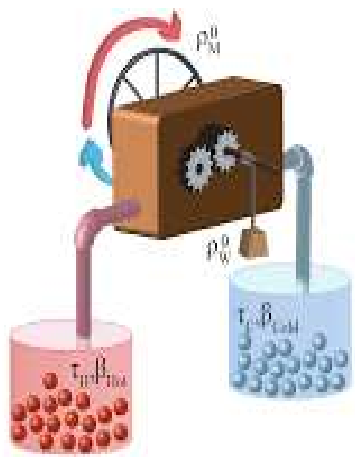
We need a fluctuating formulation!

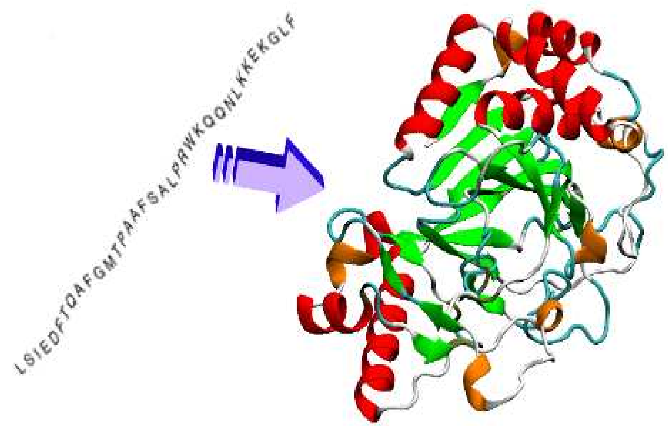
- "Statistical" (probability depends on "local microstates")
- Dynamics with fluctuations, time evolution of β_μ distribution

So we need

- a similarly probabilistic definition of $\hat{\Pi}^{\mu\nu} = \hat{T}^{\mu\nu} - \hat{T}_0^{\mu\nu}$ as an operator!!
- Probabilistic dynamics, to update $\hat{\Pi}_{\mu\nu}, \hat{T}_{\mu\nu}$!

Crooks fluctuation theorem!





Relates fluctuations, entropy in small fluctuating systems (Nano,proteins)

Crooks fluctuation theorem!

$$P(W)/P(-W) = \exp[\Delta S]$$

P(W) Probability of a system doing some work in its usual thermal evolution

P(-W) Probability of the same system "running in reverse" and decreasing entropy due to a <u>thermal fluctuation</u>

ΔS Entropy produced by $P(W)$

Looks obvious but...

Is valid for systems very far from equilibrium (nano-machines, protein folding and so on)

Proven for Markovian processes and fluctuating systems in contact with thermal bath

Leads to irreducible fluctuation/dissipation: TUR (more later!)

Applying it to locally equilibrium systems within Zubarev's formalism is straight-forward

How is Crooks theorem useful for what we did? Guarnieri et al, arXiv:1901.10428 (PRX) derive Thermodynamic uncertainity relations from

$$\hat{\rho}_{ness} \simeq \hat{\rho}_{les}(\lambda) e^{\hat{\Sigma}} \frac{Z_{les}}{Z_{ness}} \quad , \quad \hat{\rho}_{les} = \frac{1}{Z_{les}} \exp\left[-\frac{\hat{H}}{T} \right]$$

$\hat{\rho}_{les}$ is Zubarev operator while Σ is calculated with a Kubo-like formula

$$\hat{\Sigma} = \delta_\beta \Delta \hat{H}_+ \quad , \quad \hat{H}_+ = \lim_{\epsilon \to 0^+} \epsilon \int dt e^{\epsilon t} e^{-\hat{H} t} \Delta \hat{H} e^{\hat{H} t}$$

Relies on

$$\lim_{w \to 0} \left\langle \left[\hat{\Sigma}, \hat{H} \right] \right\rangle \to 0 \equiv \lim_{t \to \infty} \left\langle \left[\hat{\Sigma(t)}, \hat{H}(0) \right] \right\rangle \to 0$$

This "infinite" is "small" w.r.t. hydro gradients. $\equiv$ Markovian as in Hydro with $l_{mfp} \to \partial$ but with operators$\to$ carries all fluctuations with it!

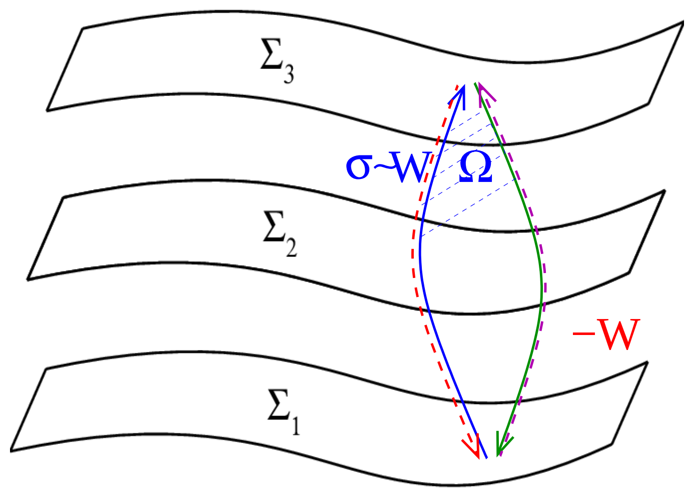
$$P(W)/P(-W) = \exp[\Delta S] \quad \text{Vs} \quad S_{eff} = \ln Z$$

KMS condition reduces the functional integral to a Metropolis type weighting, $\equiv$ periodic time at rest with β_μ

Markovian systems exhibit Crooks theorem, two adjacent cells interaction outcome probability proportional to number of ways of reaching outcome Same hyerarchy as normal gradient expansion, but operator level

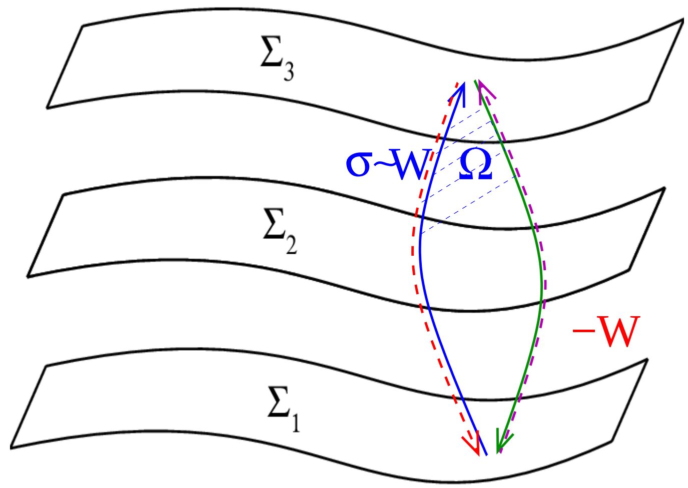
Crooks theorem's computation of $\ln Z_{zubarev}$ like "lattice weighting" by Wilson lines.
Equivalent to Jarzynski's theorem, used on lattice (Caselle et al, 1604.05544)

Applying Crooks theorem to Zubarev hydrodynamics: Stokes theorem



$$-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) = -\int_{\Sigma(\tau')} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) + \int_\Omega \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu\right),$$

true for "any" fluctuating configuration.



Let us now invert one foliation so it goes "backwards in time" assuming Crooks theorem means

$$\frac{\exp\left[-\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]}{\exp\left[-\int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]} = \exp\left[\frac{1}{2}\int_\Omega d\Omega^\mu_\mu \left[\frac{\hat{\Pi}^{\alpha\beta}}{T}\right]\partial_\beta \beta_\alpha\right]$$

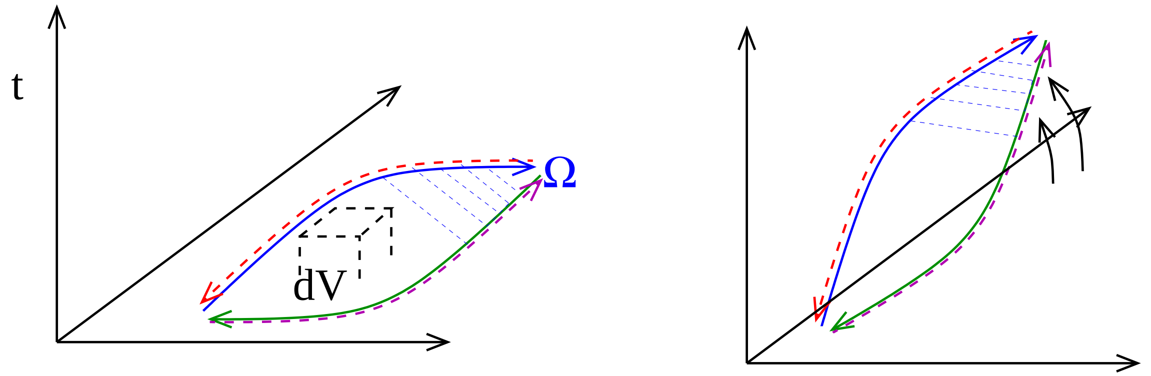
Small loop limit

$$\left\langle \exp\left[\oint d\Sigma_\mu \omega^{\mu\nu} \beta^\alpha \hat{T}_{\alpha\nu}\right]\right\rangle = \left\langle \exp\left[\int \frac{1}{2} d\Sigma_\mu \beta^\mu \hat{\Pi}^{\alpha\beta} \partial_\alpha \beta_\beta\right]\right\rangle$$

A non-perturbative operator equation...

$$\left.\frac{\hat{\Pi}^{\mu\nu}}{T}\right|_\sigma = \left(\frac{1}{\partial_\mu \beta_\nu}\right) \frac{\delta}{\delta\sigma}\left[\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu} - \int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]$$

Similar to a Wilson line in Gauge theory!



A sanity check: For a an equilibrium spacelike $d\Sigma_\mu = (dV, \vec{0})$ (left-panel) we recover Boltzmann's

$$\Pi^{\mu\nu} \Rightarrow \Delta S = \frac{dQ}{T} = \ln\left(\frac{N_1}{N_2}\right)$$

Crooks theorem: thermodynamic uncertainity relations

Andr M. Timpanaro, Giacomo Guarnieri, John Goold, and Gabriel T. Landi
Phys. Rev. Lett. 123, 090604

$$\frac{\left\langle (\Delta Q)^2 \right\rangle}{\left\langle Q \right\rangle^2} \geq \frac{2}{\Delta S(W)}$$

Valid locally in time!

$$\frac{d}{d\tau}\Delta S \geq \frac{1}{2}\frac{d}{d\tau}\frac{\left\langle Q \right\rangle^2}{\langle (\Delta Q)^2 \rangle}$$

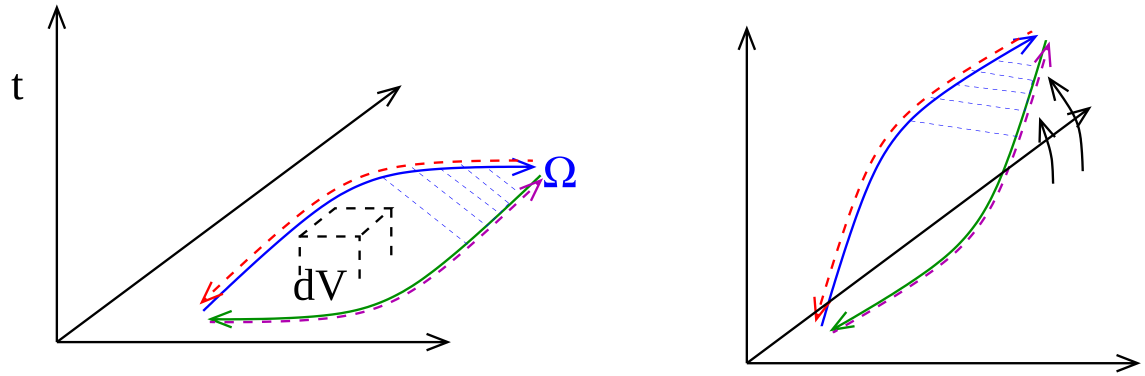
Relates thermal fluctuations and dissipation, producing an <u>irreducible</u> uncertainity

COnsequences: Hydro-TUR? Separate flow into potential and vortical part

$$\beta_\mu = \partial_\mu \phi + \zeta_\mu \quad , \quad n_\mu \to T\partial_\mu \phi \quad , \quad \omega_{\mu\nu} = g_{\mu\nu}$$

A likely TUR is

$$\frac{\langle [T_{\mu\gamma}, T^{\gamma}_{\nu}] \rangle}{\langle T^{\mu\nu} \rangle^2} \geq \frac{\mathcal{C} \epsilon_{\mu\gamma\kappa} \langle T^{\gamma\kappa} \rangle \beta^\mu}{\Pi^{\alpha\beta} \partial_\beta \zeta_\alpha} \quad , \quad \mathcal{C} \sim \mathcal{O}(1)$$

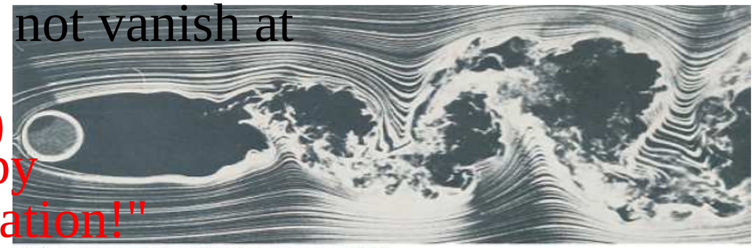


Deform the equilibrium contour and get Kubo formula! (right panel)

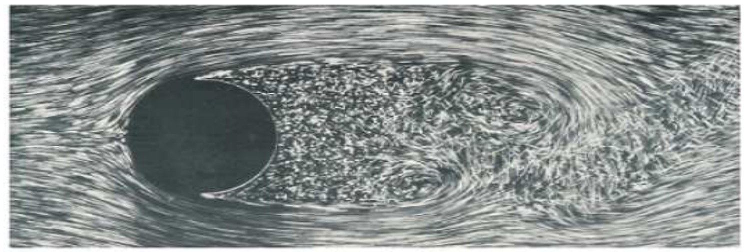
$$\mathcal{C} = \lim_{w \to 0} \frac{\mathrm{Re}\left[F(w)\right]}{\mathrm{Im}\left[F(w)\right]} \quad , \quad F(w) = \int d^3x dt \left\langle T^{xy}(x) T^{xy}(0) \right\rangle e^{i(kx - wt)}$$

−dissipation does not vanish at zero viscosity

Vlad Vicol (talk) "will be proven by a different generation!"



(a) Visualizing turbulent cylinder wake at Re = 10000
[Courtesy: Thomas Corke and Hassan Nagib; *from An Album of Fluid Motion by van Dyke (1982)*]



(b) a closer look at Re = 2000 - patterns are identical as in (a)
[Courtesy: ONERA pic. Werle & Gallon (1972) *from An Album of Fluid Motion by van Dyke (1982)*]

$$\frac{\langle [T_{\mu\gamma}, T_{\nu}^{\gamma}] \rangle}{\langle T^{\mu\nu} \rangle^2} \geq \frac{\mathcal{O}(1)\, \epsilon_{\mu\gamma\kappa} \langle T^{\gamma\kappa} \rangle \beta^{\mu}}{\Pi^{\alpha\beta} \partial_{\beta} \zeta_{\alpha}}$$

Fluctuations+Low viscosity ⇒ Turbulence ⇒ high vorticity ⇒ dissipation!
(usually mathematicians consider incompressible fluids, non-relativistic!)

Towards equations: Gravitational Ward identity!

$$\partial^\alpha \left\{ \left\langle \left[\hat{T}_{\mu\nu}(x), \hat{T}_{\alpha\beta}(x') \right] \right\rangle - \right.$$

$$\left. -\delta(x-x') \left(g_{\beta\mu} \left\langle \hat{T}_{\alpha\nu}(x') \right\rangle + g_{\beta\nu} \left\langle \hat{T}_{\alpha\mu}(x') \right\rangle - g_{\beta\alpha} \left\langle \hat{T}_{\mu\nu}(x') \right\rangle \right) \right\} = 0$$

Small change in $T_{\mu\nu}$ related to infinitesimal shift! Conservation of momentum!

Can be used to fix one component of $\beta_\mu = u_\mu / T$, so $u_\mu u^\mu = -1$ and $(\beta_\mu \beta^\mu)^{-1/2} = T$ weights $\hat{\Pi}^{\mu\nu}$ in a way that conserves $\hat{\Pi}^{\mu\nu} + \hat{T}_0^{\mu\nu}$

Putting everything together: Dynamics at Z level

$$\langle T_{\mu\nu} \rangle = \frac{2}{\sqrt{-g}} \frac{\delta \ln Z}{\delta g^{\mu\nu}} = \langle T_0 \rangle^{\mu\nu} + \Pi^{\mu\nu}$$

$$\langle T_0^{\mu\nu} \rangle = \frac{\delta^2 \ln Z}{\delta \beta_\mu d n_\nu} \quad , \quad \langle \Pi^{\mu\nu} \rangle = \frac{1}{\partial_\mu \beta_\nu} \partial_\gamma \frac{d}{d \ln(\beta_\alpha \beta^\alpha)} \left[\beta^\gamma \ln Z \right]$$

$$\partial_\alpha \left[\frac{2}{\sqrt{-g}} \frac{\delta^2 \ln Z}{\delta g_{\mu\nu} \delta g_{\alpha\beta}} - \delta(x - x') \frac{2}{\sqrt{-g}} \left(g_{\beta\mu} \frac{\delta \ln Z}{\delta g_{\alpha\nu}} + g_{\beta\nu} \frac{\delta \ln Z}{\delta g_{\alpha\mu}} - g_{\beta\alpha} \frac{\delta \ln Z}{\delta g_{\nu\mu}} \right) \right] = 0$$

and, finally, Crook's theorem

$$\frac{\delta^2}{\delta g^{\mu\nu} \delta g^{\alpha\beta}} \ln Z = \frac{\sqrt{-g}}{2} \frac{\beta_\kappa}{2 \omega^{\mu\nu} \beta^\alpha} \partial_\beta n^\kappa \partial_\gamma \frac{d}{d \ln(\beta_\alpha \beta^\alpha)} \left[\beta^\gamma \ln Z \right]$$

Ito process

$$\hat{T}_{\mu\nu}(t) = \hat{T}_{\mu\nu}(t_0) + \int \Delta^{\alpha\beta}\left[\hat{T}_{\mu\alpha}\hat{T}_{\beta\nu}\right] + \int \frac{1}{2} d\Sigma_\mu \beta_\nu \hat{\Pi}_{\alpha\beta}\partial^\alpha \beta^\beta$$

$$\ln Z|_{t+dt} = \int \mathcal{D} g_{\mu\nu}(x) T^{\mu\nu}|_{t+dt} \quad , \quad \beta_\mu|_{t+dt} = \frac{\delta \ln Z|_{t+dt}}{\delta T_{\mu\nu}} n_\nu$$

At every point in a foliation, dynamics is regulated by a stochastic term and a dissipation term. Can be done numerically with montecarlo with an ensemble of configurations at every point in time...

A numerical formulation

Define a field β_μ field and n_μ

Generate an ensemble of

$$\ln Z|_{t+dt} = \int \mathcal{D}g_{\mu\nu}(x) T^{\mu\nu}|_{t+dt} \quad , \quad \beta_\mu|_{t+dt} = \frac{\delta \ln Z|_{t+dt}}{\delta T_{\mu\nu}} n_\nu$$

According to a Metropolis algorithm ran via Crooks theorem

Reconstruct the new β and $\Pi_{\mu\nu}$. The Ward identity will make sure $\beta_\mu \beta^\mu = -1/T^2$

A semiclassical limit?

$$\partial_\mu \left\langle \hat{T}^{\mu\nu} \right\rangle = 0 \quad , \quad \partial_\mu \left\langle \hat{T}_0^{\mu\nu} \right\rangle = -\partial_\mu \left\langle \hat{\Pi}^{\mu\nu} \right\rangle$$

Integrating by parts the second term over a time scale of many $\Delta_{\mu\nu}$ gives, in a frame comoving with $d\Sigma_\mu$

$$\int_0^\tau d\tau' \left\langle \hat{\Pi}_{\mu\nu} \right\rangle \partial^\mu \beta^\nu \sim \beta^\mu \partial_\mu \left\langle \hat{\Pi}_{\mu\nu} \right\rangle + \left\langle \hat{\Pi}_{\mu\nu} \right\rangle = F(\partial^{n\geq 1}\beta_\mu, ...)$$

where $F(\beta_\mu)$ is independent of $\Pi_{\mu\nu}$. (Because local entropy is maximized at vanishing viscosity $F()$ depends on gradients. Israel-Stewart

However , results of, e.g., Gavassino 2006.09843 and Shokri 2002.04719 suggest that fluctuations with decreasing entropy have a role at first order in gradient!

Polarization,Chemical potential and gauge symmetries

$$\beta_\mu T^{\mu\nu} \to \beta_\mu T^{\mu\nu} + \mu N^\mu + \mathcal{W}\mathcal{J}^\mu$$

Approach changes very little! but

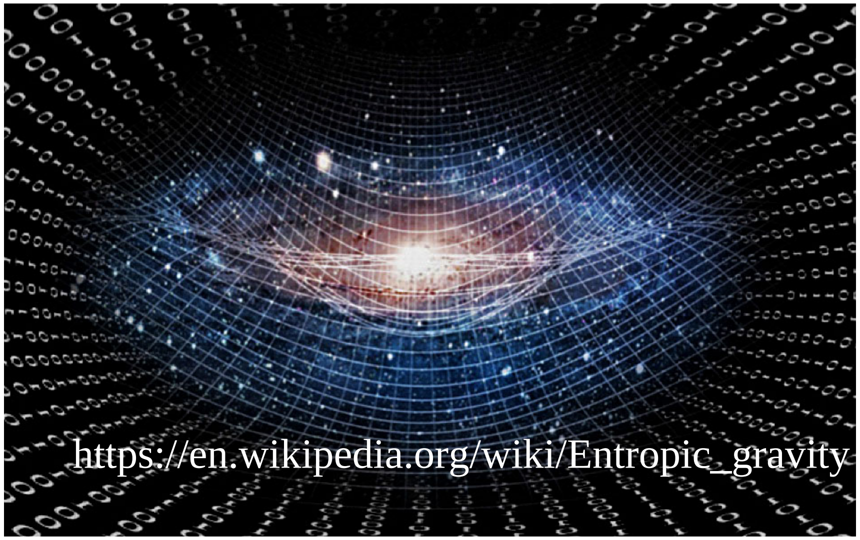
- $\mathcal{W}\mathcal{J}^\mu$ might need relaxation, with $\mathcal{S}^\mu \to \mathcal{J}^\mu$
- Gauge potentials will lead to non-local correlations, $N^\mu \to N^\mu + U\partial^\mu U$

Was motivation to look for this work, 1810.12468,1807.02796

Wild speculations

General relativity/Theory of everything T.Jacobson,gr-qc/9504004

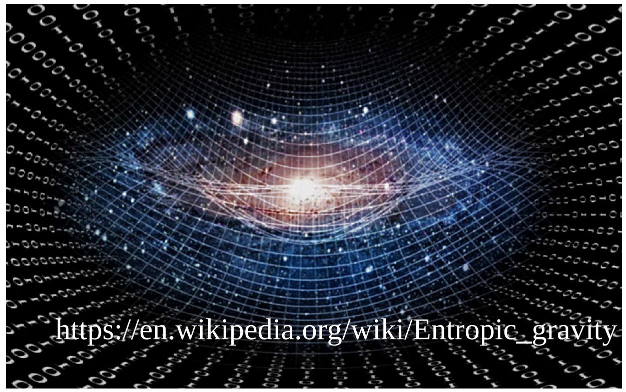
$$dS \propto d\mathcal{A} \quad , \quad + \quad , \quad dQ = TdS \Rightarrow G_{\mu\nu} \propto T_{\mu\nu}$$



T.Jacobson, gr−qc/9504004

T.Padmanabhan 0911.5004

E.Verlinde, 1001.0785



T.Jacobson, gr−qc/9504004

T.Padmanabhan 0911.5004

E.Verlinde, 1001.0785

Started the field of “entropic gravity”

- gravity is emergent and spacetime is a thermalized state
- "Quantum dynamics" is actually fluctuating equilibrium state
- Difficoulty of quantizing gravity makes it an interesting idea, but nothing concrete

Combining Crooks theorem with relativistic field theory

$$S = \int d\mathcal{A} + Tr\left[\rho \ln \rho\right]$$

Dynamics of the geometry given by

$$\exp\left[\Delta S\right] = P(W)/P(-W)$$

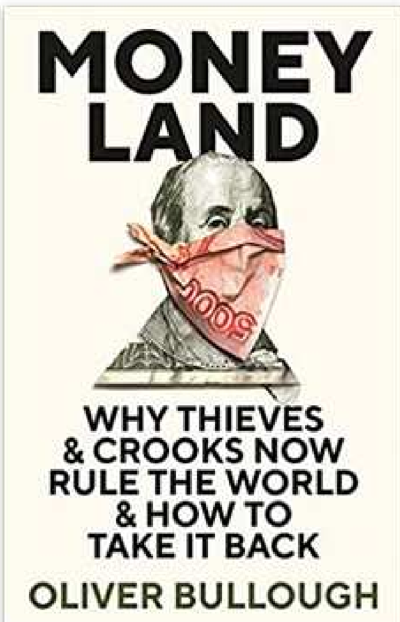
$P(W)$ given by the density matrix , $P(W) = Tr[W.\hat{\rho}]$

$$\hat{\rho} = \frac{1}{Z}\int \mathcal{D}\phi < \phi|\Psi >< \Psi|\phi >$$

Could lead to self-consistent way to update density matrix. Substitution time-horizon and fluctuation/dissipation could ensure general covariance (1501.00435)

The theory of everything...Is the universe governed by Crooks?

Many authors and lots of experimental evidence!



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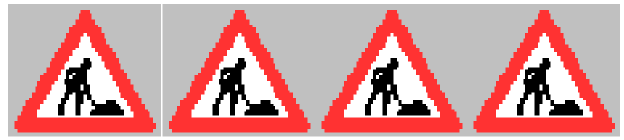
Seriously... some conclusions

Riassumindo
issos 5 anos



- Fluctuations force us to go beyond transport and perturbation theory
- Zubarev hydrodynamics and Crooks fluctuation theorem naturally provide us with a way!
- Lots to do but lots of potential!