قوانین فاینمن در نظریه میدان غیرآبلی :

$$S = S_f + S_g + S_{g.f.} + S_{ghost}$$

in a $SU(N)$ non-Abelian

$$S_f = \int d^4x \quad \bar{\Psi}^a \, (i\not{\partial} - m)^{ab} \, \psi^b$$

$$S_g = -\frac{1}{2} \int d^4x \; tr\left(F_{\mu\nu} F^{\mu\nu}\right) = -\frac{1}{4} \int d^4x \; F_{\mu\nu}{}^a F^{\mu\nu,a}$$

$$S_{g.f.} = -\frac{1}{2\xi} \int d^4x \; (\partial_\mu A^{\mu,a})^2 = \frac{1}{2\xi} \int d^4x \; A^{\mu,a} \partial_\mu \partial_\nu A^{\nu,a}$$

$$S_{ghost} = -\int d^4x \; \bar{c}^a \; \partial_\mu D^{\mu,ab} \, c^b$$

$$= -\int d^4x \; \bar{c}^a \, \partial^\mu \left(\delta^{ab} \partial_\mu - g f^{abc} A_\mu{}^c \right) c^b$$

$$= -\int d^4x \; \bar{c}^a \left(\partial^\mu \partial_\mu c^a - g f^{abc} \partial^\mu (A_\mu{}^c c^b) \right)$$

$$= +\int d^4x \, \partial^\mu \bar{c}^a \partial_\mu c^a - g f^{abc} \int d^4x \; \partial^\mu \bar{c}^a \; A_\mu{}^c c^b$$

Feynman Rules:

a) $$\int d^4x \quad \bar{\Psi}^{i,\alpha} \left(i (\gamma^\mu)_{\alpha\beta} \partial_\mu - m \, \delta_{\alpha\beta} \right) \Psi^{j,\beta} \, \delta_{ij}$$

Fermion Propagator:

$$\longrightarrow \langle \Psi_{j,\beta}(x) \, \bar{\Psi}_{i,\alpha}(y) \rangle = \underset{\beta}{\overset{j}{\bullet}} \longrightarrow \underset{\alpha}{\overset{i}{\bullet}} = \int \frac{d^4p}{(2\pi)^4} \, \delta_{ij} \left(\frac{i}{\not{p} - m} \right)_{\beta\alpha} e^{-ip(x-y)}$$

$i, j = 1, \ldots, d(r)$

↳ dimension of the representation of the symmetry group.

Gauge Kinetic Term:

(b1) $$-\frac{1}{4} \int F_{\mu\nu}{}^a F^{\mu\nu,a} d^4x = -\frac{1}{4} \int d^4x \left(\partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a - g f^{abc} A_\mu^b A_\nu^c \right)$$
$$\times \left(\partial^\mu A^{\nu,a} - \partial^\nu A^{\mu,a} - g f^{ade} A^{\mu,d} A^{\nu,e} \right)$$

$$= -\frac{1}{4} \int d^4x \left(\partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a \right) \left(\partial^\mu A^{\nu,a} - \partial^\nu A^{\mu,a} \right) \longrightarrow \text{Propagator}$$

$$-\frac{1}{4} \times 2 \int d^4x \left(\partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a \right) \left(-g f^{ade} A^{\mu,d} A^{\nu,e} \right) \longrightarrow \text{Vertex 1}$$

$$-\frac{1}{4} \int d^4x \left(-g f^{abc} A_\mu{}^b A_\nu{}^c \right) \left(-g f^{ade} A^{\mu,d} A^{\nu,e} \right) \longrightarrow \text{Vertex 2}$$

Gluon - Propagator:

$$-\frac{1}{4} \int d^4x \left(\partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a \right) \left(\partial^\mu A^{\nu,a} - \partial^\nu A^{\mu,a} \right) + \frac{1}{2\xi} \int d^4x \; A^{\mu,a} (\partial_\mu \partial_\nu) A^{\nu,a}$$

$\rightarrow$

$$= -\frac{1}{4}\times 2\int \left(\partial_\mu A_\nu{}^a \partial^\mu A^{\nu,a} - \partial_\mu A_\nu{}^a \partial^\nu A^{\mu,a}\right) + \frac{1}{2\xi}\int A^{\mu,a}\,\partial_\mu\partial_\nu A^{\nu,a}$$

$$= \frac{1}{2}\int d^4x\; A^{\mu,a}\left(g_{\mu\nu}\,\partial_\rho\partial^\rho - \partial_\mu\partial_\nu\right)A^{\nu,a} + \frac{1}{2\xi}\int d^4x\; A^{\mu,a}\,\partial_\mu\partial_\nu A^{\nu,a}$$

$$= \frac{1}{2}\int d^4x\; A^{\mu,a}\left[g_{\mu\nu}\Box - \left(1-\frac{1}{\xi}\right)\partial_\mu\partial_\nu\right]A^{\nu,a}$$

$$\longrightarrow \langle A_\mu{}^a(x)\, A_\nu{}^b(y)\rangle = \underset{a\qquad b}{\overset{\mu\qquad \nu}{\bullet\!\sim\!\sim\!\sim\!\bullet}} = \int\frac{d^4k}{(2\pi)^4}\left(\frac{i}{k^2}\right)\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab} \times e^{-ik(x-y)}$$

c) <u>Ghost kinetic term:</u>

$$S_{ghost} = \int d^4x\; \partial_\mu \bar{c}^a(x)\,\partial^\mu c^a(x) = -\int d^4x\; \bar{c}^a(x)\,\Box\, c^a(x)$$

$$\langle c^a(x)\,\bar{c}^b(y)\rangle = \underset{a\qquad b}{\bullet - - - \bullet} = \int\frac{d^4l}{(2\pi)^4}\;\delta^{ab}\;\frac{e^{-il(x-y)}}{l^2}$$

$$a, b = 1, \dots, d(G)$$

$d(G) =$ # of generators of the symmetry group G.

<u>Vertices:</u>

(1) $$-\frac{1}{2}\int\left(\partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a\right)\left(-g f^{abc} A^{\mu,b} A^{\nu,c}\right)$$

$$= \frac{g}{2}\int \partial_\mu A_\nu{}^a A^{\mu,b} A^{\nu,c} f^{abc}\, d^4x$$

$$- \frac{g}{2}\int \partial_\nu A_\mu{}^a A^{\mu,b} A^{\nu,c} f^{abc}\, d^4x$$

در جمله دوم $\nearrow$ $\quad \mu\leftrightarrow\nu \quad b\leftrightarrow c \nearrow$ $$-\frac{g}{2}\int \partial_\mu A_\nu{}^a A^{\nu,c} A^{\mu,b} f^{acb}\, d^4x$$

$$= \frac{g}{2}\int \partial_\mu A_\nu{}^a A^{\nu,c} A^{\mu,b} f^{abc}\, d^4x$$

$$\longrightarrow \boxed{(1) = g\int \partial_\mu A_\nu{}^a A^{\mu,b} A^{\nu,c} f^{abc}\, d^4x}$$

(2) $$\boxed{-\frac{1}{4}g^2\int f^{abc} f^{ade} A_\mu{}^b A_\nu{}^c A^{\mu,d} A^{\nu,e}}$$

(1) $$g\int d^4x\; \partial_\mu A_\nu{}^a A^{\mu,b} A^{\nu,c} f^{abc}$$

$$= g\int d^4x\; f^{abc}\int_{k,p,q}\underbrace{\left(\partial_\mu e^{ikx}\right)}_{ik_\mu e^{ikx}} e^{ipx} e^{iqx}\;\tilde{A}_\nu{}^a(k)\,\tilde{A}^{\mu,b}(p)\,\tilde{A}^{\nu,c}(q)$$

$$= g\int_{k,p,q}\underbrace{\int d^4x\; e^{i(p+q+k)x}}_{=(2\pi)^4\delta^4(k+p+q)} f^{abc}\, k_\mu\, \tilde{A}^{\nu,a}(k)\,\tilde{A}^{\mu,b}\,\tilde{A}_\nu{}^c(q)$$

برای بدست آوردن راس مربوطه باید درجات آزادی را به درستی شمرده "متقارن سازی" کنیم:

Vertex 1: $A_{\mu'}^{a'}(p')\quad A_{\nu'}^{b'}(k')\quad A_{\rho'}^{c'}(q')$

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$$ig \quad f^{abc}\, k^{\mu} A_{\nu}{}^{a}(k)\, A_{\mu}{}^{b}(p)\quad A^{\rho, c}(q)\quad g_{\rho\nu}$$

123   1) $a = a'$ $b = b'$ $c = c'$ $igf^{a'b'c'}\; p'^{\nu'}\; g_{\rho'\mu'}$

$\nu = \mu'$ $\mu = \nu'$ $\rho = \rho'$

$k = p'$ $p = k'$ $q = q'$

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132   2) $a = a'$ $b = c'$ $c = b'$ $ig\, \underbrace{f^{a'c'b'}}_{= -f^{a'b'c'}}\; p^{\rho'}\; g_{\nu'\mu'}$

$\nu = \mu'$ $\mu = \rho'$ $\rho = \nu'$

$k = p'$ $p = q'$ $q = k'$

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213   3) $a = b'$ $b = a'$ $c = c'$ $ig\, \underbrace{f^{b'a'c'}}_{= -f^{a'b'c'}}\; k'^{\mu'}\; g_{\rho'\nu'}$

$\nu = \nu'$ $\mu = \mu'$ $\rho = \rho'$

$k = k'$ $p = p'$ $q = q'$

---

231   4) $a = b'$ $b = c'$ $c = a'$ $ig\, \underbrace{f^{b'c'a'}}_{= +f^{a'b'c'}}\; k'^{\rho'}\; g_{\mu'\nu'}$

$\nu = \nu'$ $\mu = \rho'$ $\rho = \mu'$

$k = k'$ $p = q'$ $q = p'$

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312   5) $a = c'$ $b = a'$ $c = b'$ $ig\, \underbrace{f^{c'a'b'}}_{= +f^{a'b'c'}}\; q'^{\mu'}\; g_{\nu'\rho'}$

$\nu = \rho'$ $\mu = \mu'$ $\rho = \nu'$

$k = q'$ $p = p'$ $q = k'$

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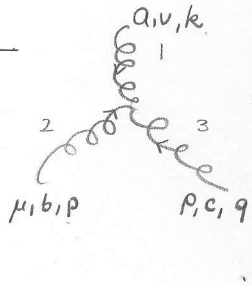
321   6) $a = c'$ $b = b'$ $c = a'$ $ig\, \underbrace{f^{c'b'a'}}_{= -f^{a'b'c'}}\; q'^{\nu'}\; g_{\mu'\rho'}$

$\nu = \rho'$ $\mu = \nu'$ $\rho = \mu'$

$k = q'$ $p = k'$ $q = p'$

$$\longrightarrow \quad \boxed{ig\, f^{abc}\left[(p-q)^{\nu} g_{\mu\rho} + g_{\mu\nu}(k-p)^{\rho} + g_{\nu\rho}\,(q-k)^{\mu}\right]}$$

2 − 3   23   12   1 − 2   13   3 − 1



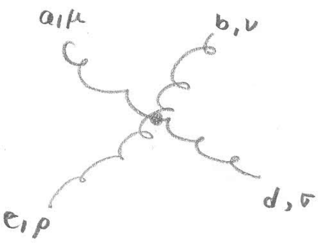
Vertex 2:

$$-\frac{g^2}{4}\int f^{abc} f^{ade}\, A_\mu^b(p)\, A_\nu^c(q)\, A_\rho^d(k)\, A_\sigma^e(\ell)\, g^{\nu\sigma}\, g^{\rho\mu}$$

$$\xrightarrow{\text{contraction}} \quad -\frac{g^2}{4}\int A_{\mu'}^{b'}(p')\, A_{\nu'}^{c'}(q')\, A_{\rho'}^{d'}(k')\, A_{\sigma'}^{e'}(\ell')\, g^{\nu'\sigma'}\, g^{\rho'\mu'}$$

$\longrightarrow$ 24 terms:

$$ig^2 \Big\{ f^{abe} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}\right) + f^{ace} f^{bde} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}\right) + f^{ade} f^{bce} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}\right)\Big\}$$

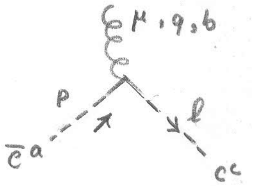


Ghost - Gluon Vertex:

$$-g\int f^{abc}\, \partial_\mu \bar{c}^a\, A^{\mu,b}\, c^c(x)\, d^4x = S_{ghost}.$$

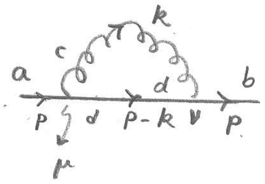
$$= -g \int_{p,q,\ell} f^{abc} (ip_\mu)\, \tilde{\bar{c}}^a(p)\, \tilde{A}^{\mu,b}(q)\, \tilde{c}^c(\ell)\, \underbrace{\int e^{-ipx+iqx+i\ell x} d^4x}_{=(2\pi)^4\delta(q+\ell-p)}$$

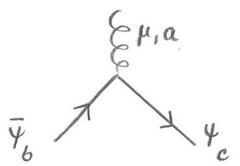
$$-ig\, p_\mu\, f^{abc}$$



One-loop-Renormalization:

a) Fermion - Self - Energy:





$$-g\, \bar{\psi}\, \gamma^\mu A_\mu^a t^a \psi \longrightarrow -ig\, \gamma^\mu (t^a)_{bc}$$

$\xi = 1$

$$-i\Sigma^{ab}(p) = (-ig)^2 \left(\mu^{\epsilon/2}\right)^2 \int \frac{d^dk}{(2\pi)^d}\, \gamma_\mu \frac{i}{\not p - \not k - m} \gamma_\nu \left(\frac{-ig^{\mu\nu}}{k^2}\right) \times (t^c)_{ad}\, (t^c)_{db}$$

$$(t^c)_{ad}\, (t^c)_{db} = (t^c t^c)_{ab} = C_2(r)\, \delta_{ab} \qquad t^a = \frac{\lambda^a}{2}$$

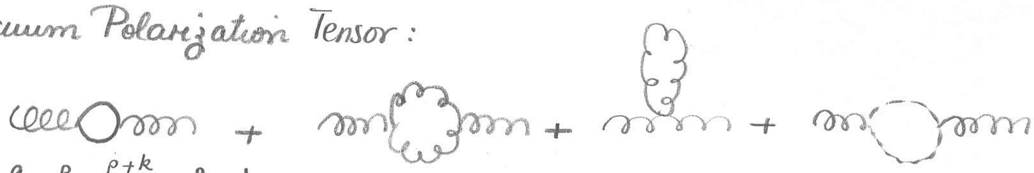
quadratic Casimir of the group.

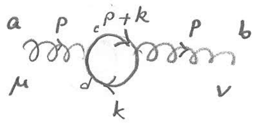
$\lambda^a =$ Gell-Mann matrices $a = 1, \dots 8$

$$C_2(r) \begin{cases} = \dfrac{N^2-1}{2N} & \text{for } SU(N) \\ = \dfrac{4}{3} & \text{for } SU(3) \end{cases}$$

$$\longrightarrow \quad \Sigma^{ab}(p) = \frac{g^2}{8\pi^2\epsilon}\, C_2(r)\, \delta^{ab}\, (-\not p + 4m) + \text{finite}.$$

b) Vacuum Polarization Tensor :



(b1) 

$$= -\,(t^a)_{dc}\,(t^b)_{cd}\,(-ig)^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\,\mathrm{tr}_f\left(\gamma_\mu\frac{i}{\not p-\not k-m}\,\gamma_\nu\,\frac{i}{\not k-m}\right)$$

$$= \frac{+g^2}{6\pi^2\epsilon}\,\underbrace{\mathrm{tr}(t^at^b)}_{=C(r)\,\delta^{ab}}\,(p_\mu p_\nu-p^2g_{\mu\nu})\times \underset{\mathrm{tr}_f}{N_f} + \text{finite}$$

$\mathrm{tr}(t^at^b) = C(r)\,\delta^{ab}$ $\qquad C(r) = \dfrac{d(r)}{d(G)}\,C_2(r)$

$d(r)$ dim of representation $\rightarrow$ 3 for fundamental quarks

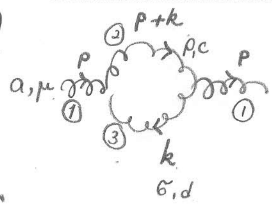
$d(G)$ dim of the group = # of generators $\rightarrow$ 8 for SU(3)

$C_2(r)$ = quadratic Casimir of the group $\rightarrow \frac{4}{3}$ for SU(3)

$\rightarrow C(r) = \frac{3}{8}\,\frac{4}{3} = \frac{1}{2}$ for $SU(3_c)$ $\qquad N_c = 3$

$\Longrightarrow$

 $$\boxed{\;= \frac{g^2}{6\pi^2\epsilon}\,N_f\,C(r)\,\delta^{ab}\,(p_\mu p_\nu - p^2 g_{\mu\nu})}$$

(b2) 

$$= \underset{\text{sym factor}}{+\tfrac{1}{2}}\,(g^2\mu^\epsilon)\,f^{acd}\,f^{bcd}\int\frac{d^dk}{(2\pi)^d}\,\frac{E_{\mu\nu}}{(p+k)^2\,k^2}$$

(البته در هر راس همه k ها باید به سمت داخل باشند)

$$E^{\mu,\rho\sigma}_{(1)} = g^{\rho\sigma}(-p-k-k)^\mu + g^{\mu\rho}(p+p+k)^\sigma + g^{\mu\sigma}(k-p)^\rho$$
$$= -g^{\rho\sigma}(2k+p)^\mu + g^{\mu\rho}(2p+k)^\sigma + g^{\mu\sigma}(k-p)^\rho$$

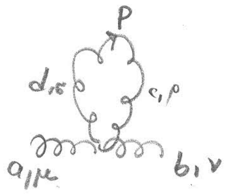
$$E^{\nu,\rho\sigma}_{(2)} = g^{\rho\sigma}(p+k+k)^\nu + g^{\nu\rho}(-p-p-k)^\sigma + g^{\nu\sigma}(-k+p)^\rho$$
$$= g^{\rho\sigma}(p+2k)^\nu - g^{\nu\rho}(2p+k)^\sigma - g^{\nu\sigma}(k-p)^\rho$$

$$E^{\mu\nu} = E^{\mu}_{\ \rho\sigma}\,E^{\nu,\rho\sigma}$$

$$f^{acd}\,f^{bcd} = C_2(G)\,\delta^{ab}$$

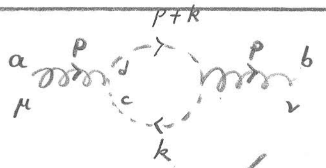
بعد از محاسبه طولانی

$$\longrightarrow\ \boxed{\frac{-g^2}{16\pi^2\epsilon}\,C_2(G)\,\delta^{ab}\left(\frac{11}{3}p_\mu p_\nu - \frac{19}{6}p^2 g_{\mu\nu}\right)}$$



$$= \frac{1}{2} \int \frac{d^dp}{(2\pi)^d} \left(\frac{-i g_{\rho\sigma}}{p^2} \right) \delta^{cd} \left(-i g \mu^{4-d} \right)$$
$$\times \Big(f^{ade} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right)$$
$$+ f^{ace} f^{bde} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho} \right)$$
$$+ f^{ade} f^{bce} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} \right) \Big) \int \frac{d^dp}{(2\pi)^d} \frac{1}{p^2} = 0$$

for $d \to 4$



$$= (-ig)^2 \, f^{cad} \, f^{dbc} \int \frac{(p+k)_\nu \, (k_\mu)}{(p+k)^2 \, k^2} (-1)$$

منفی برای loop اشباح بخاطر گراسمنی بودن میدانها منفی داریم (ولی tr نداریم زیرا ماتریسی در فضای دیراک در این مورد وجود ندارد)

$dbc = bcd$

$$f^{cad} f^{dbc} = \left(-f^{acd} \right) \left(f^{bcd} \right) = - C_2(G) \delta^{ab}$$

$$\longrightarrow \boxed{ \frac{g^2}{16\pi^2 \epsilon} C_2(G) \left(\frac{1}{3} p_\mu p_\nu + \frac{1}{6} g_{\mu\nu} p^2 \right) = }$$ 

Together:

$$= \frac{g^2}{6\pi^2 \epsilon} N_f \, C(r) \, \delta^{ab} \left(p_\mu p_\nu - p^2 g_{\mu\nu} \right)$$
$$+ \frac{g^2}{16\pi^2 \epsilon} C_2(G) \delta^{ab} \underbrace{\left(-\frac{11}{3} p_\mu p_\nu + \frac{19}{6} p^2 g_{\mu\nu} + \frac{1}{3} p_\mu p_\nu + \frac{1}{6} g_{\mu\nu} p^2 \right)}_{= \left(\frac{10}{3} p^2 g_{\mu\nu} - \frac{10}{3} p_\mu p_\nu \right)}$$

$$= \frac{g^2}{6\pi^2 \epsilon} \delta^{ab} \left(p_\mu p_\nu - p^2 g_{\mu\nu} \right) \left(N_f C(r) - \frac{5}{4} C_2(G) \right)$$

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for $SU(3)$: $\qquad C(r) = \frac{1}{2} \qquad C_2(G) = 3$

$$\left(N_f C(r) - \frac{5}{4} C_2(G) \right) = \frac{3}{4} \left(\frac{2}{3} N_f - 5 \right) \quad \checkmark$$

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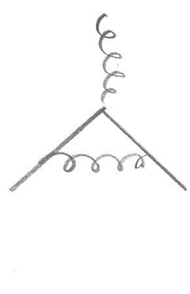
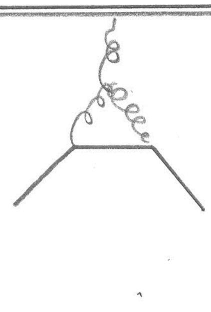
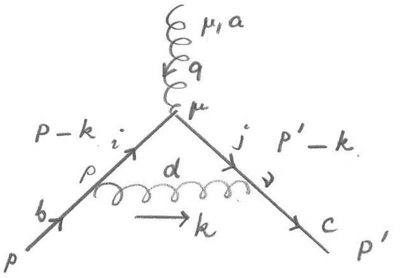
c) Vertex function :

 $+$ 



$$= (-ig)(\mu^{\frac{\epsilon}{2}})(\Lambda_\mu{}^a)_{bc} =$$

$$= (-ig\mu^{\epsilon/2})^3 \int \frac{d^dk}{(2\pi)^d} (\gamma_\nu)(t^d)_{jc} \frac{i}{(\not p' - \not k) - m}$$

$$\times (\gamma_\mu)(t^a)_{ij} \frac{i}{\not p - \not k - m} (t^d)_{bi} \gamma_\rho \left(\frac{-ig_{\rho\nu}}{k^2}\right)$$

$$t^d \, t^a \, t^d \, (\Lambda_\mu)_{QED} = \Lambda_\mu{}^a$$

$$t^d t^a t^d = t^d t^d t^a + t^d [t^a, t^d]$$

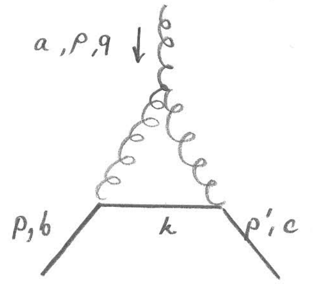
$$= C_2(r)\, t^a + i f^{adc}\, t^d t^c$$

$$= C_2(r)\, t^a + i f^{adc} \Big(\underbrace{\tfrac{1}{2} \{t^d, t^c\}}_{\substack{=0 \\ \text{sym} \times \text{antisym in } d\&c}} + \underbrace{\tfrac{1}{2} [t^d, t^c]}_{i f^{dce} t^e} \Big)$$

$dce = edc$

$$= C_2(r)\, t^a - \frac{1}{2} \underbrace{f^{adc} f^{dce}}_{= C_2(G)\, \delta^{ae}} t^e = \Big(C_2(r) - \frac{1}{2} C_2(G) \Big) t^a$$

$$\longrightarrow \boxed{\Lambda_\mu{}^a = \frac{g^2}{8\pi^2 \epsilon} \Big(C_2(r) - \frac{1}{2} C_2(G) \Big) t^a \gamma_\mu}$$



$$\sim f^{abc}\, t^b t^c = f^{abc} \Big(\tfrac{1}{2} \cancel{\{t^b, t^c\}} + \tfrac{1}{2} [t^b, t^c] \Big)$$

$$= \frac{i}{2} f^{abc} f^{ebc}\, t^e$$

$$= \frac{i}{2} \delta^{ae} t^e\, C_2(G) = \frac{i}{2} C_2(G)\, t^a$$

$$\longrightarrow \frac{g^2}{8\pi^2 \epsilon} \frac{3}{2} C_2(G)\, \gamma_\mu t^a$$

<u>together:</u> $\dfrac{g^2}{8\pi^2 \epsilon} \Big(C_2(G) + C_2(r) \Big) \gamma_\mu t^a$

<u>نتیجه</u>

$$\Sigma^{ab} = \frac{g^2}{8\pi^2 \epsilon} C_2(r)\, \delta^{ab} (-\not p + 4m) + \text{finite}$$

$$\Pi_{\mu\nu}{}^{ab} = \frac{g^2}{6\pi^2 \epsilon} \delta^{ab} (p_\mu p_\nu - p^2 g_{\mu\nu}) \Big(N_f\, C(r) - \frac{5}{4} C_2(G) \Big) + f.$$

$$\Lambda_\mu{}^a = \frac{g^2}{8\pi^2 \epsilon} \Big(C_2(r) + C_2(G) \Big) \gamma_\mu t^a + \text{finite}$$

$$\longrightarrow$$

<u>Renormalization:</u>

$$\mathcal{L} = \bar{\psi}\, i\not{\partial}\psi - m\bar{\psi}\psi - g\mu^{\epsilon/2}\,\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}^{a}F^{\mu\nu,a}$$

$$\mathcal{L}_{CT} = B\,\bar{\psi}\, i\not{\partial}\psi - A\bar{\psi}\psi - g\mu^{\epsilon/2}\, D\bar{\psi}\not{A}\psi - \frac{1}{4}\, C F_{\mu\nu}^{a}F^{\mu\nu,a}$$

$$\mathcal{L}_B = \mathcal{L}_{CT} + \mathcal{L}$$

$$= \underbrace{(1+B)}_{=Z_2}\bar{\psi}(i\not{\partial})\psi - (m+A)\bar{\psi}\psi - g\mu^{\epsilon/2}\underbrace{(1+D)}_{=Z_1}\bar{\psi}\not{A}\psi - \frac{1}{4}\underbrace{(1+C)}_{=Z_3}F_{\mu\nu}^{a}F^{\mu\nu,a}$$

$$\psi_B = Z_2^{1/2}\psi$$

! $A_\mu^{B} = Z_3^{1/2}A_\mu$ تمرین از جمله مانند QED :

$$g_B = g\mu^{\epsilon/2} Z_1 Z_3^{-1/2} Z_2^{-1}$$

$$m_B = (m+A)Z_2^{-1}$$

$i\not{\partial} = \not{p}$

$$Z_2 = 1 + B = 1 - \frac{g^2}{8\pi^2\epsilon}C_2(r)$$

$$Z_1 = 1 + D = 1 - \frac{g^2}{8\pi^2\epsilon}\left(C_2(r) + C_2(G)\right)$$

$$Z_3 = 1 + C = 1 - \frac{g^2}{6\pi^2\epsilon}\left(N_f C(r) - \frac{5}{4}C_2(G)\right) \qquad Z_1 \neq Z_2 \;!$$

$$g_B = g\mu^{\epsilon/2}\left(1 - \frac{g^2}{8\pi^2\epsilon}\left(C_2(r) + C_2(G)\right)\right)\left(1 + \frac{g^2}{12\pi^2\epsilon}\left(N_f C(r) - \frac{5}{4}C_2(G)\right)\right) \times \left(1 + \frac{g^2}{8\pi^2\epsilon}C_2(r)\right)$$

$$\simeq g\mu^{\epsilon/2}\left(1 - \frac{g^2}{48\pi^2\epsilon}\left[6C_2(r) + 6C_2(G) - 4N_f C(r) + 5C_2(G) - 6C_2(r)\right]\right)$$

$$\boxed{g_B \simeq g\mu^{\epsilon/2}\left\{1 - \frac{g^2}{48\pi^2\epsilon}\left(-4N_f C(r) + 11\, C_2(G)\right)\right\} + O(g^4)}$$

for SU(3) $\quad C(r) = \frac{1}{2}\,, \quad C_2(G) = 3$

$$g_B = g\mu^{\epsilon/2}\left\{1 - \frac{g^2}{16\pi^2\epsilon}\left(11 - \frac{2}{3}N_f\right)\right\} \quad \text{for } SU(3) \text{ at one-loop}$$

QCD – β-function :

$$\mu\frac{\partial}{\partial\mu}g_B = 0 = \mu\frac{\partial}{\partial\mu}\left(g\mu^{\epsilon/2}\left(1 + \frac{Ag^2}{16\pi^2\epsilon}\right)\right) \qquad A = -\left(11 - \frac{2}{3}N_f\right) \text{ for } SU(3)$$

$$\rightarrow \quad A = -\left(\frac{11}{3}C_2(G) - \frac{4}{3}C(r)N_f\right)$$

for $SU(N_c)$

with N_f flavors.

$$0=\mu\frac{\partial}{\partial\mu}\left(\mu^{\epsilon/2}g\left(1+\frac{Ag^2}{16\pi^2\epsilon}\right)\right)$$

$$=\frac{\epsilon}{2}\mu^{\epsilon/2}g\left(1+\frac{Ag^2}{16\pi^2\epsilon}\right)+\mu^{\epsilon/2}\beta(g)\left(1+\frac{Ag^2}{16\pi^2\epsilon}\right)$$

$$+\mu^{\epsilon/2}\frac{2Ag^2}{16\pi^2\epsilon}\beta(g)$$

$$=\frac{\epsilon}{2}\mu^{\epsilon/2}g\left(1+\frac{Ag^2}{16\pi^2\epsilon}\right)+\beta(g)\mu^{\epsilon/2}\left(1+\frac{Ag^2}{16\pi^2\epsilon}+\frac{2Ag^2}{16\pi^2\epsilon}\right)$$

$$\rightarrow\ \beta(g)\cong-g\frac{\epsilon}{2}\left(1+\frac{Ag^2}{16\pi^2\epsilon}\right)\left(1-\frac{3Ag^2}{16\pi^2\epsilon}\right)$$

$$=-g\frac{\epsilon}{2}\left(1-\frac{2Ag^2}{16\pi^2\epsilon}\right)=-g\frac{\epsilon}{2}+\frac{Ag^3}{16\pi^2}$$

$\epsilon\rightarrow 0$

$$\beta(g)=\frac{Ag^3}{16\pi^2}=\frac{-g^3}{16\pi^2}\left(11-\frac{2}{3}N_f\right)\quad \text{for } SU(3_c)$$

$$=\frac{-g^3}{16\pi^2}\left(\frac{11}{3}C_2(G)-\frac{4}{3}N_f\,C(r)\right)\quad \text{for } SU(N)$$

In $SU(3)$ $\quad 11-\frac{2}{3}N_f>0 \rightarrow N_f<\frac{33}{2}\rightarrow \beta<0$

in our world $\quad N_f=6 \Longrightarrow$ QCD has a negative β-function

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به همان روش قبلی بدست بیاید $\quad \Lambda_{QCD}=\mu_0\exp\left(-\frac{1}{2Ag^2(\mu_0)}\right)=\mu\exp\left(\frac{-1}{2Ag^2(\mu)}\right)$