Background Field Perturbation :

idea: $\quad Z = \int \mathcal{D}\varphi \; e^{i\int \mathcal{L} d^4x} \qquad \varphi = \underbrace{\varphi_c}_{\text{classical Field}} + \overbrace{\eta}^{\text{quantum fluctuation}}$ $\qquad$ یادآوری:

a) Background Field:

$$Z = \int \mathcal{D}\eta \; e^{i\int d^4x \, (\mathcal{L}_{cl} + \mathcal{L}(\eta))} = \int \mathcal{D}\eta \exp\Big(i\int \mathcal{L}(\varphi_{cl}) d^4x$$

$$+ \; i\int d^4x\, d^4y \, \frac{\delta \mathcal{L}(\varphi_c)}{\partial \eta(y)} \eta(y) + \frac{i}{2}\int d^4x\, d^4y\, d^4z \; \eta(y)\eta(z) \frac{\delta^2 \mathcal{L}(\varphi_{cl})}{\delta\eta(y)\delta\eta(z)} + \cdots \Big)$$

$$\boxed{e^{i\Gamma[\varphi_c]} = e^{i\int \mathcal{L}[\varphi_c] d^4x} \underbrace{\det\left(- \frac{\delta^2 \mathcal{L}[\varphi_c]}{\delta\varphi(x)\delta\varphi(y)} \right)}_{\text{one-loop correction}} + \cdots}$$

b) Functional determinant

A_μ is a background field: $\int \mathcal{D}\psi \mathcal{D}\bar\psi \exp\left(i\int d^4x \; \bar\psi (i\not{D} - m)\psi \right) = \det(i\not{D} - m)$

$$\det(i\not{D} - m) = \det(i\not\partial - g\not{A} - m) = \det(i\not\partial - m) \det\left(1 - \frac{g\not{A}}{i\not\partial - m}\right)$$

$$\sim \det\left(1 - \frac{i}{(i\not\partial - m)} (-ig\not{A}) \right)$$

$$\det A = \exp(\log \det A)$$

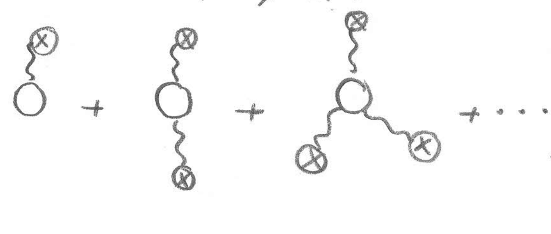
$$= \exp(\mathrm{Tr} \log A)$$

$$\log\det(i\not{D} - m) \sim \log\det\left(1 - \frac{i}{i\not\partial - m}(-ig\not{A})\right) = \mathrm{Tr}\log\left(1 - \frac{i}{i\not\partial - m}(-ig\not{A})\right)$$

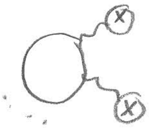
$$= \mathrm{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{-i}{i\not\partial - m} \right)^n (-ig\not{A})^n$$

$$= - \mathrm{Tr} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{i}{i\not\partial - m} \right)^n (-ig\not{A})^n$$

$$\det(i\not{D} - m) = \exp\left[\sum_{n=1}^{\infty} \left(\frac{-1}{n}\right) \mathrm{Tr}\left\{ \left(\frac{i}{i\not\partial - m} \right)^n (-ig\not{A})^n \right\} \right]$$

$= \exp \Big($  $+ \cdots \Big)$

n-th diagram in the series:



$$\boxed{\begin{aligned} &= -\frac{1}{n} \mathrm{Tr} \left(\frac{i}{i\not\partial - m} (-ig\not{A}) \right)^n = \\ &= -\frac{1}{n} \int d^dx_1 \ldots d^dx_n \; \mathrm{tr}\Big\{ (-ig\not{A}(x_1)) S_F(x_1 - x_2) (-ig\not{A}(x_2)) \ldots \\ &\qquad \times (-ig\not{A}(x_n)) S_F(x_n - x_1) \Big\} \end{aligned}}$$

β- function of QCD (Non-Abelian Gauge theory)

$$A_\mu \rightarrow A_\mu^{cl} + \mathcal{A}_\mu$$

A_μ^{cl}: Background field

$\mathcal{A}_\mu$: Quantum fluctuation

idea: Classi action $\longrightarrow$ Effective Action

$$S_{cl} = \int d^dx \; \mathcal{L}_{cl}[A_\mu, \psi, \bar\psi, c, \bar c] \longrightarrow \Gamma[A_\mu^{cl}] = \int d^dx \left\{ \mathcal{L}[A_\mu^{cl}] + \delta\mathcal{L}_{CT} + \text{loop corr.} \right\}$$

Loop corrections will include finite + divergent parts.

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu,a} + \bar\psi (i\not{D})\psi + \mathcal{L}_{g.f.} + \mathcal{L}_{ghost} + \delta\mathcal{L}_{CT}$$

$$A_\mu \longrightarrow A_\mu^{cl} + \mathcal{A}_\mu$$

$$\Longrightarrow \frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu,a} \xrightarrow{\quad * \quad} \frac{1}{4g_{eff}^2} F_{\mu\nu}^a[A_\mu^{cl}] F^{\mu\nu,a}[A_\mu^{cl}]$$

$$F_{\mu\nu}^a[A_\mu^{cl}] = \partial_\mu A_\nu^{cl,a} - \partial_\nu A_\mu^{cl,a} + f^{abc} A_\mu^{b,cl} A_\nu^{c,cl}$$

میتوان نشان داد : $\dfrac{1}{g_{eff}^2} = \dfrac{1}{g^2} + \text{corrections} \longrightarrow \beta\text{- function}$ □

* **Keypoint:** We perform an integration over the fluctuating fields $\mathcal{A}_\mu^a$ $\longrightarrow$ and arrive at a relation between g_{eff} & g

$\rightarrow$ Running coupling

1st Step: How does $\mathcal{L}$ changes after $A_\mu \longrightarrow A_\mu^{cl} + \mathcal{A}_\mu$

1) $$\bar\psi(i\not{D})\psi = \bar\psi\left(i\not\partial + \gamma^\mu (A_\mu^a + \mathcal{A}_\mu^a)t^a\right)\psi$$
$$= \boxed{\bar\psi(i\not\partial + \gamma^\mu A_\mu^a t^a)\psi + \bar\psi \mathcal{A}_\mu^a \gamma^\mu t^a \psi}$$

$$iD_\mu[A_\mu] \equiv i\partial_\mu + A_\mu^a t^a$$

$$= \bar\psi(i\not{D}[A_\mu])\psi + \bar\psi \mathcal{A}_\mu^a \gamma^\mu t^a \psi$$

2) $$F_{\mu\nu}^a = \partial_\mu(A_\nu^a + \mathcal{A}_\nu^a) - \partial_\nu(A_\mu^a + \mathcal{A}_\mu^a) + f^{abc}(A_\mu^b + \mathcal{A}_\mu^b)(A_\nu^c + \mathcal{A}_\nu^c)$$
$$= (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c) + (\partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a + f^{abc}\mathcal{A}_\mu^b \mathcal{A}_\nu^c)$$
$$+ \Big(f^{abc} A_\mu^b \mathcal{A}_\nu^c + f^{abc} \underbrace{\mathcal{A}_\mu^b A_\nu^c}_{b \leftrightarrow c}\Big) \qquad f^{acb} = -f^{abc}$$

$\longrightarrow$

$$F_{\mu\nu}{}^a[A_\mu+\mathcal{A}_\mu] = F_{\mu\nu}{}^a[A_\mu] + \left(\partial_\mu \mathcal{A}_\nu{}^a + f^{abc} A_\mu{}^b \mathcal{A}_\nu{}^c\right) - \left(\partial_\nu \mathcal{A}_\mu{}^a + f^{abc} A_\nu{}^b \mathcal{A}_\mu{}^c\right)$$
$$+ f^{abc} \mathcal{A}_\mu{}^b \mathcal{A}_\nu{}^c$$

در رابطه $(\partial_\mu \mathcal{A}_\nu{}^a + f^{abc} A_\mu{}^b \mathcal{A}_\nu{}^c)$ داریم

$$\partial_\mu \mathcal{A}_\nu{}^a + f^{abc} A_\mu{}^b \mathcal{A}_\nu{}^c = \left(\partial_\mu \delta^{ac} + f^{abc} A_\mu{}^b\right)\mathcal{A}_\nu{}^c = D_\mu[A]\,\mathcal{A}_\nu{}^c$$

به این ترتیب داریم:

$$\boxed{F^a_{\mu\nu}[A_\mu+\mathcal{A}_\mu] = F_{\mu\nu}{}^a[A_\mu] + D_\mu \mathcal{A}_\nu{}^a - D_\nu \mathcal{A}_\mu{}^a + f^{abc} \mathcal{A}_\mu{}^b \mathcal{A}_\nu{}^c}$$

به این ترتیب در $\mathcal{L}$ خواهیم داشت:

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}{}^a F^{\mu\nu,a} + \bar{\psi}(i\not{D})\psi$$
$$\mathcal{L}[A+\mathcal{A}] = -\frac{1}{4g^2}\left\{F_{\mu\nu}{}^a[A] + (D_\mu \mathcal{A}_\nu)^a - (D_\nu \mathcal{A}_\mu)^a + f^{abc}\mathcal{A}_\mu{}^b \mathcal{A}_\nu{}^c\right\}^2$$
$$+ \bar{\psi}\,(i\not{D}[A])\psi + \bar{\psi}\,(\mathcal{A}_\mu^a \gamma^\mu) t^a \psi$$

تمرین:

$$A_\mu{}^a \to \mathcal{A}_\mu{}^a + (D_\mu\alpha)^a + f^{abc}\mathcal{A}_\mu{}^b \alpha^c \quad \text{with} \quad (D_\mu\alpha) = \partial_\mu\alpha + f^{abc}\cancel{A_\mu{}^b}\alpha^c$$

می‌توان نشان داد که $\mathcal{L}[A+\mathcal{A}]$ تحت تبدیلات میدان پیمانه‌ای پس‌زمینه ناوردا است:

(background gauge transformation)

برای تعریف انتگرال روی گلوئون: ← gauge fixing ← روش Faddeev-Popov را در این فرمالیسم میدان
پس‌زمینه تکرار می‌کنیم:

یادآوری

$$\Delta_F[A^\alpha] = \det\left(\frac{\delta F^a[A]}{\delta\alpha_b}\right) = \det(-\partial_\mu D^\mu)_{ab}$$
$$\delta_\alpha A_\mu{}^a = -(D_\mu\alpha)^a$$
$$\det(-\partial_\mu D^\mu) = \int \mathcal{D}c\,\mathcal{D}\bar{c}\,\exp\left(i\int d^4x\; \bar{c}^a\,(\partial_\mu D^\mu)^{ab}\,c^b\right) \to \text{ghost term}$$
$$Z = N\int \mathcal{D}\alpha \int \mathcal{D}\omega \int \mathcal{D}A_\mu\;\Delta_F\;\delta(F^a[A]-\omega^a)\exp\left(\frac{-i}{2\xi}\int d^4x\,\omega^2 + iS_g\right)$$

$\longrightarrow$ gauge fixing term.

• در نظریه میدان در حضور میدان پس‌زمینه:

$$A_\mu{}^a \to A_\mu{}^a + D_\mu[A]\,\alpha^a + f^{abc}\mathcal{A}_\mu{}^b\alpha^c$$
$$\mathcal{A}_\mu{}^a \to \mathcal{A}_\mu{}^a + \partial_\mu\alpha^a + f^{abc}A_\mu{}^b\alpha^c + f^{abc}\mathcal{A}_\mu{}^b\alpha^c$$

قید پیمانه‌ای را به صورت زیر تعریف می‌کنیم

$$G^a[\mathcal{A}] = D^\mu[A]\,\mathcal{A}_\mu{}^a - \omega^a$$

به این ترتیب جملات g.f. و ghost تعیین می‌شوند:

$$\Delta_F\,[\mathcal{A}_\mu{}^a] = \det\left(\frac{\delta G^a[\mathcal{A}_\mu{}^a]}{\delta\alpha^d}\right)$$

$$G^a[\overset{a}{\mathcal{A}}] = D^\mu\left(\mathcal{A}_\mu{}^a + D_\mu[A]\,\alpha^a + f^{abc}\,\mathcal{A}_\mu{}^b\alpha^c\right)$$

$$= D^\mu\mathcal{A}_\mu{}^a + D^\mu(D_\mu\alpha^a) + f^{abc}\,\underbrace{D^\mu(\mathcal{A}_\mu{}^b\alpha^c)}_{=(D^\mu\mathcal{A}_\mu{}^b)\alpha^c + \mathcal{A}_\mu{}^b D^\mu\alpha^c}$$

$$\frac{\delta}{\delta\alpha^d}\,\overset{a}{G}[\mathcal{A}_\mu{}^a] = (D^\mu D_\mu)^{ad} + f^{abc}\,(D^\mu\mathcal{A}_\mu{}^b) + f^{abc}\,\mathcal{A}_\mu{}^b\,D^\mu$$

$$\boxed{\begin{aligned}\longrightarrow\ \mathcal{L}_{g.f.} &= \frac{-1}{2\xi}\left(G^a[\mathcal{A}]\right)^2 = \frac{-1}{2\xi}\left(D^\mu[A]\,\mathcal{A}_\mu{}^a\right)^2\\ \mathcal{L}_{ghost} &= -\,\bar{c}^a\,(D_\mu D^\mu)^{ad}\,c^d - f^{abc}c^a\,D^\mu\,(\mathcal{A}_\mu{}^b c^c)\end{aligned}}$$

به این ترتیب $\mathcal{L}$ کل عبارت خواهد بود از:

$$\mathcal{L}_{tot}\,(A_\mu + \mathcal{A}_\mu) = \frac{-1}{4g^2}\left(F_{\mu\nu}{}^a[A] + (D_\mu\mathcal{A}_\nu{}^a) - (D_\nu\mathcal{A}_\mu{}^a) + f^{abc}\mathcal{A}_\mu{}^b\mathcal{A}_\nu{}^c\right)^2$$

$$+\ \bar{\psi}\left(i\,\not{D}[A] + \not{\mathcal{A}}^a t^a\right)\psi$$

$$-\ \frac{1}{2g^2}\left(D_\mu\,\mathcal{A}^{\mu,a}\right)^2 \qquad \longleftarrow\ \xi = 1$$

$$-\ \bar{c}^a\,(D_\mu D^\mu)^{ad}\,c^d$$

$$-\ \bar{c}^a\,D^\mu\left(f^{abd}\,\mathcal{A}_\mu{}^b c^d\right)$$

بخصوص در جمله اول انواع و اقسام جمله‌ها در جمع کنش ظاهر می‌شوند، ما فقط به جمله‌هایی توجه داریم که در میدان‌های ψ، $\bar{\psi}$، c، $\bar{c}$ و $\mathcal{A}_\mu$ مربعی باشند (زیرا می‌توان از آنها انتگرال گاوسی گرفت و از روی کنش مؤثر به دست آمده تابع β را محاسبه کرد):

<u>Aim:</u> $\quad \mathcal{L}_{quadr} = \sum_{r,j} \varphi_{r,j}\,\Delta_{r,j}\,\varphi_{r,j}$

$$\Delta_{r,j} = -\,D^2[A] + 2\left(\tfrac{1}{2}\,F_{\rho\sigma}{}^b\,\mathcal{J}^{\rho\sigma}\right)t_r{}^b$$

for Ghosts: $\quad j=0 \quad \mathcal{J}^{\rho\sigma}=0 \quad$ for $s=0$ (spin 0 particle)

Gluons: $\quad j=1 \quad (\mathcal{J}^{\rho\sigma})_{\mu\nu} = i\left(\delta^\rho{}_\mu\,\delta^\sigma{}_\nu - \delta^\rho{}_\nu\,\delta^\sigma{}_\mu\right)$ for $s=1$ (spin 1 - particle)

Fermions: $\quad j=\frac{1}{2} \quad (\mathcal{J}^{\rho\sigma})_{\alpha\beta} = (\Sigma^{\rho\sigma})_{\alpha\beta} = \frac{i}{4}\,[\gamma^\rho,\gamma^\sigma]_{\alpha\beta}$ for $s=\frac{1}{2}$ (Spin $\frac{1}{2}$ - particle)

$r =$ fundamental representation for matter fields

$G =$ adjoint representation for gluons & ghosts.

2nd Step: One loop correction to the effective action:

هدف: بدست آوردن جملات مرتبه دوم میدانهای ψ، $\bar{\psi}$، c، $\bar{c}$، A_μ که باید روی آنها انتگرال گیری شود:

$$\mathcal{L}_{tot} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_{g.f.} + \mathcal{L}_{ghost}$$

a) $$\underbrace{\mathcal{L}_g}_{= \mathcal{L}_{gauge} + \mathcal{L}_{g.f}} = \frac{-1}{4g^2}\left(F_{\mu\nu}^{a}[A] + (D_\mu A_\nu)^a - (D_\nu A_\mu)^a + f^{abc} A_\mu^b A_\nu^c\right)^2 - \frac{1}{2g^2}\left(D_\mu A^{\mu,a}\right)^2$$

جمله پس زمینه در ادامه آن را در نظر نمی‌گیریم، در انتها مجدداً به آن برمی‌گردیم.

$$\rightarrow \mathcal{L}_A = \frac{-1}{2g^2}\left\{\frac{1}{2}\left[D_\mu A_\nu^a - D_\nu A_\mu^a\right]^2 + F_{\mu\nu}^{a}[A] f^{abc} A_\mu^b A_\nu^c - \left(D_\mu [A] A^{\mu,a}\right)^2\right\} + O(A^4) \qquad (1)$$

b) $$\mathcal{L}_\psi = \bar{\psi}\,(i \not{D}[A])\psi + \text{terms linear in } A$$

c) $$\mathcal{L}_c = \bar{c}^a (-D^2)^{ad} c^d + \text{terms linear in } A$$

در ادامه می‌توان $\mathcal{L}_A$ ، $\mathcal{L}_\psi$ را ساده‌تر نوشت:

(a) نشان می‌دهیم:
$$\mathcal{L}_A = \frac{-1}{2g^2}\left(A_\mu^a\left[(-D^2)^{ac} g^{\mu\nu} - 2 f^{abc} F^{\mu\nu,b}\right] A_\nu^c\right)$$

اثبات:

(جمله اول a) (1)
$$\frac{1}{2}\left(D_\mu A_\nu^a - D_\nu A_\mu^a\right)^2 = \frac{1}{2}\left(D_\mu A_\nu^a D^\mu A^{\nu,a} - D_\mu A_\nu^a \widetilde{D^\nu A^{\mu,a}} - D_\nu A_\mu^a D^\mu A^{\nu,a} + D_\nu A_\mu^a D^\nu A^{\mu,a}\right)$$

$$* = D_\mu A_\nu^a D^\mu A^{\nu,a} - D_\mu A_\nu^a D^\nu A^{\mu,a}$$

(a1) جمله اول *
$$\int d^4x \left(D_\mu A_\nu^a D^\mu A^{\nu,a}\right) = \int d^4x \left(\partial_\mu A_\nu^a + f^{abc} A_\mu^b A_\nu^c\right) D^\mu A^{\nu,a}$$

$$= \int d^4x \left\{A_\nu^a \left(-\partial_\mu D^\mu A^{\nu,a}\right) + f^{abc} A_\nu^c \quad A_\mu^b D^\mu A^{\nu,a}\right\}$$

$c \to a \quad b \to b \quad a \to c \quad f^{abc} = -f^{cba}$

$$= \int d^4x \left\{A_\nu^a \left(-\partial_\mu D^\mu \delta^{ac} - f^{abc} \quad A_\mu^b D^\mu\right) A^{\nu,c}\right\}$$

$$= \int d^4x \; A_\nu^a \left\{-\underbrace{\left(\partial_\mu \delta^{ac} + f^{abc} \quad A_\mu^b\right)}_{= (D_\mu)^{ac}} D^\mu A^{\nu,c}\right.$$

$$= \int d^4x \quad A_\nu^a \quad \left(-D_\mu D^\mu\right)^{ac} A^{\nu,c}$$

(a2 جمله دوم) * $\quad - \int d^4x \; D_\mu A_\nu^{\ a} D^\nu A^{\mu,a} = \int d^4x \; \underline{A_\nu^{\ a} (D_\mu D^\nu)^{ac} A^{\mu,c}}$ بهمین ترتیب

(b جمله سوم) (1) $\quad \int d^4x \; (D_\mu A^{\mu,a})^2 = - \int d^4x \; A^{\mu,a} (D_\mu D_\nu)^{ab} A^{\nu,b}$

جملات بالا باید در $-\frac{1}{2g^2}$ ضرب شوند

$$(1) \Rightarrow \mathcal{L}_A = \frac{-1}{2g^2} \left\{ A_\mu^{\ a} [-D^2 g^{\mu\nu} + \underbrace{D^\nu D^\mu - D^\mu D^\nu}_{= -[D^\mu, D^\nu]^{ac} = -f^{abc} F_{\mu\nu}^{\ b}}]^{ac} A_\nu^{\ c} + \underbrace{f^{abc} A_\mu^{\ b} F^{\mu\nu,a} A_\nu^{\ c}}_{*} \right.$$

$$[D_\mu, D_\nu] = -i F_{\mu\nu}^{\ a} t^a$$

$$[D_\mu, D_\nu]^{ac} = -i F_{\mu\nu}^{\ b} (t^b)^{ac} = -i F_{\mu\nu}^{\ b} (i f^{abc}) = f^{abc} F_{\mu\nu}^{\ b}$$

$$* \quad f^{abc} A_\mu^{\ b} F^{\mu\nu,a} A_\nu^{\ c} \underset{a \leftrightarrow b}{=\!=} f^{bac} A_\mu^{\ a} F^{\mu\nu,b} A_\nu^{\ c} = - f^{abc} A_\mu^{\ a} F^{\mu\nu,b} A_\nu^{\ c}$$

$$\rightarrow \boxed{\mathcal{L}_A = \frac{-1}{2g^2} \left\{ A_\mu^{\ a} \left[(-D^2)^{ac} g^{\mu\nu} - 2 f^{abc} F^{\mu\nu,b} \right] \right\} A_\nu^{\ c}}$$

• می‌خواهیم در مورد $\mathcal{L}_A$ ادعای دیگری کنیم:

ادعا:

$$\mathcal{L}_A = -\frac{1}{2g^2} \left(A_\mu^{\ a} [(-D^2)^{ac} g^{\mu\nu} - 2 \left(\tfrac{1}{2} F_{\rho\sigma}^{\ b} \mathcal{J}^{\rho\sigma} \right)^{\mu\nu} (t_G^{\ b})^{ac}] A_\nu^{\ c} \right)$$

with $\quad (\mathcal{J}^{\rho\sigma})_{\mu\nu} = i \left(\delta^\rho_{\ \mu} \delta^\sigma_{\ \nu} - \delta^\rho_{\ \nu} \delta^\sigma_{\ \mu} \right)$

اثبات:

a) $\quad i \left(F_{\rho\sigma}^{\ b} \mathcal{J}^{\rho\sigma} \right)^{\mu\nu} = i^2 F_{\rho\sigma}^{\ b} \left(g^{\rho\mu} g^{\sigma\nu} - g^{\rho\nu} g^{\sigma\mu} \right) = - F^{\mu\nu,b} + F^{\nu\mu,b} = -2F^{\mu\nu,b}$

$$i f^{abc} = (t^b)^{ac} \quad \rightarrow \quad f^{abc} = -i (t^b)^{ac}$$

$$\Rightarrow \; -2 f^{abc} F^{\mu\nu,b} = -2(-i)(t^b)^{ac} \left(-\frac{i}{2} F_{\rho\sigma}^{\ b} (\mathcal{J}^{\rho\sigma})^{\mu\nu} \right)$$

$$= 2 \left(\tfrac{1}{2} (t^b)^{ac} \right) F_{\rho\sigma}^{\ b} (\mathcal{J}^{\rho\sigma})^{\mu\nu}$$

و این ادعای ما را اثبات می‌کند

$$\Rightarrow \boxed{\mathcal{L}_A = \frac{-1}{2g^2} \left(A_\mu^{\ a} [(-D^2)^{ac} g^{\mu\nu} - 2 \left(\tfrac{1}{2} F_{\rho\sigma}^{\ b} \mathcal{J}^{\rho\sigma} \right)^{\mu\nu} (t^b)^{ac}] A_\nu^{\ c} \right)}$$

b) <u>Quadratic Lagrangian for fermions:</u>

$$\mathcal{L}_\psi = \bar{\psi} (i \not{D}) \psi$$

$$(i\not{D})^2 = -\gamma^\mu \gamma^\nu D_\mu D_\nu = -\left(\tfrac{1}{2} [\gamma^\mu, \gamma^\nu] D_\mu D_\nu + \tfrac{1}{2} \overbrace{\{\gamma^\mu, \gamma^\nu\}}^{2g^{\mu\nu}} D_\mu D_\nu \right)$$

$$= 2i \underbrace{\left(\tfrac{i}{4} [\gamma^\mu, \gamma^\nu] \right)}_{\Sigma^{\mu\nu}} D_\mu D_\nu - D^2$$

ادامه

$$(i\not{D})^2 = -D^2 + 2i \underbrace{\Sigma^{\mu\nu}}_{\substack{\text{antisym}\\ \mu\leftrightarrow\nu}} \Big(\frac{1}{2} \underbrace{[D_\mu, D_\nu]}_{-iF^a_{\mu\nu} t^a} + \frac{1}{2} \underbrace{\cancel{\{D_\mu, D_\nu\}}}_{\text{sym } \mu\leftrightarrow\nu} \Big)$$

$$\boxed{(-i\not{D})^2 = -D^2 + 2\left(\frac{1}{2} F^a_{\mu\nu} \Sigma^{\mu\nu}\right) t^a}$$

این درست است وقتی ماده فقط $\mathcal{L}_\psi$ $(i\not{D})$ داشتیم:

Remember:

$$\int \mathcal{D}\psi \mathcal{D}\bar\psi \; e^{i\int \bar\psi (i\not{D})\psi} = \left(\det (i\not{D})\right)^{N_f} = \left(\det \underbrace{(i\not{D})^2}\right)^{N_f/2}$$

این همان صورت بالا را دارد

c) Quadratic form of ghost Lagrangian:

$$\boxed{\mathcal{L}_c = \bar{c}^a \, (-D^2)^{ab} \, c^b}$$

ghost = Grassmanian with spin 0

به این ترتیب خواهیم داشت:

$$\underbrace{e^{i\Gamma}}_{\text{کنش موثر}} \sim \int \underbrace{\mathcal{D}\psi \mathcal{D}\bar\psi \mathcal{D}c \mathcal{D}\bar{c} \; \mathcal{D}A_\mu}_{\text{به طور کلی } \mathcal{D}\varphi} \; e^{\,i\int d^4x \sum_{r,j} \varphi_{r,j} \Delta_{r,j} \varphi_{r,j} + i \underbrace{S[A]}_{\text{کنش کلاسیک برای میدان پیمانه‌ای}}}$$

$$\Delta_{r,j} = -D^2 + 2\left(\frac{1}{2} F^b_{\rho\sigma} \mathcal{J}^{\rho\sigma}\right)(t^b)_r$$ ← r نوع representation

ghosts: $j=0$ $\quad \mathcal{J}^{\rho\sigma}=0$ for $s=0$ (spinor) $\quad r=G$ (adjoint)

gluons: $j=1$ $\quad (\mathcal{J}^{\rho\sigma})_{\mu\nu} = i\left(\delta^\rho{}_\mu \delta^\sigma{}_\nu - \delta^\rho{}_\nu \delta^\sigma{}_\mu\right)$ for $s=1$ $\quad r=G$

Fermions: $j=\frac{1}{2}$ $\quad (\mathcal{J}^{\rho\sigma})_{\alpha\beta} = (\Sigma^{\rho\sigma})_{\alpha\beta} = \frac{i}{4}[\gamma^\rho, \gamma^\sigma]_{\alpha\beta}$ for $s=\frac{1}{2}$, r

r = fundamental representation for fermions

G = adjoint " for gaugefields & ghosts

⟶ نتیجه اولیه