Lecture 16-QM_II (22/08/00)

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$H_0 = \frac{\vec{P}^2}{2m_e} - \frac{Ze^2}{r}$ $\qquad E_n^{(0)} = -\frac{R_y}{n^2}$ $\qquad \psi_{nlm_l}(\vec{r}) = R_{nl}(r)\, Y_{lm}(\theta,\varphi)$

$$H = H_0 + H_1 + H_2 + H_3$$

$H_1 = \frac{-(\vec{P}^2)^2}{8 m_e^3 c^2}$ $\qquad E_n^{(1)} = \langle H_1 \rangle$

$H_2 = -\vec{\mu}_s \cdot \vec{B}$ $\qquad \vec{B} = -\frac{\vec{v}\times\vec{E}}{c}$ $\qquad \vec{v} = \frac{\vec{P}}{m_e}$

$\boxed{H_2 = \frac{Ze^2 g_s}{2 m_e c^2 r^3}\, \vec{S}\cdot\vec{L}}$ $\qquad \vec{E} = -\vec{\nabla}\varphi = -\vec{\nabla}\left(+\frac{Ze}{r}\right) = \frac{Ze}{r^3}\vec{r}$ $\qquad V(r) = e\varphi(r)$

$\boxed{\vec{B} = \frac{Ze}{m_e c r^3}\vec{L}}$

$E_n^{(1)} = \langle H_2 \rangle$ $\qquad \vec{\mu}_s = -\frac{\mu_B g_s}{\hbar}\vec{S}$ $\qquad \mu_B = \frac{e\hbar}{2m_e c}$

$|n l m_l\rangle\, |s, m_s\rangle$

$\{H_0, \vec{L}^2, L_3, \vec{S}^2, S_3\} \longrightarrow \{H_0, \vec{J}^2, J_3, \vec{S}^2, \vec{L}^2\}$

$\vec{J} = \vec{L} + \vec{S}$

$s = \frac{1}{2}$ $\qquad m_s = \pm\frac{1}{2}$ $\qquad j = l - \frac{1}{2},\ l + \frac{1}{2}$

$m_j = m_l \pm \frac{1}{2}$

$\rightarrow \quad |j\, m_j;\, l, s=\tfrac{1}{2}\rangle = \alpha_\pm\, |l, m_l = m_j - \tfrac{1}{2}\rangle |\uparrow\rangle + \beta_\pm\, |l, m_l = m_j + \tfrac{1}{2}\rangle |\downarrow\rangle$

$$\langle H_2 \rangle = \left\langle \frac{Ze^2 g_s}{2m_e c^2}\, \frac{1}{r^3}\, \vec{S}\cdot\vec{L} \right\rangle$$

$\langle \vec{r} | l, m_l \rangle = R_{nl}(\vec{r})\, Y_{lm}(\theta,\varphi)$ $\qquad \sim \frac{1}{a_B^3 n^3} f(l)$ $\qquad \underline{l \geq 1}$ $\qquad (Z\alpha)^{\cdots}$

$l = 0$

- $H_3 = \frac{\pi}{2}\, \frac{Ze^2\hbar^2}{m_e^2 c^2}\, \delta(\vec{r})$

لرزش

Zitterbewegung

$\langle V(\vec{x} + \delta\vec{x}) \rangle = V(\vec{x}) + \frac{1}{6}\langle (\delta\vec{x})^2 \rangle\, \Delta V(\vec{x}) + \cdots$

$V(\vec{x}) = -\frac{Ze^2}{r}$ $\qquad \delta\vec{x} \sim \lambda_c = \frac{\hbar}{m_e c}$

$\Delta \frac{1}{r} = -4\pi\delta^3(\vec{r})$ $\qquad |Y_{00}(\theta,\varphi)|^2$

$\langle H_3 \rangle = \int |R_{nl}(r)|^2\, \delta^3(\vec{r})\, d^3r\, |Y_{lm}(\theta,\varphi)|^2$ $\qquad d^3r = r^2 dr\, d(\cos\theta)\, d\varphi$

$l = 0 \rightarrow m_l = 0$

$|R_{n0}(\vec{0})|^2\, \delta_{l0}\, \delta_{m_l 0}$

$= \frac{(Z\alpha)^4}{n^3}\, \frac{1}{2} m_e c^2$ $\qquad (Z\alpha)^2$ $\qquad = \ldots \alpha \ln\alpha$

تصحیحات تابشی

خود انرژی الکترون

Electron Self-energy

$n = 2$

$^2S_{1/2}\quad ^2P_{1/2}\quad ^2P_{3/2}$ $\qquad ^2P_{3/2}$ $\qquad ^2S_{1/2}$ $\qquad ^2P_{1/2}$ $\qquad \langle H_2 \rangle$ $\qquad 2P_{3/2}$ $\qquad ^2S_{1/2}\quad ^2P_{1/2}$ $\qquad ^2S_{1/2}$ $\qquad ^2P_{1/2}$

$^n l_j$

$l = 0 \qquad s = \frac{1}{2} \qquad j = \frac{1}{2}$

x_j $\qquad$ $\langle H_2 \rangle$ $\qquad$ ${}^2S_{1/2} \quad {}^2P_{1/2}$ $\qquad$ ${}_{1/2}$ $\quad {}^2P_{1/2}$

$l=0 \quad s=\frac{1}{2} \quad j=\frac{1}{2}$

$l=1$ $\qquad$ $\langle H_1 + H_2 \rangle$ $\qquad$ Lamb Shift.

H_4 $\qquad$ $H_2 = -\vec{\mu}_s \cdot \vec{B}$ $\qquad$ ساختار فوق ریز

$$\boxed{\vec{A} = -\frac{1}{4\pi}\left(\vec{\mu}_I \times \vec{\nabla}\right)\frac{1}{r}}$$

(هسته) $\vec{\mu}_I = g_I \frac{\mu_N}{\hbar} \vec{I}$ $\qquad$ (الکترون) $\vec{\mu}_s = -g_s \frac{\mu_B}{\hbar}\vec{S}$

مگنتون هسته $\mu_N = \frac{Ze\hbar}{2m_N c}$, $\quad g_I \sim 5.56$ $\qquad$ مگنتون بوهر $\mu_B = \frac{e\hbar}{2m_e c}$, $\quad g_s \sim 2$

m_N $\quad$ جرم هسته

پروتون $\mu_p = 2.793\,\mu_N$ $\qquad$ μ_{quark} $\qquad$ $p \to uud$

نوترون $\mu_n = -1.913\,\mu_N$ $\qquad$ پارتون parton

$$\vec{\nabla}\times\vec{A} = \vec{B}_N = -\frac{1}{4\pi}\vec{\nabla}\times\left(\vec{\mu}_I \times \vec{\nabla}\right)\frac{1}{r}$$

$$\boxed{\vec{B}_N = \frac{1}{4\pi}\left(\left(\vec{\mu}_I\cdot\vec{\nabla}\right)\vec{\nabla}\frac{1}{r} - \vec{\mu}_I\,\vec{\nabla}^2\frac{1}{r}\right)}$$

$$B_i = \frac{1}{4\pi}\gamma_N\left(I_j\,\partial_j\,\partial_i\frac{1}{r} - I_i\,\partial_j^2\frac{1}{r}\right)$$

$\vec{\mu}_I = \gamma_N \vec{I}$

$\gamma_N = \frac{Ze\,g_N}{2m_{N_p}c}$

$\vec{\mu}_s = \gamma_e \vec{S}$

$\gamma_e = -\frac{e g_s}{2m_e c}$

$$H_4 = -\vec{\mu}_s \cdot \vec{B}_N$$

$$\boxed{H_4 = -\frac{\gamma_e\gamma_N}{4\pi}\left(I_j S_i\,\partial_j\partial_i\frac{1}{r} - \underbrace{S_i I_i}_{\vec{S}\cdot\vec{I}}\,\partial_j^2\frac{1}{r}\right)}$$

$$E_n^{(1)} = \langle H_4 \rangle$$

تحلیل‌ها $\quad$ جمله اول $\langle \partial_i\partial_j \frac{1}{r} \rangle$ $\qquad$ $\langle \partial_j^2 \frac{1}{r} \rangle$ $\qquad$ $\frac{1}{3}\nabla^2\frac{1}{r}$

فرض: $(l=0)$ $\quad \partial_x^2\frac{1}{r} = \partial_y^2\frac{1}{r} = \partial_z^2\frac{1}{r}$

جمله اول $\displaystyle\int |R_{n0}(r)|^2\, \cancel{|Y_{00}(\theta,\varphi)|^2}\; \underbrace{\partial_i\partial_j\frac{1}{r}}_{\sim\frac{1}{3}\partial_i^2\frac{1}{r}}\; \underbrace{d^3r}_{r^2dr\, \cancel{d(\cos\theta)\,d\varphi}}$ $\quad (l=0$ for $n0)$

$$= \frac{1}{3}\int \delta_{ij}\left(-4\pi\delta^3(\vec{r})\right)|R_{n0}(r)|^2\,dr\,r^2 = -\frac{4\pi}{3}|R_{n0}(0)|^2\,\delta_{ij}$$

جمله دوم $\displaystyle\int |R_{n0}(r)|^2\,\partial_j^2\frac{1}{r}\,r^2dr = -4\pi\,|R_{n0}(0)|^2$

در جمله اول $\displaystyle -\frac{\gamma_e\gamma_N}{4\pi}\left(-\frac{4\pi}{3}\right)|R_{n0}(0)|^2\, I_j S_i\,\delta_{ij}$

$$= +\frac{1}{3}\gamma_e\gamma_N\,|R_{n0}(0)|^2\,\vec{I}\cdot\vec{S}$$

در جمله دوم $\displaystyle = +\frac{\gamma_e\gamma_N}{4\pi}(-4\pi)\,|R_{n0}(0)|^2\,\vec{I}\cdot\vec{S} =$

$$E_n^{(1)} = \langle H_4 \rangle = \underbrace{\left(-1+\frac{1}{3}\right)}_{-\frac{2}{3}}|R_{n0}(0)|^2\langle \vec{I}\cdot\vec{S}\rangle\,\gamma_e\gamma_N$$

$l=0$

$-\frac{2}{3}$

$$\boxed{\begin{array}{l} \ell = 0 \\ \quad E_n^{(1)} = \frac{4}{3}\, g_N\, \frac{m_e}{m_N}\, (m_e c^2)\, \frac{1}{\hbar^2}\, (Z\alpha)^4\, \frac{1}{n^3}\, \langle \vec{S}\cdot\vec{I} \rangle \end{array}}$$

$$\langle(\vec{S}\cdot\vec{L})\rangle = \frac{\hbar^2}{2} \begin{cases} \ell & j = \ell + \frac{1}{2} \\ -\ell - 1 & j = \ell - \frac{1}{2} \end{cases}$$