QM-2-L5-14041217

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Spin

اثر زیمان

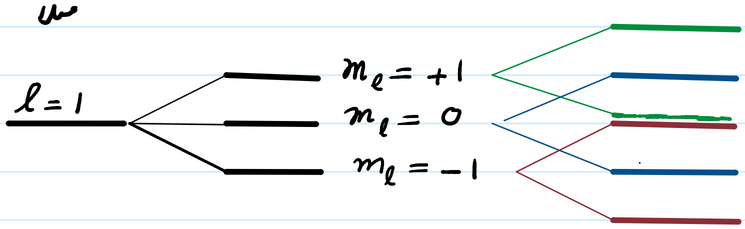
$$H = H_0 + H_B^{(1)}$$

$$H_0 = -\frac{\hbar^2}{2m_e}\vec{\nabla}^2 - \frac{Ze^2}{r}$$

$$H_B^{(1)} = -\vec{\mu}\cdot\vec{B} = -\mu_B \frac{\vec{L}\cdot\vec{B}}{\hbar} \quad \text{with} \quad \mu_B = \frac{-e\hbar}{2m_e c}$$

$$E_{n,m_\ell} = E_n + \hbar\omega_L m_\ell \qquad \omega_L = \text{Larmor frequency} = \frac{eB}{2m_e c}$$

مثال



$\Delta E = 2\hbar\omega_L$

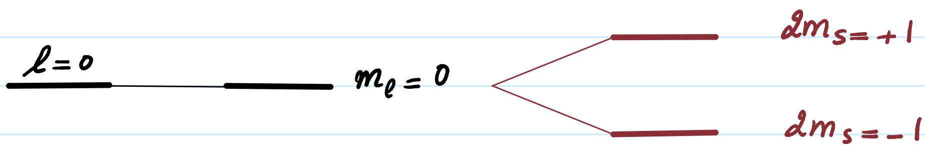
$\Delta E = \hbar\omega_L$

$\Delta E = 0 \quad 0 = \Delta E$

$\Delta E = -\hbar\omega_L$

$\Delta E = -2\hbar\omega_L$

Paschen - Back Effect



$$\Delta E = \hbar\omega_L (m_\ell + 2m_s) \qquad 2m_s = \pm 1 \;\curvearrowright\; m_s = \pm\frac{1}{2}$$

$\ell = 0 \qquad m_\ell = 0 \longrightarrow \Delta E = \pm\hbar\omega_L$

$\ell = 1$

$m_\ell = 1 \longrightarrow \Delta E = \hbar\omega_L (1 \pm 1) \longrightarrow \Delta E = 2\hbar\omega_L \quad \Delta E = 0$

$m_\ell = 0 \longrightarrow \Delta E = \hbar\omega_L (\pm 1) \longrightarrow \Delta E = +\hbar\omega_L \quad \Delta E = -\hbar\omega_L$

$m_\ell = -1 \longrightarrow \Delta E = \hbar\omega_L (-1 \pm 1) \longrightarrow \Delta E = 0 \quad \Delta E = -2\hbar\omega_L$

خطوط طیفی نو وقتی بوجود میاید که گشتاور مغناطیسی را به صورت زیر تصحیح کنیم :

$$\boxed{\begin{aligned} &\vec{\mu}_B = \vec{\mu}_L + \vec{\mu}_S \\ &\vec{\mu}_L = g_L \frac{\mu_B}{\hbar}\vec{L} = \frac{\mu_B}{\hbar}\vec{L} \\ &\vec{\mu}_S = g_S \frac{\mu_B}{\hbar}\vec{S} = 2\frac{\mu_B}{\hbar}\vec{S} \end{aligned}}$$

$\left.\begin{array}{l} g_L = 1 \\ g_S = 2 \end{array}\right\}$ ضریب ژیرومغناطیسی

فرمول بندی اسپین :

$$\vec{L} = (L_x, L_y, L_z)$$

$$[L_i, L_j] = i\hbar\,\epsilon_{ijk} L_k$$

ℓ, m_ℓ درجه آزادی

$$\vec{L}^2 |\ell, m_\ell\rangle = \hbar\ell(\ell+1)|\ell, m_\ell\rangle$$

$[L_i, L_j] = i\hbar \epsilon_{ijk} L_k$ $\qquad l, m_l$ دو عدد کوانتومی

$\vec{L}^2 |l, m_l\rangle = \hbar l(l+1) |l, m_l\rangle$

$L_z |l, m_l\rangle = \hbar m_l |l, m_l\rangle$

به همین ترتیب

$$\boxed{\begin{aligned} &\vec{S} = (S_x, S_y, S_z) \\ &[S_i, S_j] = i\hbar \epsilon_{ijk} S_k \\ &\vec{S}^2 |s, m_s\rangle = \hbar^2 s(s+1) |s, m_s\rangle \\ &S_z |s, m_s\rangle = \hbar m_s |s, m_s\rangle \end{aligned}}$$

s, m_s دو عدد کوانتومی

برای ذرات اسپین $\frac{1}{2}$ (مثل الکترون‌ها): $s = \frac{1}{2}, \quad m_s = \pm \frac{1}{2}$

نمادگذاری

$$|s, m_s\rangle = \begin{cases} |\uparrow\rangle = |s = \frac{1}{2}, m_s = +\frac{1}{2}\rangle \\ |\downarrow\rangle = |s = \frac{1}{2}, m_s = -\frac{1}{2}\rangle \end{cases}$$

$S_z |\uparrow\rangle = +\frac{\hbar}{2} |\uparrow\rangle$

$S_z |\downarrow\rangle = -\frac{\hbar}{2} |\downarrow\rangle$

$$\boxed{\begin{aligned} &S_x |\uparrow\rangle = ? \quad S_x |\downarrow\rangle = ? \\ &S_y |\uparrow\rangle = ? \quad S_y |\downarrow\rangle = ? \end{aligned}}$$

$S^2 |\uparrow\rangle = \hbar^2 (\frac{1}{2})(\frac{1}{2}+1) |\uparrow\rangle = \frac{3\hbar^2}{4} |\uparrow\rangle$

$S^2 |\downarrow\rangle = \frac{3\hbar^2}{4} |\downarrow\rangle$

جبر $SU(2)$ $\quad [S_i, S_j] = i\epsilon_{ijk} S_k$

خاصیت راست‌بهنجارش Orthonormality

$\langle \uparrow | \uparrow \rangle = 1$

$\langle \downarrow | \downarrow \rangle = 1$

$\langle \uparrow | \downarrow \rangle = 0$

$\langle \downarrow | \uparrow \rangle = 0$

حالت‌های $|\uparrow\rangle$ و $|\downarrow\rangle$ به یک نرم شده‌اند و بر هم عمود اند

عملگرهای بالابر و پایین‌بر:

$S_\pm = S_x \pm i S_y$

شبیه

$L_\pm = L_x \pm i L_y$

$$\boxed{\begin{aligned} &L_\pm |l, m_l\rangle = \hbar \sqrt{l(l+1) - m_l(m_l \pm 1)} \; |l, m_l \pm 1\rangle \\ &S_\pm |s, m_s\rangle = \hbar \sqrt{s(s+1) - m_s(m_s \pm 1)} \; |s, m_s \pm 1\rangle \end{aligned}}$$

$S_+ |\uparrow\rangle = S_+ | s = \frac{1}{2}, m_s = +\frac{1}{2}\rangle =$

$= \hbar \cancel{\sqrt{\frac{1}{2}(\frac{1}{2}+1) - (\frac{1}{2})(\frac{1}{2}+1)}} \; |s = \frac{1}{2}, \underbrace{\frac{1}{2}+1}_{\text{نداریم}}\rangle$

$= 0$

نداریم

$= 0$

$$\Rightarrow \boxed{S_+ |\uparrow\rangle = 0}$$

$$S_+ |\downarrow\rangle = S_+ \left| s = \tfrac{1}{2}, m_s = -\tfrac{1}{2} \right\rangle$$

$$= \hbar \sqrt{\underbrace{\tfrac{1}{2}\left(\tfrac{1}{2}+1\right)}_{\frac{3}{4}} - \underbrace{\left(\tfrac{-1}{2}\right)\left(\tfrac{-1}{2}+1\right)}_{-\frac{1}{4}}} \left| s = \tfrac{1}{2}, \underbrace{\tfrac{-1}{2}+1}_{\frac{1}{2}} \right\rangle$$

$$= \hbar |\uparrow\rangle$$

$$\Rightarrow \boxed{S_+ |\downarrow\rangle = \hbar |\uparrow\rangle}$$

S_+ عملگر بالابرنده اسپین $\frac{1}{2}$

به همین ترتیب

$$\boxed{\begin{aligned} S_- |\downarrow\rangle &= 0 \\ S_- |\uparrow\rangle &= \hbar S_+ \end{aligned}}$$

حال با توجه به اینکه

$$\left.\begin{aligned} S_+ &= S_x + i S_y \\ S_- &= S_x - i S_y \end{aligned}\right\} \longrightarrow \begin{aligned} S_x &= \tfrac{1}{2}(S_+ + S_-) \\ S_y &= \tfrac{1}{2i}(S_+ - S_-) \end{aligned}$$

$$S_x |\uparrow\rangle = \tfrac{1}{2}(S_+ + S_-)|\uparrow\rangle = \tfrac{1}{2}(0 + \hbar)|\downarrow\rangle = \tfrac{\hbar}{2}|\downarrow\rangle$$
$$S_x |\downarrow\rangle = \tfrac{1}{2}(S_+ + S_-)|\downarrow\rangle = \tfrac{1}{2}(\hbar + 0)|\uparrow\rangle = \tfrac{\hbar}{2}|\uparrow\rangle$$
$$S_y |\uparrow\rangle = \tfrac{1}{2i}(S_+ - S_-)|\uparrow\rangle = \tfrac{1}{2i}(0 - \hbar)|\downarrow\rangle = +\tfrac{i\hbar}{2}|\downarrow\rangle$$
$$S_y |\downarrow\rangle = \tfrac{1}{2i}(S_+ - S_-)|\downarrow\rangle = \tfrac{1}{2i}(\hbar + 0)|\uparrow\rangle = -\tfrac{i\hbar}{2}|\uparrow\rangle$$

$$S_z |\uparrow\rangle = \tfrac{\hbar}{2}|\uparrow\rangle$$
$$S_z |\downarrow\rangle = -\tfrac{\hbar}{2}|\downarrow\rangle$$

نمایش ماتریسی $\vec{S}$

$$S_i = \begin{pmatrix} \langle\uparrow|S_i|\uparrow\rangle & \langle\uparrow|S_i|\downarrow\rangle \\ \langle\downarrow|S_i|\uparrow\rangle & \langle\downarrow|S_i|\downarrow\rangle \end{pmatrix}$$

$$S_x = \begin{pmatrix} \langle\uparrow|S_x|\uparrow\rangle & \langle\uparrow|S_x|\downarrow\rangle \\ \langle\downarrow|S_x|\uparrow\rangle & \langle\downarrow|S_x|\downarrow\rangle \end{pmatrix} = \begin{pmatrix} \tfrac{\hbar}{2}\langle\uparrow|\downarrow\rangle & \tfrac{\hbar}{2}\langle\uparrow|\uparrow\rangle \\ \tfrac{\hbar}{2}\langle\downarrow|\downarrow\rangle & \tfrac{\hbar}{2}\langle\downarrow|\uparrow\rangle \end{pmatrix}$$

$$= \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \frac{\hbar}{2}\sigma_x$$

تعریف

$$\boxed{S_x = \tfrac{\hbar}{2}\sigma_x \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

به همین ترتیب

$$S_y = \tfrac{\hbar}{2}\sigma_y \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ \cdot & \cdot \end{pmatrix}$$

به همین ترتیب

$$\boxed{\begin{array}{ll} S_y = \frac{\hbar}{2}\sigma_y & \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ S_z = \frac{\hbar}{2}\sigma_z & \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array}}$$

$$S_+ = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \equiv \hbar\sigma_+ \qquad \sigma_+ = \sigma_x + i\sigma_y$$

$$S_- = \hbar \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \equiv \hbar\sigma_- \qquad \sigma_- = \sigma_x - i\sigma_y$$

$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$ $\quad \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ $\quad$ Pauli Matrizen $\quad SU(2)$ جبر

$[\sigma_x, \sigma_y] = 2i\sigma_z$ $\quad$ or

1) $[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$

2) $\sigma_i^2 = 1$

3) $\{\sigma_i, \sigma_j\} = 2\delta_{ij}\mathbb{1}_{2\times2}$

4) $\sigma_i\sigma_j = \frac{1}{2}\left(\{\sigma_i,\sigma_j\} + [\sigma_i,\sigma_j]\right) = \frac{1}{2}\left(2\delta_{ij} + 2i\epsilon_{ijk}\sigma_k\right)$

$\quad = \delta_{ij} + i\epsilon_{ijk}\sigma_k$

5) $Tr\,\sigma_i = 0 \quad i = 1,2,3$

6) $\det\sigma_i = -1 \quad i = 1,2,3$

7) $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \sigma_i\sigma_j a_i b_j = (\delta_{ij} + i\epsilon_{ijk}\sigma_k)a_i b_j$

$\quad = \mathbb{1}_{2\times2}\,\vec{a}\cdot\vec{b} + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

اسپینورها : ویژه‌حالت‌های ماتریس $\vec{S}$ (نمایش برداری اسپین بالا $|\uparrow\rangle$ و اسپین پایین $|\downarrow\rangle$)

a) $S_z = \frac{\hbar}{2}\sigma_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$S_z\begin{pmatrix} u \\ v \end{pmatrix} = +\frac{\hbar}{2}\begin{pmatrix} u \\ v \end{pmatrix} \curvearrowright \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} u \\ v \end{pmatrix}$

$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ بدون نقض کلیت و به دلیل نرمالیزاسیون $\leftarrow \begin{pmatrix} u \\ -v \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$

$\hookrightarrow +\frac{\hbar}{2}$ ویژه‌حالت عملگر S_z به ازای ویژه‌مقدار $\quad |\uparrow\rangle = \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |s=\frac{1}{2}, m_s=+\frac{1}{2}\rangle$

b) $S_z\begin{pmatrix} u \\ v \end{pmatrix} = -\frac{\hbar}{2}\begin{pmatrix} u \\ v \end{pmatrix} \curvearrowright \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = -\frac{\hbar}{2}\begin{pmatrix} u \\ v \end{pmatrix}$

$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ بدون نقض کلیت $\leftarrow \begin{pmatrix} u \\ -v \end{pmatrix} = \begin{pmatrix} -u \\ -v \end{pmatrix}$

$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ بدون نقض کلیت $\leftarrow$ $\begin{pmatrix} u \\ -v \end{pmatrix} = \begin{pmatrix} -u \\ -v \end{pmatrix}$

$\hookrightarrow -\frac{\hbar}{2}$ ویژه‌حالت عملگر S_z به ازای ویژه‌مقدار $|\downarrow\rangle \equiv \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |s=\frac{1}{2}, m_s=-\frac{1}{2}\rangle$

Spin up $\quad |\uparrow\rangle = |s=\frac{1}{2}, m_s=+\frac{1}{2}\rangle \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \chi_+$

Spin down $\quad |\downarrow\rangle = |s=\frac{1}{2}, m_s=-\frac{1}{2}\rangle \Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \chi_-$

دیراک

$$|\uparrow\rangle = |\tfrac{1}{2}, +\tfrac{1}{2}\rangle$$
$$|\downarrow\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$