The mass Spectrum :

$$D_\mu \varphi_i = \partial_\mu \varphi_i - ig\, A_\mu^a \tau^a \varphi_i - \frac{ig'}{2} B_\mu \varphi_i$$

$A_\mu^a \quad a = 1,2,3 \qquad SU(2)$ gauge bosons

$B_\mu \qquad U(1)$ gauge boson.

g, g' are coupling constants corresponding to $SU(2)$ and $U(1)$, respectively.

$$\langle \vec\varphi \rangle = \vec\varphi_0 = \frac{1}{\sqrt2}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \vec\varphi = \vec\varphi' + \vec\varphi_0$$

We consider :

$$\left| -ig\, A_\mu^a \tau^a \vec\varphi^0 - \frac{ig'}{2} B_\mu \vec\varphi^0 \right|^2$$

ناشی از shift

$$\Delta\mathcal{L} = (i)(-i)\left(\frac{1}{\sqrt2}\right)^2 (0 \ \ v)\left(g A_\mu^a \tau^a + \frac{g'}{2} B_\mu\right)\left(g A^{\mu, b}\tau^b + \frac{g'}{2} B^\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{2}\left(\frac{1}{4}\right)(0 \ \ v)\left(g A_\mu^a \sigma^a + g' B_\mu\right)\left(g A^{\mu, b}\sigma^b + g' B^\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Delta\mathcal{L} = \frac{1}{8}(0 \ \ v)\begin{pmatrix} g A_\mu^{(3)} + g' B_\mu & ig A_\mu^{(2)} + g A_\mu^{(1)} \\ -ig A_\mu^{(2)} + g A_\mu^{(1)} & -g A_\mu^{(3)} + g' B_\mu \end{pmatrix}$$

$$\times \begin{pmatrix} g A^{\mu(3)} + g' B^\mu & ig A^{\mu(2)} + g A^{\mu(1)} \\ -ig A^{\mu(2)} + g A^{\mu(1)} & -g A^{\mu(3)} + g' B^\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{v^2}{8}\left(-ig A_\mu^{(2)} + g A_\mu^{(1)} \qquad -g A_\mu^{(3)} + g' B_\mu\right)\begin{pmatrix} ig A^{\mu(2)} + g A^{\mu(1)} \\ -g A^{\mu(3)} + g' B^\mu \end{pmatrix}$$

$$= \boxed{\frac{v^2}{8}\left\{ g^2 \left[(A_\mu^{(1)})^2 + (A_\mu^{(2)})^2\right] + \left[-g A_\mu^{(3)} + g' B_\mu\right]^2 \right\}} \quad \star$$

Let us now define :

$$W_\mu^\pm = \frac{1}{\sqrt2}\left(A_\mu^{(1)} \mp i A_\mu^{(2)}\right)$$

$$Z_\mu = \frac{g A_\mu^{(3)} - g' B_\mu}{\sqrt{g^2 + g'^2}}$$

(a) $$m_{W^\pm}^2 \left(W_\mu^\pm\right)^\dagger \left(W^{\mu\pm}\right) = \frac{1}{2} m_{W^\pm}^2 \left(A_\mu^{(1)} \mp i A_\mu^{(2)}\right)\left(A^{\mu(1)} \pm i A^{\mu(2)}\right)$$
$$= \frac{1}{2} m_{W^\pm}^2 \left(\left(A_\mu^{(1)}\right)^2 + \left(A_\mu^{(2)}\right)^2\right)$$

$$\boxed{m_{W^\pm}^2 = \frac{g^2 v^2}{4}} \qquad \boxed{m_{W^\pm} = \frac{gv}{2}}$$

b) $$\frac{1}{2} m_{Z^0}^2 Z_\mu Z^\mu = \frac{1}{2} m_{Z^0}^2 \frac{\left(gA_\mu^{(3)} - g' B_\mu\right)\left(gA^{\mu(3)} - g' B^\mu\right)}{g^2 + g'^2}$$

$$\Rightarrow \quad m_{Z^0}^2 = \frac{v^2}{4}\left(g^2 + g'^2\right) \;\curvearrowright\; \boxed{m_{Z^0} = \frac{v}{2}\sqrt{g^2 + g'^2}}$$

We say $W_\mu^\pm$ & Z_μ are fields corresponding to these mass Eigenstates.

سؤال: حال باید تعریف فوتون روشن شود.

$$D_\mu = \partial_\mu - ig A_\mu^a T^a - ig' Y B_\mu$$

T^a are in a general representation of SU(2)

Y: U(1) hypercharge

Let us introduce:

a) $T^\pm = T^{(1)} \pm i T^{(2)}$ or $T^\pm = \frac{1}{2}\left(\sigma^1 \pm i\sigma^2\right) = \sigma^\pm$ & $T^3 = \frac{1}{2}\sigma^3$

$\underbrace{\text{Def:}}_{\text{ضریب برهمکنش الکترودینامیک کلاسیک}}$ $$e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}}$$

مشتق هموردا را می‌توان به این شکل نوشت.

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - \frac{i}{\sqrt{g^2 + g'^2}} Z_\mu^0 \left(g^2 T^3 - g'^2 Y\right)$$
$$- \frac{igg'}{\sqrt{g^2 + g'^2}} A_\mu \left(T^3 + Y\right) \qquad **$$

$$A_\mu = \frac{g' A_\mu^{(3)} + g B_\mu}{\sqrt{g^2 + g'^2}}$$

فوتون

A_μ is massless (combination of $A_\mu^{(3)}$ and B_μ)

جمله آخر در **

$$\underbrace{\frac{-igg'}{\sqrt{g^2 + g'^2}}}_{-ie} \underbrace{\left(T^3 + Y\right)}_{Q = -1} A_\mu = -ieQ A_\mu = ie A_\mu$$

Q is the electric charge spectrum number.

Mixing Angle:

Angles that appear when we change the basis from $(A_\mu^{(3)}; B_\mu) \to (Z_\mu^0, A_\mu)$

$$\begin{pmatrix} Z_\mu^0 \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_\omega & -\sin\theta_\omega \\ +\sin\theta_\omega & \cos\theta_\omega \end{pmatrix} \begin{pmatrix} A_\mu^{(3)} \\ B_\mu \end{pmatrix}$$

$$Z_\mu^0 = A_\mu^{(3)} \cos\theta_\omega - B_\mu \sin\theta_\omega = \frac{g A_\mu^{(3)} - g' B_\mu}{\sqrt{g^2+g'^2}}$$

$$\boxed{\cos\theta_\omega = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_\omega = \frac{g'}{\sqrt{g^2+g'^2}}}$$

$$A_\mu = A_\mu^{(3)} \sin\theta_\omega + B_\mu \cos\theta_\omega = \frac{g' A_\mu^{(3)} + g B_\mu}{\sqrt{g^2+g'^2}}$$

The above choice of $\sin$ & $\cos\theta_\omega$ is consistent.

1) $$e = \frac{g g'}{\sqrt{g^2+g'^2}} = g' \cos\theta_\omega = g \sin\theta_\omega \to \boxed{\frac{e}{\sin\theta_\omega} = g}$$

2) Using $Q = T^3 + Y \to Y = Q - T^3$

در فرمول Z_μ^0 در ** فرمول به توجه کنیم

$$g^2 T^3 - g'^2 Y \underset{Q = T^3 + Y}{=} g^2 T^3 - g'^2 (Q - T^3) = (g^2 + g'^2) T^3 - g'^2 Q$$

دو طرف را به $\sqrt{g^2+g'^2}$ تقسیم می‌کنیم:

$$\frac{g^2 T^3 - g'^2 Y}{\sqrt{g^2+g'^2}} = \sqrt{g^2+g'^2}\, T^3 - \frac{g'^2}{\sqrt{g^2+g'^2}} Q = \frac{g}{\cos\theta_\omega} (T^3 - Q \sin^2\theta_\omega)$$

$$\sqrt{g^2+g'^2} = \frac{g}{\cos\theta_\omega}$$

$$\frac{g'^2}{\sqrt{g^2+g'^2}} = \sin^2\theta_\omega \frac{g}{\cos\theta_\omega}$$

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} (W_\mu^+ T^+ + W_\mu^- T^-) - \frac{i}{\sqrt{g^2+g'^2}} Z_\mu^0 (g^2 T^3 - g'^2 Y) - \frac{i g g'}{\sqrt{g^2+g'^2}} A_\mu (T^3 + Y)$$

$$\boxed{D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} (W_\mu^+ T^+ + W_\mu^- T^-) - i Z_\mu^0 \frac{g}{\cos\theta_\omega} (T^3 - Q \sin^2\theta_\omega) - ie Q A_\mu}$$

Results :

$$g = \frac{e}{\sin\theta_\omega}$$

برهمکنش بوزون‌های جرم‌دار $W_\mu^\pm$ و Z_μ^0 با دو پارامتر توصیف می‌شود:

$$\begin{cases} \theta_\omega & \text{Weinberg Angle} \\ e & \text{Well measured election charge} \end{cases}$$

- $m_{\omega^\pm} = \frac{gv}{2} \qquad m_{Z^0} = \sqrt{g^2+g'^2}\;\frac{v}{2}$
- $\frac{m_{\omega^\pm}}{m_{Z^0}} = \frac{g}{\sqrt{g^2+g'^2}} = \cos\theta_\omega \rightarrow \theta_\omega = \arccos\frac{g}{\sqrt{g^2+g'^2}} = \arccos\frac{m_{\omega^\pm}}{m_{Z^0}}$

جرم $W_\mu^\pm$ و Z_μ^0 را می‌توان به طور مستقل در آزمایشگاه بدست آورد. و به این ترتیب θ_ω را تعیین نمود.

Coupling to fermions:

Once the quantum number of fermions are fixed:

1) We know in which representation they are
2) What is $Q = T^3 + Y$

$\rightarrow$ The covariant derivative D_μ determines uniquely the coupling of $\omega_\mu^\pm$ & Z_μ^0 to fermions:

Fact: W bosons couple only to left_handed quarks & leptons.

In chiral basis:

left handed	Right handed
$T^3 = \pm\frac{1}{2}$ $Q = T^3 + Y$	$T^3 = 0$ $Q = Y =$ electric charge
$E_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} = \begin{pmatrix} \frac{1}{2} - \frac{1}{2} = 0 \\ -\frac{1}{2} - \frac{1}{2} = -1 \end{pmatrix}$ left-handed leptons	$\nu_R \quad Q = Y = 0$ $e_R^- \quad Q = Y = -1$
Quark multiplet $Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \frac{1}{6} = \frac{2}{3} \\ -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3} \end{pmatrix}$	$u_R \quad Q = \frac{2}{3} = Y$ $d_R \quad Q = -\frac{1}{3} = Y$

$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$T^3 = \frac{1}{2}\sigma^3 =$

Eigenvalue $T^3 \rightarrow \pm\frac{1}{3}$

$$\mathcal{L} = \bar{E}_L (i\not{D}) E_L + \bar{Q}_L (i\not{D}) Q_L + \bar{e}_R (i\not{D}) e_R + \bar{u}_R (i\not{D}) u_R + \bar{d}_R (i\not{D}) d_R$$

پس

$$\bar{Q}_L (i\not{D}) Q_L = \bar{Q}_L \, i\gamma^\mu \left(\partial_\mu - ig A_\mu^a \tau^a - Y i g' B_\mu\right) Q_L$$

$$Y = \frac{1}{6}$$

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} \left(W_\mu^+ T^+ + W_\mu^- T^-\right) - \frac{ig}{\cos\theta_\omega} Z_\mu^0 \left(T^3 - Q \sin^2\theta_\omega\right) - ieQA_\mu$$

اگر $\mathcal{L}$ را باز کنیم :

$$\mathcal{L} = \bar{E}_L (i\not{\partial}) E_L + \bar{e}_R (i\not{\partial}) e_R + \bar{Q}_L (i\not{\partial}) Q_L + \bar{u}_R (i\not{\partial}) u_R + \bar{d}_R (i\not{\partial}) d_R$$
$$+ g \left(W_\mu^+ J^\mu_{W+} + W_\mu^- J^\mu_{W-} + Z_\mu^0 J^\mu_{Z^0}\right) + e A_\mu J^\mu_{EM}$$

$$J^\mu_{W+} = \frac{1}{\sqrt{2}} \left(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L\right)$$

$$J^\mu_{W-} = \frac{1}{\sqrt{2}} \left(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L\right)$$

$$J^\mu_{EM} = \bar{e} \gamma^\mu (-1) e + \bar{u} \gamma^\mu \left(\tfrac{2}{3}\right) u + \bar{d} \gamma^\mu \left(\tfrac{-1}{3}\right) d$$

$$J^\mu_Z = \frac{1}{\cos\theta_\omega} \Big(\bar{\nu}_L \gamma^\mu \tfrac{1}{2} \nu_L + \bar{e}_L \gamma^\mu \left(\tfrac{-1}{2} + \sin^2\theta_\omega\right) e_L + \bar{e}_R \gamma^\mu \sin^2\theta_\omega e_R$$
$$+ \bar{u}_L \gamma^\mu \left(\tfrac{1}{2} - \tfrac{2}{3} \sin^2\theta_\omega\right) u_L + \bar{d}_L \gamma^\mu \left(\tfrac{-1}{2} + \tfrac{1}{3} \sin^2\theta_\omega\right) d_L$$
$$+ \bar{u}_R \gamma^\mu \left(\tfrac{-2}{3}\right) \sin^2\theta_\omega u_R + \bar{d}_R \gamma^\mu \left(\tfrac{1}{3}\right) \sin^2\theta_\omega d_R$$

$$T^+ = T^1 - iT^2 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$T^- = T^1 + iT^2 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

مثال: نوع لپتونی

a) $$\frac{-i}{\sqrt{2}} \left(\bar{\nu}_L \; \bar{e}_L\right) W_\mu^+ T^+ \gamma^\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$
$$= \frac{-i}{\sqrt{2}} \left(\bar{\nu}_L \; \bar{e}_L\right) \gamma^\mu \begin{pmatrix} e_L \\ 0 \end{pmatrix} = \frac{-i}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L$$

نوع کوارکی

b) $$\frac{-i}{\sqrt{2}} \left(\bar{u}_L \; \bar{d}_L\right) W_\mu^+ T^+ \gamma^\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$
$$= \frac{-i}{\sqrt{2}} \left(\bar{u}_L \; \bar{d}_L\right) \gamma^\mu \begin{pmatrix} d_L \\ 0 \end{pmatrix} = \frac{-i}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L$$

به این ترتیب جفت‌شدگی W_μ^+ به راست‌گردها صفر است.

Fermion Mass Term:

- A coupling to an arbitrary scalar field must exist;

$$\langle \varphi \rangle = \varphi_0 \neq 0 \longrightarrow \text{ SSB of the corresponding global symmetry.}$$

Aim: We have to give a mass to: $W_\mu^\pm$, Z_μ, Q_L, E_L, u_R, d_R, e_R

We want that this scalar particle is in a spinor representation:

$$\vec{\varphi}_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

1) Ansatz: $\Delta \mathcal{L}_e = -\lambda_e \left(\bar{E}_L \cdot \vec{\varphi}^0 \right) e_R + h.c.$

$$\bar{E}_L = \left(\bar{\nu}_L \;\; \bar{e}_L \right), \quad e_R, \quad \vec{\varphi}^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Delta \mathcal{L}_e = -\frac{1}{2} \lambda_e v \; \bar{e}_L e_R + h.c.$$

$$-m\bar{\psi}\psi = -m \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right)$$

$$m_e = \frac{v \lambda_e}{\sqrt{2}} \qquad \text{Electron mass.}$$

$$m_{Z^0} = \frac{v}{2} \sqrt{g^2 + g'^2}$$

$$m_{W^\pm} = \frac{g v}{2}$$

$$\frac{m_e}{m_W} \simeq 6 \times 10^{-6} = \frac{\frac{\lambda_e v}{\sqrt{2}}}{\frac{g v}{2}} = \sqrt{2} \, \frac{\lambda_e}{g}$$

$e = 0.5\,\text{MeV}$
$\mu \sim 105.6\,\text{MeV}$
$\tau \sim 1.7\,\text{GeV}$
$W^\pm \sim 80.2\,\text{GeV}$
$Z^0 \sim 91.19\,\text{GeV}$
$p \sim 938.3\,\text{MeV}$
$n \sim 939.6\,\text{MeV}$
$\pi^\pm \sim 139.6\,\text{MeV}$
$\pi^0 \sim 135\,\text{MeV}$

$\alpha_s(m_Z) \sim 0.12$
$\sin^2\theta_W = 0.23$

The GWS theory allows the electron to be very light, but it cannot explain why the electron is so light compared to $W_\mu^\pm$ bosons $\longrightarrow$ Hierarchy Problem.

2) $\Delta \mathcal{L}_q = -\lambda_d \, \bar{Q}_L \cdot \vec{\varphi}^0 d_R - \lambda_u \, \epsilon^{ab} \, \bar{Q}_L^a \, \vec{\varphi}_0^{+b} \, u_R + h.c$

جمله اول.

$$-\lambda_d \bar{Q}_L \cdot \vec{\varphi}^0 d_R = -\lambda_d \left(\bar{u}_L \;\; \bar{d}_L \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}} d_R = -\frac{\lambda_d v}{\sqrt{2}} \bar{d}_L d_R$$

$$m_d = \frac{\lambda_d v}{\sqrt{2}}$$

جمله دوم

$$\lambda_u \epsilon^{ab} \bar{Q}_L^a \cdot \vec{\varphi}_0^{+b} u_R = \lambda_u \epsilon^{12} \bar{Q}_L^{(1)} \vec{\varphi}_0^{(2)+} u_R + \epsilon^{21} \bar{Q}_L^{(2)} \vec{\varphi}_0^{(1)+} u_R \lambda_u$$

$$= \lambda_u \bar{u}_L v u_R \frac{1}{\sqrt{2}} = \frac{v \lambda_u}{\sqrt{2}} \bar{u}_L u_R$$

$$m_u = \frac{v \lambda_u}{\sqrt{2}}$$