

$$H_0 = \frac{\vec{p}^2}{2m_e} - \frac{Ze^2}{r}$$

$$E_n^{(0)} = -\frac{Ry}{n^2}$$

$$\psi_{n\ell m}(\vec{r}) = R_{n\ell}(r) Y_{\ell m}(\theta, \varphi)$$

$$H = H_0 + H_1 + H_2 + H_3$$

$$H_1 = \frac{-(\vec{p}^2)^2}{8m_e^3 c^2}$$

$$E_n^{(1)} = \langle H_1 \rangle$$

$$H_2 = -\vec{\mu}_s \cdot \vec{B}$$

$$\vec{B} = -\frac{\nabla \times \vec{E}}{c} \quad \vec{v} = \frac{\vec{p}}{m_e}$$

$$H_2 = \frac{Ze^2 g_s}{2m_e c^2 r^3} \vec{S} \cdot \vec{L}$$

$$\vec{E} = -\nabla \varphi = -\nabla \left(\frac{Ze}{r} \right) = \frac{Ze}{r^3} \vec{r} \quad V(r) = e\varphi(r)$$

$$\vec{B} = \frac{Ze}{m_e c^3} \vec{L}$$

$$E_n^{(1)} = \langle H_2 \rangle$$

$$\vec{\mu}_s = \frac{\mu_B g_s}{\hbar} \vec{S}$$

$$\mu_B = \frac{e\hbar}{2m_e c}$$

$$|n\ell m_\ell\rangle |s, m_s\rangle$$

$$\{H_0, \vec{L}^2, L_z, \vec{S}^2, S_z\} \rightarrow \{H_0, \vec{J}^2, J_z, \vec{S}^2, \vec{L}^2\}$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$s = \frac{1}{2} \quad m_s = \pm \frac{1}{2}$$

$$j = l - \frac{1}{2}, l + \frac{1}{2}$$

$$m_j = m_\ell \pm \frac{1}{2}$$

$$\rightarrow |j, m_j; l, s = \frac{1}{2}\rangle = \alpha_\pm |l, m_\ell = m_j - \frac{1}{2}\rangle |\uparrow\rangle + \beta_\pm |l, m_\ell = m_j + \frac{1}{2}\rangle |\downarrow\rangle$$

$$\langle H_2 \rangle = \left\langle \frac{Ze^2 g_s}{2m_e c^2} \frac{1}{r^3} \vec{S} \cdot \vec{L} \right\rangle$$

$$\langle \vec{r} | l, m_\ell \rangle = R_{n\ell}(\vec{r}) Y_{\ell m}(\theta, \varphi) \sim \frac{1}{r^3} f(l)$$

$l \geq 1$
 $\sim \frac{1}{r^3} \frac{1}{n^3} f(l)$
 $(Z\alpha)^2$

$l=0$

$$H_3 = \frac{\pi}{2} \frac{Ze^2 \hbar^2}{m_e^2 c^2} \delta(\vec{r})$$

• \leftarrow زitterbewegung

$$\langle V(\vec{x} + \delta\vec{x}) \rangle = V(\vec{x}) + \frac{1}{6} \langle (\delta\vec{x})^2 \rangle \Delta V(\vec{x}) + \dots$$

$$V(\vec{x}) = -\frac{Ze^2}{r}$$

$$\delta\vec{x} \sim \lambda_c = \frac{\hbar}{m_e c}$$

$$\Delta \frac{1}{r} = -4\pi \delta^3(\vec{r})$$

$$|Y_{00}(\theta, \varphi)|^2$$

$$\langle H_3 \rangle = \int |R_{n\ell}(r)|^2 \delta^3(\vec{r}) \frac{d^3r}{r^2 dr d(\cos\theta) d\varphi} |Y_{\ell m}(\theta, \varphi)|^2$$

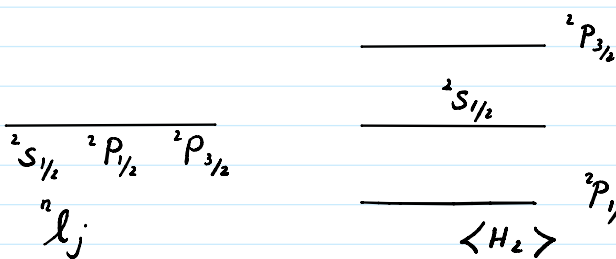
$$l=0 \rightarrow m_\ell=0$$

$$|R_{n0}(0)|^2 \delta_{l0} \delta_{m_\ell 0}$$

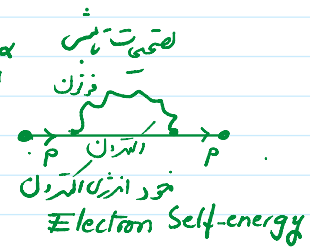
$$= \frac{(Z\alpha)^4}{n^3} \frac{1}{2} m_e c^2$$

$$= \dots \alpha \ln \alpha$$

$n=2$



$$l=0 \quad s = \frac{1}{2} \quad j = \frac{1}{2}$$



$$l=0 \quad E_n^{(1)} = \frac{4}{3} g_N \frac{m_e}{m_N} (m_e c^2) \frac{1}{\hbar^2} (Z\alpha)^4 \frac{1}{n^3} \langle \vec{S} \cdot \vec{I} \rangle$$

$$\langle (\vec{S} \cdot \vec{L}) \rangle = \frac{\hbar^2}{2} \begin{cases} l & j = l + \frac{1}{2} \\ -l-1 & j = l - \frac{1}{2} \end{cases}$$