BRST - Symmetry (Becchi - Rouet - Stora - Thytin 1974 - 1976)

Faddeev - Popov خلاصه روش

$$\mathcal{Z} = \int \mathcal{D} A_\mu \, e^{iS_g} \quad ; \quad 1 = \int \mathcal{D}\alpha \;\; \delta\left[F^a[A_\mu^\alpha]\right] \Delta_F\left[A_\mu^\alpha\right]$$

$$\rightarrow \mathcal{Z} = \int \mathcal{D} A_\mu \; e^{iS_g} \underbrace{\int \mathcal{D}\alpha \;\; \delta\left[F^a[A_\mu^\alpha]\right] \Delta_F\left[A_\mu^\alpha\right]}_{=1}$$

$$A_\mu^\alpha = A_\mu - \frac{1}{g}\,\partial_\mu \alpha(x) \qquad \text{for QED}$$

$$A_\mu^\alpha = A_\mu - \frac{1}{g}\,D_\mu \alpha(x) \qquad \text{for QCD (in general } SU(N) \text{ theory)}$$

قید $F^a[A_\mu^\alpha] = 0 \;\curvearrowright\; \partial_\mu A^{\mu, a} = 0 \;\curvearrowright\; \begin{cases} \partial_\mu A^{\mu,\alpha} = \partial_\mu A^\mu - \frac{1}{g}\Box\alpha(x) & \text{for } U(1) \\ \partial_\mu A^{\mu,a,\alpha} = \partial_\mu A^{\mu,a} - \frac{1}{g}\partial_\mu D^\mu \alpha & \text{for } SU(N) \end{cases}$

$$\curvearrowright \quad \Delta_F[A_\mu] = \det\left(\frac{\delta F^a[A_\mu]}{\delta\alpha^d}\right)$$

$$\longrightarrow \Delta_F[A_\mu] = \det\left(-\frac{1}{g}\Box\right) \qquad U(1)$$

$$\Delta_F[A_\mu] = \det\left(-\frac{1}{g}\partial_\mu D^\mu\right) \qquad SU(N)$$

In $SU(N)$

$$(A_\mu^\alpha)^a = A_\mu^a - \frac{1}{g}\partial_\mu \alpha^a + f^{abc} A_\mu^b \alpha^c$$

$$\partial^\mu (A_\mu^\alpha)^a = \partial^\mu A_\mu^a - \frac{1}{g}\Box\alpha^a + f^{abc}\,\partial^\mu(A_\mu^b \alpha^c)$$

$$= \partial^\mu A_\mu^a - \frac{1}{g}\,\partial_\mu\left[\left(\delta^{ac}\partial^\mu - g f^{abc} A^{\mu,b}\right)\alpha^c\right]$$

$$\curvearrowright \quad \Delta_F[A_\mu] = \det\left(\frac{\delta\,(\partial_\mu A^{\mu,\alpha})^a}{\delta\,\alpha^d}\right) = \det\left(-\frac{1}{g}(\partial_\mu D^\mu)^{ad}\right)$$

with

$$D_\mu^{ad} = \left(\delta^{ad}\partial_\mu - g f^{abd} A_\mu^b\right)$$

$$\curvearrowright \quad \Delta[A_\mu] = \det\left(M^{ad}\right)$$

روش FP :

a) $$\mathcal{Z} = \int \mathcal{D} A_\mu \; e^{iS_g[A_\mu]}$$

$$= \int \mathcal{D}\alpha \quad \underbrace{\int \mathcal{D}A_\mu^\alpha \; e^{iS_g[A_\mu^\alpha]} \;\delta\left[F^a[A_\mu^\alpha]\right] \Delta_F[A_\mu^\alpha]}_{\alpha=1}$$

$$\mathcal{D}A_\mu^\alpha = \mathcal{D}A_\mu$$

$$= \underbrace{\int \mathcal{D}\alpha}_{\infty} \quad \underbrace{\int \mathcal{D}A_\mu \; e^{iS_g[A_\mu]} \;\delta\left[F[A_\mu]\right] \Delta_F[A_\mu]}_{=\text{ finite}}$$

b) $$\mathcal{Z} = \int \mathcal{D}\alpha \int \mathcal{D}A_\mu \; e^{iS_g[A_\mu]} \int \mathcal{D}\omega \; e^{-\frac{i}{2\xi}\int d^4x\,(\omega^a)^2} \;\delta\left[F^a - \omega^a\right] \Delta_F[A_\mu]$$

$$= \int \mathcal{D}\alpha \int \mathcal{D}A_\mu \; \exp\left(iS_g[A_\mu] + iS_{g.f.}[A_\mu]\right) \Delta_F[A_\mu]$$

$$S_{g.f} = -\frac{1}{2\xi}\int d^4x\,(\partial_\mu A^\mu)^2 = -\frac{1}{2\xi}\int d^4x\,(F^a)^2$$

ξ [illegible]

c) $\Delta_F[A] = \det(M) = \int \mathcal{D}c\,\mathcal{D}\bar{c}\quad e^{i\int \bar{c}^a M^{ab} c^b d^4x}$

$$S_{ghost} = \int \bar{c}^a M^{ab} c^b$$

$$F^a = \partial_\mu A^{\mu,a} \qquad M^{ab} = \begin{cases} U(1) & -\frac{1}{g}\Box \\ SU(N) & -\frac{1}{g}(\partial_\mu D^\mu)^{ab} \end{cases}$$

$$\bar{c}^a (\partial_\mu D^\mu)^{ab} c^b = \bar{c}^a \partial_\mu \left(\delta^{ab}\partial^\mu - g f^{acb} A^{\mu,c}\right) c^b$$
$$= \bar{c}^a \partial_\mu \partial^\mu c^a - g f^{acb}\ \bar{c}^a \partial_\mu (A^{\mu,c} c^b)$$

$$\Delta_F[A_\mu{}^\alpha] = \underbrace{\det\left(\frac{\delta F^a[A_\mu{}^\alpha]}{\delta \alpha^d}\right)_{\alpha=0}}_{?} = \det(M^{ad})$$

We can expand $F^a[A_\mu^\alpha]$ in the orders of α :

$$F^a[A_\mu{}^\alpha] = F^a[A_\mu] + \int \underbrace{\left(\frac{\delta F^a[A_\mu{}^\alpha]}{\delta \alpha^b(y)}\right)_{\alpha=0}}_{\sim M^{ab}(x,y)} \alpha^b(y)\, d^4y + \dots$$

$$M^{ab}(x,y) = \left(\frac{\delta F^a[A_\mu{}^\alpha]}{\delta \alpha^b(y)}\right)_{\alpha=0} = \int d^4z\ \frac{\delta F^a[A_\mu{}^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\ \frac{\delta A_\nu^{\alpha,c}(z)}{\delta \alpha^b(y)}\Bigg|_{\alpha=0}$$

$$= -\frac{1}{g}\int d^4z\ \frac{\delta F^a[A_\mu{}^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\Bigg|_{\alpha=0} \frac{\delta}{\delta\alpha^b(y)} \left(\partial_\nu^z \alpha^c(z) - g f^{cdm} A_\nu^d(z)\, \alpha^m(z)\right)\Bigg|_{\alpha=0}$$

$$= \frac{-1}{g}\int d^4z\ \frac{\delta F^a[A_\mu{}^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\Bigg|_{\alpha=0} \left(\partial_\nu^z \delta^4(z-y)\,\delta^{bc} - g f^{cdm} A_\nu^d(z)\, \delta(y-z)\,\delta^{bm}\right)\Bigg|_{\alpha=0}$$

$$= \frac{-1}{g}\int d^4z\ \frac{\delta F^a[A_\mu{}^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\Bigg|_{\alpha=0} \left(\partial_\nu^z \delta^{bc} - g f^{cdb} A_\nu^d(z)\right)\delta(y-z)$$

$$\boxed{M^{ab}(x,y) = -\frac{1}{g}\int d^4z\ \frac{\delta F^a[A_\mu{}^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\Bigg|_{\alpha=0} (D_\nu^z)^{bc}\ \delta(y-z)}$$

$$Z_{ghost} = \int \mathcal{D}c\,\mathcal{D}\bar{c}\quad \exp\left(i\int d^4x\, d^4y\ \bar{c}^a(x)\, M^{ab}(x,y)\, c^b(y)\right)$$

$$M^{ab}c^b = (Mc)^a_{(x)} = \int d^4y\ M^{ab}(x,y)\, c^b(y) \curvearrowright$$

$$(Mc)^a(x) = \int d^4y \int d^4z \left(\frac{\delta F^a[A_\mu^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\right)\Bigg|_{\alpha=0} \left(-\frac{1}{g}\left(D_\nu^{z}\right)^{cb}(z)\,\delta(y-z)\right) c^b(y)$$

فعلاً D فقط روی δ اثر می‌کند

$$= -\frac{1}{g}\int d^4y \int d^4z \left(\frac{\delta F^a[A_\mu^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\right)_{\alpha=0} \left[\left(\partial_\nu^z\,\delta^{cb} - g\, f^{cdb} A_\nu^d(z)\right)\delta(y-z)\right] c^b(y)$$

$$= -\frac{1}{g}\int d^4y\, d^4z \left(\frac{\delta F^a[A_\mu^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\right)_{\alpha=0} \left[\underbrace{\delta^{cb}\,\partial_\nu^z\,\delta(y-z)}_{*} - g f^{cdb} A_\nu^d(z)\,\delta(y-z)\right] c^b(y)$$

$$= -\frac{1}{g}\int d^4z \left(\frac{\delta F^a[A_\mu^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\right)_{\alpha=0} \underbrace{\left(\partial_\nu^z\,\delta^{cb} - g f^{cdb} A_\nu^d(z)\right)}_{D_\nu^{cb}(z)} c^b(z)$$

$$(1) \qquad \boxed{(Mc)^a(x) = -\frac{1}{g}\int d^4z \left(\frac{\delta F^a[A_\mu^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\right)_{\alpha=0} D_\nu^{cb}(z)\, c^b(z)}$$

$$\rightarrow \boxed{Z_{ghost} = \int \mathcal{D}c\, \mathcal{D}\bar{c}\, \exp\left(\frac{-i}{g}\int d^4x\, d^4z \left(\frac{\delta F^a[A_\mu^\alpha(x)]}{\delta A_\nu^{\alpha,c}(z)}\right)_{\alpha=0} D_\nu^{cb}(z)\, c^b(z)\right)}$$

$$* \quad \int dx\, dy\; \partial_y\left(\delta(x-y) f(x)\right) = \int dy\; \partial_y\left(\int \delta(x-y) f(x)\, dx\right) = \int dy\, \frac{\partial f(y)}{\partial y}$$

رابطهٔ (۱) رابطه‌ای است که بعداً به آن نیاز داریم.

از طرفی به تغییرات A_μ در SU(N) توجه می‌کنیم:

$$\delta_\alpha A_\mu^c = -\frac{1}{g}\left(D_\mu \alpha(z)\right)^c = -\frac{1}{g}\left(\delta^{cb}\,\partial_\mu^z - g f^{cdb} A_\mu^d(z)\right)\alpha^b(z)$$

حال اگر فرض کنیم

$$\alpha^b(y) = c^b(y)\,\theta$$

ghost تابع گراسمنی

a global (infinitesimal) transformation parameter

θ is Grassmanian.

خواهیم داشت:

$$\longrightarrow \delta_\alpha A_\mu^c(z) = -\frac{1}{g}\left(\partial_\mu c^c(z) - g f^{cdb} A_\mu^d(z)\, c^b(z)\right)\theta =$$

$$= -\frac{1}{g}\left(D_\mu c(z)\right)^c \theta = +\theta\, \frac{1}{g}\left(D_\mu c(z)\right)^c$$

به این ترتیب خواهیم داشت:

$$\boxed{\begin{aligned}\delta_\alpha A_\mu^c(z) &= \theta \left(\frac{1}{g} D_\mu c\right)^c(z) \\ &= \theta \; s A_\mu^c(z) \equiv \delta_{BRS} \; A_\mu^c(z)\end{aligned}}$$

New look at (1)

$$(Mc)^a(x) = \int d^4 z \; \frac{\delta F^a[A_\mu(x)]}{\delta A_\mu^c(z)} \underbrace{\left(-\frac{1}{g} \; D_\nu^{cb}(z)\, c^b(z)\right)}_{= -s A_\nu^c(z)}$$

$$\boxed{(Mc)^a(x) = - s \;\; F^a[A_\mu(x)]}$$

$$\begin{aligned} Z_{ghost} &= \int \mathcal{D}c \, \mathcal{D}\bar{c} \;\; \exp\left(-i \int d^4 z \;\; \bar{c}^a(z) \; (Mc)^a(z)\right) \\ &= \int \mathcal{D}c \, \mathcal{D}\bar{c} \;\; \exp\left(+i \int d^4 z \;\; \bar{c}^a(z) \;\; sF^a[A_\mu]\right)\end{aligned}$$

New global BRS transformation:

a) $$\delta_{BRS} \; A_\mu^a(x) = -\frac{1}{g} \; (D_\mu c(x))^a \theta \underset{\substack{\uparrow \\ c \to gc \\ \alpha^a \to g c \theta}}{=} - \; (D_\mu c)^a \theta = \theta \; (D_\mu c)^a = \theta s A_\mu^a$$

$$\rightarrow \boxed{\delta_{BRS} \;\; A_\mu^a(x) = \theta \; s A_\mu^a(x)}$$

نتیجه اینکه هرگونه تبدیل تحت تبدیل پیمانه‌ای با پارامتر $\alpha(x)$ را می‌توان به ناوردائی تحت تبدیل سراسری BRST تبدیل کرد

b) For fermions:

$$\psi \to \psi' = V(x)\psi(x) = e^{i\alpha^a t^a}\psi(x) \approx (1 + i\alpha^a t^a)\psi(x)$$

$$\delta_\alpha \psi(x) = i \, \alpha^a \, t^a \psi(x)$$

$$\delta_{BRS} \; \psi(x) = i \, g c^a \theta \; t^a \psi(x) = \theta \left(-ig \;\; c^a t^a \psi(x)\right)$$

$$= \theta \;\; (s\psi)$$

$$\boxed{\delta_{BRS} \;\; \psi(x) = \theta \; (s\psi)} \qquad \text{with} \qquad s\psi = -ig \; (c^a t^a \psi(x))$$

c) Ghosts :

$$c'(x) = V(x)\,c(x)\,V^{+}(x) = e^{i\alpha^a t^a}\, c(x)\, e^{-i\alpha^a t^a}$$

$$= (1 + i\alpha^a t^a)\, c(x)\, (1 - i\alpha^a t^a)$$

$$= c(x) + i\,[\alpha^a t^a, c(x)]$$

$$\boxed{c'^{c}(x) = c^{c}(x) - f^{abc}\,\alpha^a c^b(x)}$$

$$\delta_\alpha c^c(x) = - f^{abc}\,\alpha^a\, c^b(x) =$$

$$\underset{\alpha \to g c\theta}{=} - f^{abc}\, g c^a \theta c^b = +\theta\,(g f^{abc} c^a c^b) = \theta\,(s c^c)$$

$$\rightarrow \boxed{\delta_{BRS}\, c^c(x) = \theta\,(s c^c)} \qquad s c^c = g f^{abc} c^a c^b$$

در مورد $\bar{c}$ نیز داشتیم:

$$\delta_{BRS}\, \bar{c}^a = -\frac{1}{\xi}\,\theta F^a = \theta\,(s\bar{c}^a) \quad \rightarrow \quad s\bar{c}^a = -\frac{1}{\xi} F^a$$

حل

$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_{g.f.} + \mathcal{L}_{ghost}$ is invariant under:

$$\delta_{BRS} A_\mu^a = \theta\,(D_\mu c)^a = \theta\,(s A_\mu^a)$$

$$\delta_{BRS}\, \psi = \theta\,(-i g c^a t^a \psi(x)) = \theta\,(s\psi)$$

$$\delta_{BRS}\, c^a = \theta\left(\frac{1}{2} g f^{abc} c^a c^b\right) = \theta\,(s c^a)$$

$$\delta_{BRS}\, \bar{c}^a = \theta\left(-\frac{1}{\xi} F^a\right) = \theta\,(s\bar{c}^a)$$

یادآوری: تنها جمله‌ای که در لاگرانژی تحت تبدیل پیمانه‌ای ناوردا نبود، جمله gauge fixing بود (که به دلایلی که قبلاً گفته بودیم برای ما اهمیت زیادی داشت):

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi}\,(F^a)^2$$

$$\delta_{BRS}\, \mathcal{L}_{g.f.} = -\frac{1}{2}\, 2F^a\, \delta_{BRS} F^a = -\frac{1}{\xi} F^a\, (Mc)^a \theta \qquad (*)$$

قبلاً بدست آورده بودیم

$$s F^a = -(Mc)^a$$

$$\delta_{BRS} F^a = \theta\,(s F^a) = -\theta\,(Mc)^a = (Mc)^a \theta$$

$$\mathcal{L}_{ghost} = \bar{c}^a\,(Mc)^a$$

از طرفی

$$\delta_{BRS}\mathcal{L}_{ghost} = \underbrace{\delta_{BRS}\bar{c}^a}\,(Mc)^a + \bar{c}^a\,\underbrace{\delta_{BRS}(Mc)^a}_{=0}$$

باید ثابت شود ✓

$$= \left(-\frac{1}{\xi}\theta F^a\right)(Mc)^a = +\frac{1}{\xi}F^a(Mc)^a\,\theta$$

در مقایسه با $(*)$ خواهیم داشت:

$$\boxed{\delta_{BRS}\,\mathcal{L}_{g.f.} + \delta_{BRS}\,\mathcal{L}_{ghost} = 0}$$

در نتیجه نشان می‌دهد $\delta_{BRS}\bar{c}^a = -\frac{1}{\xi}\theta F^a$ صحیح است.

✓ در ادامه باید ثابت می‌کنیم $\delta_{BRS}(Mc)^a = 0$ است. اگر این رابطه صحیح باشد، معنی آن این است که عملگر δ_{BRS} اصطلاحاً nilpotent است.

$$\delta_{BRS}(Mc)^a = \delta_{BRS}\,\delta_{BRS}\,F^a = 0$$

با $\delta_{BRS}F^a = (Mc)^a\theta \rightsquigarrow \delta_{BRS}^2\,F^a = \delta_{BRS}(Mc)^a\,\theta = 0$

این باید اثبات شود.

درادامه نشان می‌دهیم که برای $F^a = \partial_\mu A^{\mu,a}$ (به عنوان مثال) $\delta_{BRS}^2 = 0$ است.

$$\delta_{BRS}\,F^a[A_\mu] = \int d^4z\,\frac{\delta F^a[A_\mu]}{\delta A_\mu^b(z)}\,\delta_{BRS}A_\mu^b(z) = \int d^4z\,\frac{\delta}{\delta A_\mu^b(z)}\left(\partial_\nu A^{\nu,a}(x)\right)\delta_{BRS}A_\mu^b$$

$$= \int d^4z\;\partial_\nu^z\,\delta(z-x)\,\delta^{ab}g^{\mu\nu}\,\delta_{BRS}A_\mu^b(z)$$

$$= -\,\delta_{BRS}\,\partial_\nu A_\nu^a(x) = -\partial_\nu\,\delta_{BRS}A_\nu^a(x)$$

$$\delta_{BRS}F^a = -\partial_\nu\left(\theta\,(D^\nu c)^a\right) = -\theta\,\partial_\nu\left(D^{\nu,ab}c^b(x)\right)$$

$$\boxed{\delta_{BRS}F^a = -\theta\,\partial_\nu\left(\partial^\nu c^a - g f^{acb}A^{\nu,c}c^b(x)\right)}$$

$$\delta_{BRS}^2 F^a = -\theta\,\partial_\nu\left(\partial^\nu\delta_{BRS}c^a - g f^{acb}(\delta_{BRS}A^{\nu,c})c^b - g f^{acb}A^{\nu,c}\delta_{BRS}c^b\right)$$

$$= -\theta\,\partial_\nu\Big\{\partial^\nu\left(-\frac{1}{2}g f^{abc}c^bc^c\theta\right) - g f^{acb}\left(\theta\,D^{\nu,cd}c^d\right)c^b$$

$$- g f^{acb}A^{\nu,c}\left(\frac{1}{2}g f^{bmn}c^mc^n\theta\right)\Big\}$$

جمله اول

$$\partial_\nu\left(\frac{1}{2}g f^{abc}c^bc^c\theta\right) =$$

$$= \frac{1}{2}g f^{abc}(\partial_\nu c^b)c^c\theta + \frac{1}{2}g f^{abc}c^b(\partial_\nu c^c)\theta$$

$$= \frac{1}{2}g f^{abc}(\partial_\nu c^b)c^c\theta - \underbrace{\frac{1}{2}g f^{abc}(\partial_\nu c^c)c^b\theta}_{c\leftrightarrow b \quad \text{تغییر نامگذاری}}$$

$$= -\frac{1}{2}g f^{acb}(\partial_\nu c^b)c^c\theta =$$

$$= +\frac{1}{2}g f^{abc}(\partial_\nu c^b)c^c\theta$$

$$\overset{\text{together}}{=} g f^{abc}(\partial_\nu c^b)c^c\theta$$

جمله دوم

$$-g f^{acb}(\theta D^{\nu,cd} c^d)c^b = +g f^{acb}(D^{\nu,cd}c^d)\theta c^b$$

$$= g f^{acb}(\partial^\nu c^c - g f^{cpd} A^{\nu,p} c^d)\theta c^b$$

$$= g f^{acb}\partial^\nu c^c \theta c^b - g^2 f^{acb} f^{cpd} A^{\nu,p} c^d \theta c^b$$

$$= \underbrace{-g f^{acb}\partial^\nu c^c c^b \theta}_{\text{سهم جمله اول را حذف می‌کند}} + g^2 f^{acb} f^{cpd} A^{\nu,p} c^d c^b \theta$$

$$\text{جمله اول + جمله دوم} = g^2 f^{acb} f^{cpd} A^{\nu,p} c^d c^b \theta$$

جمله سوم

$$-\frac{1}{2} g^2 f^{acb} f^{bmn} A^{\nu,c} c^m c^n \theta$$

تغییر نام‌گذاری: $c\to p$, $m\to d$, $n\to b$, $b\to c$

$$= -\frac{1}{2} g^2 f^{apc} f^{cdb} A^{\nu,p} c^d c^b \theta$$

از رابطه کلی زیر استفاده می‌کنیم:

$$f^{apc} f^{cdb} + f^{adc} f^{cbp} + f^{abc} f^{cpd} = 0$$

$$\Rightarrow f^{apc} f^{cdb} = -f^{adc} f^{cbp} - f^{abc} f^{cpd}$$

$$\text{جمله سوم} = -\frac{1}{2} g^2 (-f^{adc} f^{cbp} - f^{abc} f^{cpd}) A^{\nu,p} c^d c^b \theta$$

$$= \frac{1}{2} g^2 f^{adc} f^{cbp} A^{\nu,p} c^d c^b \theta + \underbrace{\frac{1}{2} g^2 f^{abc} f^{cpd} A^{\nu,p} c^d c^b \theta}_{= -\frac{1}{2} g^2 f^{acb} f^{cpd} A^{\nu,p} c^d c^b \theta}$$

$$\text{جمله اول + دوم + سوم} = \underbrace{\frac{1}{2} g^2 f^{adc} f^{cbp} A^{\nu,p} c^d c^b \theta}_{\text{نام‌گذاری } d \leftrightarrow b \text{ را عوض می‌کنیم}} + \frac{1}{2} g^2 f^{acb} f^{cpd} A^{\nu,p} c^d c^b \theta$$

$$= \frac{1}{2} g^2 f^{abc} f^{cdp} A^{\nu,p} c^b c^d \theta + \frac{1}{2} g^2 (-f^{abc})(-f^{cdp}) A^{\nu,p} (-c^b c^d)\theta$$

$$= \frac{1}{2} g^2 f^{abc} f^{cdp} A^{\nu,p} c^b c^d \theta - \frac{1}{2} g^2 f^{abc} f^{cdp} A^{\nu,p} c^b c^d \theta$$

$$= 0$$

و این نشان می‌دهد که $\delta_{BRS}^2 F^a = 0$ ∎

توضیح:

$$f^{apc} f^{cdb} + f^{adc} f^{cbp} + f^{abc} f^{cpd} = 0$$

for SU(2): $\epsilon^{apc}\epsilon^{cdb} + \epsilon^{adc}\epsilon^{cbp} + \epsilon^{abc}\epsilon^{cpd}$

$$= \delta^{ad}\cancel{\delta^{pb}} - \delta^{ab}\cancel{\delta^{dp}} + \delta^{ab}\cancel{\delta^{dp}} - \delta^{ap}\cancel{\delta^{db}} + \delta^{ap}\cancel{\delta^{bd}} - \delta^{ad}\cancel{\delta^{bp}} = 0$$

سؤال: آیا تقارن BRS تقارن بنیادی طبیعت است؟ (مشکل بستگی تبدیلات به ξ)

با این هدف که تقارن BRST تقارن بنیادی طبیعت باشد یک جمله به $\mathcal{L}$ اضافه می‌کنیم:

$$\mathcal{L}_{g.f.} = b^a \partial_\mu A^{\mu,a} + \frac{1}{2}\xi b^a b^a = b^a F^a + \frac{1}{2}\xi b^a b^a$$

b^a جمله جنبشی ندارد، در نتیجه میدان کمکی (auxilliary) است. معادله حرکت آن یک قید است

$$\frac{\delta \mathcal{L}}{\delta b^a} = \frac{\delta \mathcal{L}_{g.f.}}{\delta b^a} = \partial_\mu A^{\mu,a} + \xi b^a = 0 \longrightarrow b^a = -\frac{1}{\xi}\partial_\mu A^{\mu,a} = \frac{-1}{\xi} F^a$$

• اگر معادله حرکت b^a (قید بالا را) در $\mathcal{L}_{g.f.}$ جایگزین کنیم به همان $\mathcal{L}_{g.f.}$ همیشگی می‌رسیم.

تبدیلات BRST جدید

$$\delta_{BRS}\, \bar{c}^a = -\frac{1}{\xi}\theta F^a = \theta b^a = \theta(s\bar{c}^a) \longrightarrow s\bar{c}^a = b^a$$

$$\delta_{BRS}\, \mathcal{L}_{g.f.} = (\delta_{BRS} b^a) F^a + b^a \delta_{BRS} F^a + \xi b^a \delta_{BRS} b^a$$

$$= b^a \delta_{BRS} F^a \underset{b^a = -\frac{1}{\xi}F^a}{=} - b^a \delta_{BRS} b^a = 0 \longrightarrow (\delta b^a = 0)$$

$$\delta b^a = \delta(s\bar{c}^a) = \theta\, s^2 \bar{c}^a = 0$$

BRST-transformation:

به این ترتیب تبدیلات جدید به ξ بستگی ندارند:

$$\delta_{BRS} A_\mu{}^a = \theta D_\mu{}^{ab} c^b = \theta(sA_\mu{}^a)$$

$$\delta_{BRS} \psi = -ig\theta t^a c^a \psi = \theta(s\psi)$$

$$\delta_{BRS} c^a = \frac{1}{2} g\theta f^{abc} c^b c^c = \theta(sc^a)$$

$$\delta_{BRS} \bar{c}^a = \theta b^a = \theta(s\bar{c}^a)$$

$$\delta_{BRS} b^a = 0 = \theta(sb^a)$$

خواص s:

a) $s\bar{c}^a = b^a \qquad s^2\bar{c}^a = sb^a = 0 \rightsquigarrow s^2 = 0$

b) $s(AB) = (sA)B \pm A(sB)$

علامت منفی وقتی ظاهر می‌شود که A گراسمنی باشد
یا شامل تعداد فردی عناصر گراسمنی باشد.

c) $s\bar{s} + \bar{s}s = 0 \qquad \bar{s}^2 = 0$

Anti BRST _ transformation :

$$\bar{s} A_\mu^a = D_\mu^{ab} \bar{c}^b$$

$$\bar{s}\, \bar{c}^a = -\frac{1}{2} g f^{abc} \bar{c}^b \bar{c}^c$$

$$\bar{s}\, c^a = -b^a - g f^{abc} \bar{c}^b c^c$$

$$\bar{s} b^a = -g f^{abc} c^b c^c$$

$$\bar{s} \psi = i g\, t^a \bar{c}^a \psi$$

اگر F^a در $A^{\mu,a}$ خطی باشد، آنگاه $\bar{s}$ تقارن $\mathcal{L}_{tot}$ است.

بدون توجه به اینکه ما به چه دلیل c و $\bar{c}$ را در لاگرانژی وارد کردیم، اینها دو نوع میدان هستند که با A_μ کوپل می‌شوند، و نوعی تقارن سراسری جدید برای اینها بوجود میاورند.