نظریه میدان کوانتومی ذرات ۱- آبان ۹۵- صفحه ۹۲

Remember:

- Vertex function for a scalar field theory $\longleftrightarrow$ n-point Green's function

$$G_c^{(n)}(x_1, \ldots, x_n) = \left(\frac{1}{i}\right)^{n-1} \frac{\delta^n}{\delta J(x_1) \cdots \delta J(x_n)} W[J]\Big|_{J=0}$$

n-point green's function

$$W[J] = \sum_{n=1}^{\infty} \frac{(i)^{n-1}}{n!} \int d^d x_1 \ldots d^d x_n \; G_c^{(n)}(x_1, \ldots, x_n)\, J(x_1) \ldots J(x_n)$$

$$\Gamma^{(n)}(x_1, \ldots, x_n) = \frac{\delta^n}{\delta\varphi(x_1) \ldots \delta\varphi(x_n)} \Gamma[\varphi]\Big|_{\varphi=0}$$

n-point Vertex function.

$$\Gamma[\varphi] = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^d x_1 \ldots d^d x_n \; \Gamma^{(n)}(x_1, \ldots, x_n)\, \varphi(x_1) \ldots \varphi(x_n)$$

Other properties of $\Gamma^{(n)}(x_1, \ldots x_n)$ & $G_c^{(n)}(x_1, \ldots x_n)$

$$\Phi_J(x) = \frac{\delta W[J]}{\delta J(x)} \overset{1}{=} \langle \varphi(x) \rangle_J \quad ; \quad \frac{\delta \Gamma[\Phi]}{\delta \Phi(x)}\Big|_{\Phi_J} \overset{2}{=} -J(x)$$

1) $$\frac{\delta^2 W[J]}{\delta J(x)\, \delta J(y)}\Big|_{J=0} = \frac{\delta \Phi_J(x)}{\delta J(y)}\Big|_{J=0} = +i\, G_c^{(2)}(x, y) \qquad (3)$$

2) $$\frac{\delta^2 \Gamma[\Phi_J]}{\delta \Phi_J(x)\, \delta \Phi_J(z)}\Big|_{\Phi_J=0} = -\frac{\delta J(x)}{\delta \Phi_J(z)}\Big|_{\Phi_J=0} = \Gamma^{(2)}(x, z)$$

now: $$\int \Gamma^{(2)}(x, z)\, G_c^{(2)}(z, y)\, d^d z = -(-i) \int \frac{\delta J(x)}{\delta \Phi_J(z)}\Big|_{\Phi_J=0} \frac{\delta \Phi_J(z)}{\delta J(y)}\Big|_{J=0} d^d z$$
$$= i\, \delta^d(x - y)$$

$$\nearrow \quad \boxed{\int d^d z \;\; \Gamma^{(2)}(x, z)\, G_c^{(2)}(z, y) = i\, \delta^d(x - y)}$$

In mom-space $\quad \Gamma^{(2)}(p)\, G_c^{(2)}(p) = i$

با فرض $\quad \Gamma^{(2)}(P_1, P_2) = \Gamma^{(2)}(P_1)\, \delta^d(P_1 + P_2)$ and

$\quad G_c^{(2)}(P_1, P_2) = G_c^{(2)}(P_1)\, \delta^d(P_1 + P_2)$ $\qquad$ ok ✓

از روابط فوق نتیجه میگیرم :

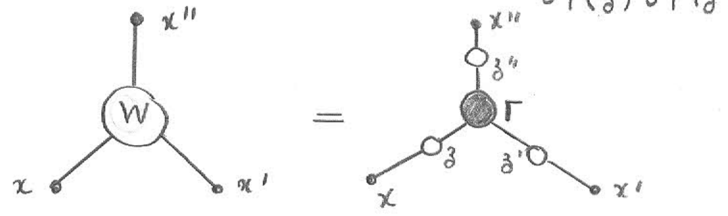
$$(*)\quad \boxed{\left.\frac{\delta}{\delta J(x)}\right|_{J=0} = \int \left.\frac{\delta \phi_J(y)}{\delta J(x)}\,\frac{\delta}{\delta \phi_J(y)}\right|_{\substack{J=0\\ \phi_J=0}} d^d y = i\int G_c^{(2)}(y-x)\left.\frac{\delta}{\delta \phi_{cl}(y)}\right|_{\phi_{cl}=0}}$$

Remember: $\phi_{J=0} = \phi_{cl}(x) = \langle \varphi(x) \rangle_{J=0}$

ادعا :

A) $$\frac{\delta^3 \Gamma[0]}{\delta\phi(y)\,\delta\phi(y')\,\delta\phi(y'')} = \int d^4x\, d^4x'\, d^4x''\; \Gamma^{(2)}(x,y)\,\Gamma^{(2)}(x',y')\,\Gamma^{(2)}(x'',y'') \times \left.\frac{\delta^3 W[J]}{\delta J(x)\,\delta J(x')\,\delta J(x'')}\right|_{J=0} = \Gamma^{(3)}(y,y',y'')$$

B) $$\frac{i\,\delta^3 W[0]}{\delta J(x)\,\delta J(x')\,\delta J(x'')} = \int d^4z\, d^4z'\, d^4z''\; G_c^{(2)}(x-z)\,G_c^{(2)}(x'-z')\,G_c^{(2)}(x''-z'') \times \left.\frac{\delta^3 \Gamma[\phi]}{\delta\phi(z)\,\delta\phi(z')\,\delta\phi(z'')}\right|_{\phi=0} = G_c^{(3)}(x,x',x'')$$



در شکل : 

$G_c^{(2)}(x-y)$

$G_c^{(3)}(x,x',x'') \leftrightarrow \Gamma^{(3)}(z,z',z'')$

$$\int d^4z\; G_c^{(2)}(x-z)\,\Gamma^{(2)}(z-z') = i\,\delta^d(x-z')$$

اثبات :

قدم اول : از دو طرف $\frac{\delta}{\delta J(x'')}$ می‌گیریم و از رابطه (*) استفاده می‌کنیم :

$$0 \overset{(3)}{=} \frac{\delta}{\delta J(x'')} \int d^d z\, (-i) \left.\frac{\delta^2 W[J]}{\delta J(x)\,\delta J(z)}\right|_{J=0} \left.\frac{\delta^2 \Gamma[\phi]}{\delta\phi(z)\,\delta\phi(z')}\right|_{\phi=0}$$

$$0 = \int d^d z\,(-i) \left.\frac{\delta^3 W[J]}{\delta J(x)\,\delta J(z)\,\delta J(x'')}\right|_{J=0} \underbrace{\frac{\delta^2\Gamma[0]}{\delta\phi(z)\,\delta\phi(z')}}_{=\Gamma^{(2)}(z-z')}$$

$$+(-i)\int d^d z\, \underbrace{\frac{\delta^2 W[0]}{\delta J(x)\,\delta J(z)}}_{i G_c^{(2)}(x-z)}\; \underbrace{\frac{\delta}{\delta J(x'')}\,\frac{\delta^2\Gamma[0]}{\delta\phi(z)\,\delta\phi(z')}}$$

$$= \int d^d z''\, \underbrace{\left.\frac{\delta\phi(z'')}{\delta J(x'')}\right|_{J=0}}_{i\,G_c^{(2)}(z''-x'')} \frac{\delta^3\Gamma[0]}{\delta\phi(z)\,\delta\phi(z')\,\delta\phi(z'')}$$

$$\int d^d z \; \Gamma^{(2)}(z,z') \frac{\delta^3 W[0]}{\delta J(x)\,\delta J(z)\,\delta J(x'')} = -i^2 \int d^d z\, d^d z'' \; G_c^{(2)}(x-z)\, G_c^{(2)}(z''-x'') \times \frac{\delta^3 \Gamma[0]}{\delta\phi(z)\,\delta\phi(z')\,\delta\phi(z'')}$$

قدم دوم: این رابطه را از دو طرف در $G_c^{(2)}(z'-x')$ ضرب می‌کنیم و روی z' انتگرال می‌گیریم:

$$\int d^d z \; \overbrace{\int d^d z' \; \Gamma^{(2)}(z,z')\, G_c^{(2)}(z'-x')}^{=\, i\,\delta^d(z-x')} \; \frac{\delta^3 W[0]}{\delta J(x)\,\delta J(x'')\,\delta J(z)}$$

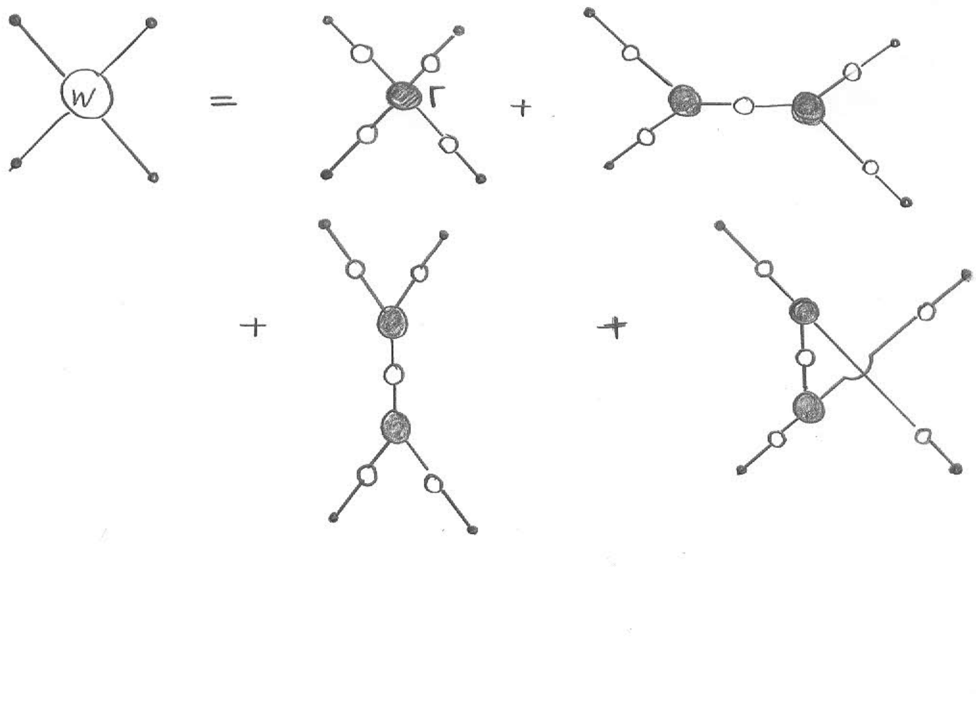
$$= \int d^d z\, d^d z'\, d^d z'' \; G_c^{(2)}(x-z)\, G_c^{(2)}(x'-z')\, G_c^{(2)}(x''-z'') \times \frac{\delta^3 \Gamma[0]}{\delta\phi(z)\,\delta\phi(z')\,\delta\phi(z'')}$$

با انتگرال‌گیری روی z

$$\boxed{\frac{i\,\delta^3 W[0]}{\delta J(x)\,\delta J(x')\,\delta J(x'')} = \int d^d z\, d^d z'\, d^d z'' \; G_c^{(2)}(x-z)\, G_c^{(2)}(x'-z')\, G_c^{(2)}(x''-z'') \times \frac{\delta^3 \Gamma[0]}{\delta\phi(z)\,\delta\phi(z')\,\delta\phi(z'')}}$$

q.e.d.

به همین ترتیب می‌توان نشان داد:



Fermions & Functional Method.

✓ در کوانتش کانونیک میدان فرمیونی ← Anticommuting Operators

$$\{\psi(\vec{x},t),\ \psi^{+}(\vec{y},t)\} \neq 0$$

✓ در روش کوانتش از طریق انتگرال مسیر، میدان‌های فرمیونی از طریق توابع anticommuting توصیف می‌شوند.

- <u>Grassmann Variables:</u>

$$\{C_i, C_j\} = C_i C_j + C_j C_i \qquad j,i = 1,\dots,n$$

<u>خواص :</u> 1) $C_i^2 = 0$

2) بسط $f(C_1,C_2)$ در C_i :

$$f(C_1,C_2) = a_0 + a_1 C_1 + a_2 C_2 + a_3 C_1 C_2 + \cancel{a_4 C_1^2} + \cancel{a_5 C_2^2} + \cancel{a_6 C_1^2 C_2} + \dots$$

$$= a_0 + a_1 C_1 + a_2 C_2 + a_3 C_1 C_2$$

$a_0 \dots a_3$ are ordinary "commuting" numbers

3) مشتق‌گیری از راست و چپ :

از چپ : $$\frac{\partial^L f(C_1,C_2)}{\partial C_1} = a_1 + a_3 C_2$$

از راست : $$\frac{\partial^R f(C_1,C_2)}{\partial C_1} = a_1 - a_3 C_2 \qquad C_1 C_2 = -C_2 C_1$$

4) $\frac{\partial}{\partial C_i}$ is also a Grassmann operator

$$\left\{C_i, \frac{\partial}{\partial C_j}\right\} = \delta_{ij} \qquad \left\{\frac{\partial}{\partial C_i}, \frac{\partial}{\partial C_j}\right\} = 0$$

ادعا : $$\left\{C_1, \frac{\partial^L}{\partial C_1}\right\} f(C_1,C_2) = f(C_1,C_2)$$

اثبات :

✓ $$C_1 \frac{\partial^L}{\partial C_1} f(C_1,C_2) = C_1 (a_1 + a_3 C_2) = a_1 C_1 + a_3 C_1 C_2$$

$$C_1 f = a_0 C_1 + \cancel{a_1 C_1^2} + a_2 C_1 C_2 + \cancel{a_3 C_1^2 C_2} = a_0 C_1 + a_2 C_1 C_2$$

✓ $$\frac{\partial^L}{\partial C_1} C_1 f = a_0 + a_2 C_2$$

$$\left\{C_1, \frac{\partial^L}{\partial C_1}\right\} f = C_1 \frac{\partial^L}{\partial C_1} f + \frac{\partial^L}{\partial C_1}(C_1 f) = a_0 + a_1 C_1 + a_2 C_2 + a_3 C_1 C_2$$

$$= f(C_1,C_2)$$

$$\rightarrow \boxed{\left\{C_1, \frac{\partial^L}{\partial C_1}\right\} f(C_1,C_2) = f(C_1,C_2)}$$

5) انتگرال

dC_i is also Grassmannian

$$\{C_i, dC_j\} = 0 \qquad \{dC_i, dC_j\} = 0$$

Multiple integral: $\int dC_1\, dC_2\, f(C_1, C_2) = \int dC_1 \left(\int dC_2\, f(C_1, C_2)\right)$

$$\left(\int dC_1\right)^2 = \int dC_1\, dC_2 = -\int dC_2\, dC_1 = -\left(\int dC_1\right)^2$$

$$\rightarrow \left(\int dC_1\right)^2 = 0 \longrightarrow \int dC_1 = 0$$

but $\int dC_1\, C_1 = 1$ قرارداد

n-dim $\int dC_i = 0 \qquad \int dC_i\, C_i = 1$ (no summation over i)

$$\int dC_1\, f(C_1, C_2) = \int dC_1 \left(a_0 + a_1 C_1 + a_2 C_2 + a_3 C_1 C_2\right) = a_1 + a_3 C_2$$

همانطور که قبلاً نشان داده شد. $= \dfrac{\partial^L f(C_1, C_2)}{\partial C_1}$

به این ترتیب انتگرال‌گیری و مشتق‌گیری هر دو یک جواب می‌دهند.

؟

$$\int dC_1 \int dC_1\, f(C_1, C_2) = \int dC_1 \left(a_1 + a_3 C_2\right) = 0 \checkmark$$

(یعنی) $\dfrac{\partial^L}{\partial C_1}\left(\dfrac{\partial^L}{\partial C_1} f(C_1, C_2)\right) = 0 \checkmark$

Complex Grassmann Variables:

$$\eta = \eta_1 + i\eta_2 \qquad \eta^* = \eta_1 - i\eta_2 \qquad \{\eta, \eta^*\} = 0$$

Gaussian integral for η, η^*

$$\int d\eta_1\, d\eta_1^*\, d\eta_2\, d\eta_2^* \cdots d\eta_n\, d\eta_n^*\, \exp\left((\eta^*, A\eta)\right) = \det A$$

$$(\eta^*, A\eta) = \sum_{i,j} \eta_i^* A_{ij} \eta_j$$

$f(x) = y$ $\qquad$ تابع گراسمنی $\leftarrow \psi(x) = \eta \hookrightarrow$ Grassmann variable

عدد معمولی

$$\{\psi(x), \psi(y)\} = 0$$

$$\frac{\delta \psi(x)}{\delta \psi(y)} = \delta(x - y) \qquad \int d\psi(x) = 0$$

$$\int d\psi(x)\, \psi(x) = 1$$

Free fermions and Generating functional:

$S_0=\int d^dx\, \mathcal{L}_0$; $\quad \mathcal{L}_0 = \bar\Psi(x)\,(i\not\partial - m)\,\Psi(x) = \bar\Psi(x)\, S_f^{-1}\, \Psi(x)$; $\quad S_f^{-1} = i\not\partial - m$

$$Z_0[\eta,\bar\eta] = \frac{\int \mathcal{D}\bar\Psi \mathcal{D}\Psi \exp\left(iS_0 + i\int \bar\Psi\eta + i\int\bar\eta\Psi\right)}{\int \mathcal{D}\bar\Psi\,\mathcal{D}\Psi\; e^{iS_0}}$$

می‌توان نشان داد:

$$\boxed{\begin{aligned} Z_0[\eta,\bar\eta] &= \mathcal{N}\int \mathcal{D}\bar\Psi\mathcal{D}\Psi \exp\left(i\int\left(\bar\Psi S_f^{-1}\Psi + \bar\Psi\eta + \bar\eta\Psi\right)d^dx\right) \\ &\overset{\frown}{=} \exp\left(-i\int d^dx\, d^dy\;\; \bar\eta_\alpha(x)\, S_f^{\alpha\beta}(x-y)\,\eta_\beta(y)\right)\end{aligned}}$$

بعد از انتگرال‌گیری تابعی روی $\bar\Psi,\Psi$

$$S_f(x-y) = \int\frac{d^dp}{(2\pi)^d}\,\frac{i\,e^{-ip(x-y)}}{\not p - m}$$

Two point function:

$$\left(\frac{1}{i}\right)^2 \frac{\delta^2}{\delta\eta_\alpha(x)\,\delta\bar\eta_\beta(y)}\, Z_0[\eta,\bar\eta]\Big|_{\eta,\bar\eta=0} = \langle 0|T(\Psi_\beta(y)\,\bar\Psi_\alpha(x))|0\rangle = i\,S^f_{\beta\alpha}(y-x)$$

a) $$\frac{\delta}{\delta\bar\eta_\beta(y)}\, Z_0[\eta,\bar\eta] = \frac{\delta}{\delta\bar\eta_\beta(y)} \exp\left(-i\int d^dz_1\, d^dz_2\; \bar\eta_\rho(z_1)\, S_f^{\rho\sigma}(z_1-z_2)\,\eta_\sigma(z_2)\right)$$

$$= \left\{-i\int d^dz_1\, d^dz_2\;\; \delta^\beta{}_\rho\;\; \delta(y-z_1)\; S_f^{\rho\sigma}(z_1-z_2)\;\eta_\sigma(z_2)\right\} Z_0$$

$$= \left\{-i\;\;\int d^dz_1\;\; S_f^{\beta\sigma}\,(y-z_2)\,\eta_\sigma(z_2)\right\} Z_0$$

b) $$\frac{\delta}{\delta\eta_\alpha(x)}\left(\frac{\delta}{\delta\bar\eta_\beta(y)}\, Z_0[\eta,\bar\eta]\right)\Big|_{\eta=\bar\eta=0} = -i\int d^dz_1\;\; S_f^{\beta\sigma}(y-z_2)\;\delta^\alpha{}_\sigma\;\delta(z_2-x)$$

$$= -i\,S_f^{\beta\alpha}\,(y-x)$$

$$\boxed{\left(\frac{1}{i}\right)^2 \frac{\delta^2 Z_0[\eta,\bar\eta]}{\delta\bar\eta_\beta(y)\,\delta\eta_\alpha(x)}\Big|_{\eta=\bar\eta=0} = i\,S_f^{\beta\alpha}(y-x) = \langle 0|T(\Psi_\beta(y)\bar\Psi_\alpha(x))|0\rangle}$$

Interacting Theories: $\qquad$ Yukawa theory:

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$ $\qquad \mathcal{L}_{int} = g\,\bar\Psi_\alpha\Psi^\alpha\varphi$ $\qquad \mathcal{L}_0 = \bar\Psi\,(i\not\partial - m_f)\Psi$

$$\begin{aligned} Z[\eta,\bar\eta,J] &= \mathcal{N}\exp\left(i\int \mathcal{L}_{int}\left[\frac{1}{i}\frac{\delta}{\delta\eta},\ \frac{1}{i}\frac{\delta}{\delta\bar\eta},\ \frac{1}{i}\frac{\delta}{\delta J}\right]\right) Z_0[J]\; Z_0[\eta,\bar\eta] \\ &= \mathcal{N}\exp\left(ig\int d^dz\;\left(\frac{1}{i}\right)^3 \frac{\delta^3}{\delta\eta_\alpha(z)\,\delta\bar\eta^\alpha(z)\,\delta J(z)}\right) Z_0[J]\; Z_0[\eta,\bar\eta]\Bigg|_{\eta=\bar\eta=J=0}\end{aligned}$$

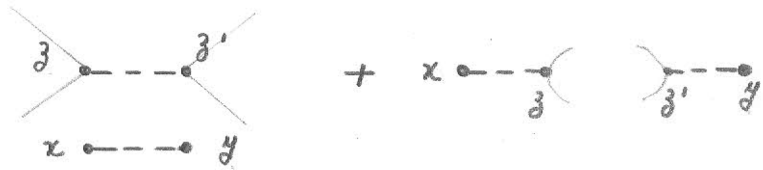
$$\langle \Omega | T(\varphi(x)\varphi(y)) | \Omega \rangle \Big|_{1-loop} = \left(\frac{1}{i}\right)^2 \frac{\delta}{\delta J(x)\, \delta J(y)} \, Z[\eta, \bar\eta, J] \Big|_{\substack{J=0 \\ \eta=\bar\eta=0}}$$

$$= \frac{(ig)^2}{2!} \left(\frac{1}{i}\right)^{6+2} \frac{\delta^2}{\delta J(x) \delta J(y)} \int d^4z\, d^4z' \; \frac{\delta^4 Z_0[J]}{\delta J(x)\delta J(y)\, \delta J(z)\, \delta J(z')} \Big|_{J=0}$$

$$\times \frac{\delta^2}{\delta \eta_\alpha(z)\, \delta \bar\eta^\alpha(z)} \; \frac{\delta^2}{\delta \eta_\beta(z')\, \delta \bar\eta^\beta(z')} \; Z_0[\eta, \bar\eta] \Big|_{\eta=\bar\eta=0}$$

The bosonic part: $\displaystyle \frac{\delta^4}{\delta J(x)\delta J(y)\delta J(z)\delta J(z')} Z_0[J]\Big|_{J=0}$

$$= \left(-i\Delta_F(y-x)\right)\left(-i\Delta_F(z-z')\right) + \left(-i\Delta_F(z-x)\right)\left(-i\Delta_F(y-z')\right)$$



The fermionic part: $\displaystyle \frac{\delta^2}{\delta\eta_\alpha(z)\,\delta\bar\eta^\alpha(z)} \; \frac{\delta^2}{\delta\eta_\beta(z')\,\delta\bar\eta^\beta(z')} \; Z_0[\eta,\bar\eta]\Big|_{\eta=\bar\eta=0}$

a) $\displaystyle \frac{\delta}{\delta\bar\eta^\beta(z')} \exp\left(-i\int d^4u\, d^4w\; \bar\eta_\gamma(u)\, S^{\gamma\gamma'}(u-w)\, \eta_{\gamma'}(w)\right)$

$$= \left(-i\int d^4w\; S_\beta{}^{\gamma'}(z'-w)\, \eta_{\gamma'}(w)\right) e^{-i\int\bar\eta S\eta}$$

b) $\displaystyle \frac{\delta}{\delta\eta_\beta(z')} (a) = \left(-i\; S_\beta{}^\beta(z'-z')\right) e^{-i\int\bar\eta S\eta}$

$$+(-1)^2 \left(-i\int d^4w\; S_\beta{}^{\gamma'}(z'-w)\,\eta_{\gamma'}(w)\right)\left(-i\int d^4u\; \bar\eta_\gamma(u)\, S^{\gamma\beta}(u-z')\right) \times e^{-i\int\bar\eta S\eta}$$

c) $\displaystyle \frac{\delta}{\delta\bar\eta^\alpha(z)} (b) = \left(-i\,S_\beta{}^\beta(z'-z')\right)\left(-i\int d^4w\; S_\alpha{}^{\gamma'}(z-w)\eta_{\gamma'}(w)\right) e^{-i\int\bar\eta S\eta}$

$$+(-1)\left(-i\int d^4w\; S_\beta{}^{\gamma'}(z'-w)\eta_{\gamma'}(w)\right)\left(-i\; S_\alpha{}^\beta(z-z')\right) e^{-i\int\bar\eta S\eta}$$

$$+(-1)^2\left(-i\int d^4w\; S_\beta{}^{\gamma'}(z'-w)\,\eta_{\gamma'}(w)\right)\left(-i\int d^4u\; \bar\eta_\gamma(u)\, S^{\gamma\beta}(u-z')\right)$$

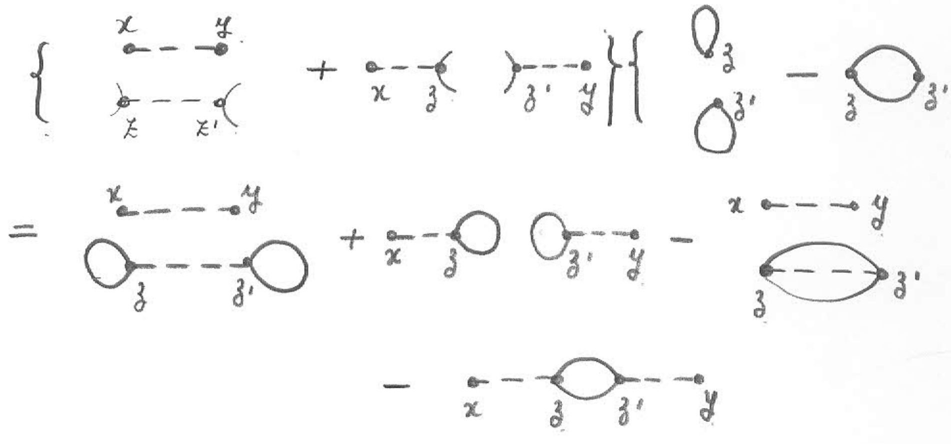
$$\times\left(-i\int d^4w\; S_\alpha{}^{\gamma'}(z-w)\,\eta_{\gamma'}(w)\right) e^{-i\int\bar\eta S\eta}$$

d) $\displaystyle \frac{\delta}{\delta\eta_\alpha(z)} (c)\Big| = \left(-i\,S_\beta{}^\beta(z'-z')\right)\left(-i\; S_\alpha{}^\alpha(z-z)\right)$

$$-\left(-i\,S_\beta{}^\alpha(z'-z)\right)\left(-i\; S_\alpha{}^\beta(z-z')\right)$$

$$\xrightarrow{\text{نتیجه}} \quad \left(-iS_\beta{}^{\beta}(0)\right)\left(-iS_\alpha{}^{\alpha}(0)\right) - \left(-iS_\beta{}^{\alpha}(z'-z)\right)\left(-iS_\alpha{}^{\beta}(z-z')\right)$$

Together:



disconnected diagrams

Connected diagram

The connected term:

$$= -\frac{g^2}{2}\int d^4z\, d^4z' \left(-i\Delta_F(z'-y)\right)\left(-i\Delta_F(x-z)\right)(-1)\left(-iS_\beta{}^{\alpha}(z'-z)\right)\left(-iS_\alpha{}^{\beta}(z-z')\right)$$

(-1) trace for closed fermion loop.

QED & Photon propagator (Maxwell theory)

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \Rightarrow \frac{1}{2}\int d^4x\, A_\mu(x)\left(g^{\mu\nu}\Box - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right)A_\nu(x) = S$$

$$Z_0[J^\mu] = \frac{\int \mathcal{D}A_\mu\, e^{iS_g + i\int J_\mu(z)A^\mu(z)\, d^dz}}{\int \mathcal{D}A_\mu\, e^{iS_g}}$$

$$\underset{\text{Integration over } A_\mu}{=} \exp\left(\frac{-i}{2}\int d^4x\, d^4y\; J_\mu(x)D^{\mu\nu}(x-y)J_\nu(y)\right)$$

$$\left(g^{\mu\nu}\Box - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right)D_{\nu\rho}(x-y) = \delta^\mu{}_\rho\; \delta^d(x-y)$$

$$\boxed{\langle 0|T(A_\mu(x)A_\nu(y))|0\rangle = \left(\frac{1}{i}\right)^2 \frac{\delta^2 Z_0[J^\mu]}{\delta J^\mu(x)\,\delta J^\nu(y)}\bigg|_{J^\mu=0}}$$