قدم دوم: در نهایت $\eta(y)$ را حول $\eta(x)$ بسط می‌دهیم

$$y = x + z$$

$$\eta(y = x+z) = \eta(x) + z_\mu \partial^\mu \eta(x) + z^\mu z^\nu \partial_\mu \partial_\nu \eta(x) + \dots$$

با جایگذاری در رابطه (A) داریم:

$$\Gamma[\phi_{cl}] = \Gamma[\phi_{cl}^{(0)}] - \frac{1}{2}\int d^4x \; M^2[\phi_{cl}]\,\eta^2(x) + \frac{1}{2}\int \chi^{\mu\nu}[\phi_{cl}^{(0)}]\,\partial_\mu \eta(x)\,\partial_\nu (x)\, d^4x + \dots$$

with
$$M^2 = -\int \frac{\delta^2 \Gamma[\phi]}{\delta\phi(0)\,\delta\phi(z)}\Big|_{\phi_{cl}^{(0)}} d^4 z$$

$$\chi^{\mu\nu} = -\frac{1}{2}\int \frac{\delta^2\Gamma[\phi]}{\delta\phi(0)\,\delta\phi(z)}\Big|_{\phi_{cl}^{(0)}} z^\mu z^\nu$$

نمایش این بطور کلی

$$\Gamma[\phi] = \int d^4x \left\{ \frac{1}{2} Z_\phi\, \partial_\mu\phi\, \partial^\mu\phi - U(\phi_{cl}^{(0)}) \right\} + \dots$$

$$\phi(x) = \phi_{cl}^{(0)} + \eta(x)$$

$$\partial_\mu \phi = \partial_\mu \eta(x)$$

$$\phi_{cl}^{(0)} = const \qquad U(\phi_{cl}^{(0)}) \quad \text{پتانسیل موثر}$$

(در ارتباط با شکست خودبخودی تقارن مورد استفاده زیادی دارد).

در اغلب موارد فرض می‌کنند min کنش موثر یک میدان ثابت است ، $\phi_{cl}^{(0)} = const$ (یعنی داریم $\eta(x) = 0$ است)

در اینصورت:

$$\Gamma[\phi_{cl}^{(0)}] = -\int d^dx \; U(\phi_{cl}^{(0)}) = -\Omega_d\, U(\phi_{cl}^{(0)})$$

Ω_d: حجم d بعدی — $U(\phi_{cl}^{(0)})$: پتانسیل موثر.

($\longrightarrow$ later Spontaneous Symmetry Breaking)

نوع دیگری از رابطه برای پتانسیل موثر:

$$\Gamma[\phi_{cl}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^dx_1 \dots d^dx_n \times \Gamma^{(n)}(x_1, \dots, x_n)\, \phi_{cl}(x_1) \dots \phi_{cl}(x_n)$$

اگر $\phi_{cl} = \phi_{cl}^{(0)}$ ثابت باشد

$$\phi_{cl}^{(0)} = const. \longrightarrow \Gamma[\phi_{cl}^{(0)}] = -\Omega_d\, U(\phi_{cl}^{(0)})$$

$$\Gamma[\phi_{cl}^{(0)}] = \sum_{n=0}^{\infty} \frac{1}{n!} \underbrace{\int d^dx_1 \dots d^dx_n \; \Gamma^{(n)}(x_1, \dots, x_n)}_{= \tilde{\Gamma}^{(n)}(p_1, \dots p_n)\big|_{p_i = 0}} (\phi_{cl}^{(0)})^n$$

zero mode of $\Gamma^{(n)}$ in mom-space.

$\Longrightarrow$

$$\Gamma[\Phi_{cl}] = \sum_{n=0}^{\infty} \frac{1}{n!} \tilde{\Gamma}^{(n)}(p_1, \dots p_n)\Big|_{\{p_i\}=0} \left(\Phi_{cl}^{(0)}\right)^n = -\Omega\, U(\Phi_{cl}^{(0)})$$

$$\longrightarrow \boxed{U(\Phi_{cl}^{(0)}) = -\frac{1}{\Omega} \sum_{n=0}^{\infty} \frac{1}{n!} \tilde{\Gamma}^{(n)}(p_1, \dots, p_n)\Big|_{p_i=0}}$$

(۲) One-loop effective action کنش مؤثر تا مرتبه تک حلقه :

Saddle point expansion :

$$f(x) = f(x_0) + (x - x_0) \frac{df(x)}{dx}\Big|_{x_0} + \frac{1}{2!}(x - x_0)^2 \frac{d^2 f}{dx^2}\Big|_{x_0} + \cdots$$

اگر $\frac{df(x)}{dx}\Big|_{x_0} = 0$ باشد، یعنی اصطلاحاً x_0 یک نقطه‌ی stationary باشد (برای $f(x)$)

$$\longrightarrow f(x) = f(x_0) + \frac{1}{2!}(x - x_0)^2 \frac{d^2 f}{dx^2}\Big|_{x_0} + \cdots$$

$$I = \int dx\, e^{-f(x)} = \int dx\; e^{-f(x_0) - \frac{1}{2!}(x-x_0)^2 f''(x_0) + \cdots}$$

$$= e^{-f(x_0)} \underbrace{\int dx\; e^{-\frac{1}{2}(x-x_0)^2 f''(x_0) + \cdots}}_{\text{Gaussian integral}}$$

ما از این ایده برای بدست آوردن کنش مؤثر در حد تک حلقه (مرتبه $\hbar^0$) استفاده می‌کنیم :

For $J=0$:

$$Z[0] = \int \mathcal{D}\varphi\; e^{iS[\varphi]} = \int \mathcal{D}\varphi\; e^{i\int d^d x\, \mathcal{L}[\varphi]}$$

$$\longrightarrow \varphi = \varphi_{cl} + \eta(x) \quad \text{with} \quad \varphi_{cl} = \text{solution of classical EOM.}$$

$$\text{i.e.} \quad \frac{\delta \mathcal{L}}{\delta \varphi}\Big|_{\varphi_{cl}} = 0$$

$$\longrightarrow \int d^d x\, \mathcal{L}[\varphi] = \int d^d x\, \mathcal{L}[\varphi_{cl} + \eta(x)]$$

$$= \int d^d x\, \mathcal{L}[\varphi_{cl}] + \cancel{\int d^d x\, \eta(x) \frac{\delta \mathcal{L}}{\delta \varphi(x)}\Big|_{\varphi_{cl}}}^{=0} +$$

بنا به فرض

$$+ \frac{1}{2!} \int \frac{\delta^2 \mathcal{L}}{\delta\varphi(x)\delta\varphi(y)}\Big|_{\varphi_{cl}} \eta(x)\eta(y)\, d^d x\, d^d y + \cdots$$

$$\Rightarrow \boxed{S[\varphi] = S[\varphi_{cl}] + \frac{1}{2!} \int d^d x\, d^d y\, \frac{\delta^2 \mathcal{L}}{\delta\varphi(x)\delta\varphi(y)}\Big|_{\varphi_{cl}} \eta(x)\eta(y) + \cdots}$$

$$Z[0] = e^{iS[\varphi_{cl}]} \int \mathcal{D}\eta\; \exp\left(\frac{i}{2} \int d^d x\, d^d y\, \frac{\delta^2 \mathcal{L}}{\delta\varphi(x)\delta\varphi(y)}\Big|_{\varphi_{cl}} \eta(x)\eta(y) + \cdots\right)$$

انتگرال روی $\eta(x)$ را می‌توان گرفت .

$$Z[0] = e^{iW[0]} = e^{i\Gamma[\varphi_{cl}]} \cong e^{iS[\varphi_{cl}]} \left[\det \left(\frac{-\delta^2 \mathcal{L}}{\delta\varphi(x)\delta\varphi(y)} \Big|_{\varphi_{cl}} \right) \right]^{-1/2}$$

$$\det A = e^{\ln\det A}$$

$$Z[0] \cong e^{iS[\varphi_{cl}]} \exp\left(-\frac{1}{2} \ln\det \left(\frac{-\delta^2 \mathcal{L}}{\delta\varphi(x)\delta\varphi(y)} \Big|_{\varphi_{cl}} \right) \right) = e^{i\Gamma[\varphi_{cl}]}$$

$$\boxed{\longrightarrow \Gamma[\varphi_{cl}] = S[\varphi_{cl}] + \frac{i}{2} \ln\det \left(\frac{-\delta^2 \mathcal{L}}{\delta\varphi(x)\delta\varphi(y)} \right) \Big|_{\varphi_{cl}} \quad \text{One-loop Effective Action.}}$$

$\hbar^0$ تا مرتبه تک حلقه

← برای مثال به چند صفحه بعد مراجعه بفرمایید.

Functional Determinant (Background Field) (۳)

Fermion Determinant:

$$I[A_\mu] = \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \exp\left(i \int d^4x \, \bar{\psi}(x) (i\partial\!\!\!/ - g A\!\!\!/ - m) \psi(x) \right)$$

A_μ یک میدان زمینه است زیرا در انتگرال مسیر روی A_μ انتگرال گیری نمی‌شود و تنها میدان‌هایی که روی آنها انتگرال مسیر گرفته می‌شود ψ ، $\bar{\psi}$ هستند.

$$I[A_\mu] = \det(iD\!\!\!\!/ - m) = \det(i\partial\!\!\!/ - m - gA\!\!\!/)$$

$$= \det\left((i\partial\!\!\!/ - m)\left[1 - \frac{gA\!\!\!/}{i\partial\!\!\!/ - m} \right] \right) = \det(i\partial\!\!\!/ - m) \det\left(1 - \frac{gA\!\!\!/}{i\partial\!\!\!/ - m} \right)$$

$$= \underbrace{\det(i\partial\!\!\!/ - m)}_{\text{is not important}} \det\left(1 - \frac{i(-igA\!\!\!/)}{i\partial\!\!\!/ - m} \right)$$

$$\sim \det\left(1 - \frac{i}{i\partial\!\!\!/ - m}(-igA\!\!\!/) \right)$$

این سمت از دترمینان فرمیونی را می‌توان به صورت یک بسط برحسب توان‌های A_μ نوشت:

$$\log\det(iD\!\!\!\!/ - m) \sim \log\det\left(1 - \frac{i}{i\partial\!\!\!/ - m}(-igA\!\!\!/) \right) = \mathrm{Tr}\log\left(1 - \frac{i}{i\partial\!\!\!/ - m}(-igA\!\!\!/) \right)$$

$$= \mathrm{Tr} \sum_{n=1}^{\infty} \frac{1}{n} (-1)^{n+1} \left(\frac{-i}{i\partial\!\!\!/ - m} (-igA\!\!\!/) \right)^n$$

$$\log(1+x) = \sum_{n=1}^{\infty} \frac{1}{n} (-1)^{n+1} x^n$$

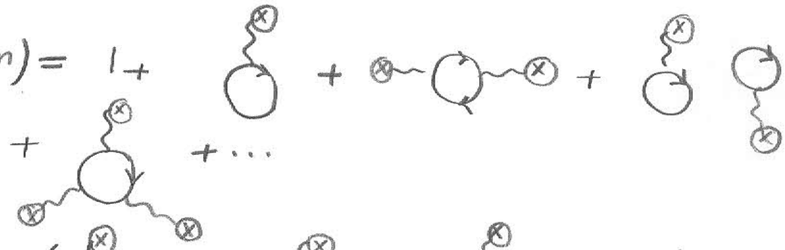
$$= -\mathrm{Tr} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{i}{i\partial\!\!\!/ - m} (-igA\!\!\!/) \right)^n$$

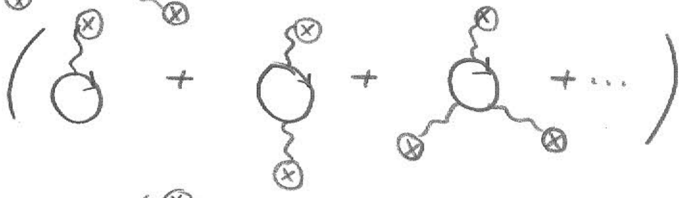
$\longrightarrow$

$$\boxed{\det(i\partial\!\!\!/ - m) \sim \exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{Tr}\left(\frac{i}{i\partial\!\!\!/ - m}(-ig A\!\!\!/)\right)^{n}\right)}$$

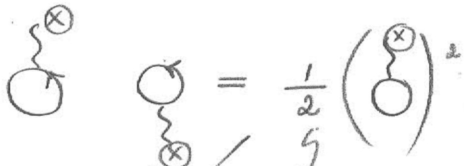
نمایش نموداری ضابطه بالا:

 $-ig\,\gamma^{\mu}\int d^{d}x\, A_{\mu}(x)$

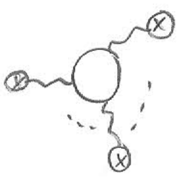
$I[A_\mu] = \det(i\partial\!\!\!/ - m) = 1 +$ 

$\overset{!}{=} \exp\Big($  $\Big)$

See Peskin - Chapter 9

Note : 1) 

تقارن ۲ به تکرار وجود دارد

2)  n - insertion

n امین نمودار در لطف بودن ما:

(−) for each fermion loop

$\left(\frac{1}{n}\right)$ for n - insertion of the external field A_μ

$\frac{1}{n}$ ضریب تقارن است. ⟵ عبارت ریاضی مربوط به نمودار در n امین رتبه:

$$-\frac{1}{n}\,\mathrm{Tr}\left(\frac{i}{i\partial\!\!\!/ - m}(-igA\!\!\!/)\right)^{n}$$

$$= \frac{-1}{n}\int d^{d}x_1 \dots d^{d}x_n \ \mathrm{tr}\Big[\left(-igA\!\!\!/(x_1)\right) S_F(x_1 - x_2)\left(-igA\!\!\!/(x_2)\right) S_F(x_2 - x_3) \times \dots \left(-igA\!\!\!/(x_n)\right) S_F(x_n - x_1)\Big]$$

A Toy Model - The Gross - Neveu Model

Aim: Computing the effective potential of this model.

- ✓ This model exhibits spontaneous symmetry breaking (Chiral)
- ✓ Asymptotically free theory (like QCD → negative 1-loop β-funct.)
- ✓ A $d=2$ dimensional theory:

$$\mathcal{L} = \bar{\psi}_i (i\not\partial)\psi_i + \frac{g^2}{2}(\bar{\psi}_i\psi_i)^2 \qquad i = 1,\dots,N_f$$

number of flavors

$$\gamma^0 = \sigma_2$$

$$\gamma^1 = i\sigma_1$$

✓ Symmetry properties:

Classical $\mathcal{L}$ has a discrete chial symmetry:

$$\psi_i \to \gamma_5\psi_i \qquad \bar{\psi}_i \to -\bar{\psi}_i\gamma_5$$

$$\begin{aligned}
\mathcal{L}' &= \bar{\psi}_i'(i\not\partial)\psi_i' + \frac{g^2}{2}(\bar{\psi}_i'\psi_i')^2 \\
&= \underbrace{(-\bar{\psi}_i\gamma_5)(i\not\partial)\gamma_5}_{-i\not\partial}\psi_i + \frac{g^2}{2}(-\bar{\psi}_i\underbrace{\gamma_5\gamma_5}_{=1}\psi_i)^2 \\
&= \bar{\psi}_i(i\not\partial)\psi_i + \frac{g^2}{2}(\bar{\psi}_i\psi_i)^2 = \mathcal{L}
\end{aligned}$$

$$\{\gamma_\mu,\gamma_5\} = 0$$

Semi-bosonized (s.b.) Lagrangian:

Introduce $\sigma = -g^2\bar{\psi}_i\psi_i$ and rewrite $\mathcal{L}$:

$$\boxed{\mathcal{L}_{s.b.} = \bar{\psi}_i(i\not\partial)\psi_i - \frac{\sigma^2}{2g^2} - \sigma\bar{\psi}_i\psi_i}$$

$\sigma(x)$ is an auxilliary field →

$$\frac{\delta\mathcal{L}_{s.b.}}{\delta\sigma(x)} = -\frac{\sigma}{g^2} - \bar{\psi}_i\psi_i = 0 \to \sigma = -g^2\bar{\psi}_i\psi_i \checkmark$$

→

Effective action of the model:

$$e^{iW[J]} = Z[J] = \int D\sigma D\psi D\bar{\psi}\, \exp\left(i\int d^dx\,\left(\mathcal{L}[\sigma,\psi,\bar{\psi}] + J\sigma\right)\right)$$

$$D\psi = \prod_{i,x} d\psi_i(x)$$

Define:

نماد تعریف

$$\frac{\delta W[J]}{\delta J(x)} = \langle\sigma(x)\rangle_J = \sigma_J(x)$$

تبدیل Legendre $\quad W[J] = \Gamma[\sigma_J] + \int J(x)\,\sigma_J(x)\,d^4x$

for $J=0$ $\quad \langle\sigma(x)\rangle_{J=0} = \sigma_{cl}(x)$

فرض $\quad \lim_{J\to 0}\sigma_J = \sigma_{cl} = a = \text{const}$

$$\Gamma[\sigma_{cl}] = -\Omega_d V_{eff}(\sigma_{cl}) \qquad V_{eff}(\sigma_{cl}) = V_{eff}(a)$$

حجم فضا ـ زمان

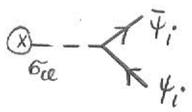
همانطور که قبلاً استدلال شد:

$$V_{eff}(\sigma_{cl}) = \sum_{n=0}^{\infty}\frac{1}{n!}\sigma_{cl}^{\,n}\,\Gamma^{(n)}(p_i=0)$$

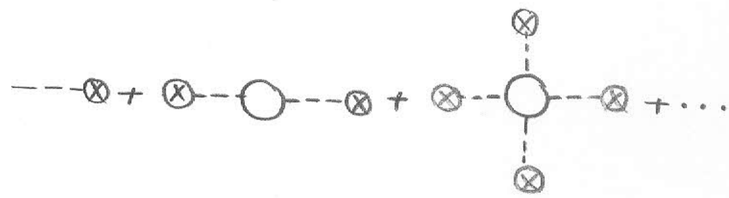
propagator

$-\frac{1}{2g^2}\sigma_{cl}^2 \quad$ ---⊗ $\quad \sim (2g^2)$

external field

$-\sigma_{cl}\bar{\psi}_i\psi_i \quad$  $\quad -1$

$\bar{\psi}_i(i\partial\!\!\!/)\psi_i \quad \longrightarrow \quad \frac{i}{p\!\!\!/} \sim \frac{ip\!\!\!/}{p^2}$



$$e^{i\Gamma[\sigma_{cl}]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\; e^{iS_{s.b.}[\psi,\bar{\psi},\sigma_{cl}]}$$

نقش جرم را ایفا می‌کند

$$= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\;\exp\left(i\int d^dx\left[\bar{\psi}_i(i\partial\!\!\!/-\underbrace{\sigma_{cl}}_{=\text{const.}})\psi_i - \frac{\sigma_{cl}^2}{2g^2}\right]\right)$$

$$= \underbrace{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\;\exp\left(i\int d^dx\;\bar{\psi}_i(i\partial\!\!\!/-\sigma_{cl})\psi_i\right)}_{(\det(i\partial\!\!\!/-\sigma_{cl}))^{N_f}}\; e^{-i\Omega\frac{\sigma_{cl}^2}{2g^2}}$$

$\Psi = (\psi_1 \cdots \psi_{N_f})$

. flavors تعداد طعم‌ها

$$e^{i\Gamma[\sigma_{cl}]} = \left(\det(i\partial\!\!\!/-\sigma_{cl})\right)^{N_f} e^{-i\Omega\frac{\sigma_{cl}^2}{2g^2}}$$

$$\Gamma[\sigma_{cl}] = -i\ln\left(\det(i\partial\!\!\!/-\sigma_{cl})\right)^{N_f} - \Omega\frac{\sigma_{cl}^2}{2g^2}$$

$$\boxed{\Gamma[\sigma_{cl}] = -iN_f\,\ln\det(i\partial\!\!\!/-\sigma_{cl}) - \Omega\frac{\sigma_{cl}^2}{2g^2}}$$

Question: $\ln\det(i\partial\!\!\!/-\sigma_{cl}) = ?$

$\det(i\partial\!\!\!/ - \sigma_{cl}) \longleftarrow$ حساب کنید برای بعد ۲ دوران $\qquad \sigma_{cl} \equiv \sigma = \text{const.}$

a) <u>Note</u>: $\det\big((i\partial\!\!\!/ + \sigma)(i\partial\!\!\!/ - \sigma)\big) = \det\big(-(\partial^2+\sigma^2)\underbrace{\mathbb{1}_{2\times 2}}_{\text{تانسور ۲ بعدی}}\big)$

$$= (\partial^2+\sigma^2)^2$$

b) $\det(i\partial\!\!\!/ - \sigma) = ?$

$$\det(i\partial\!\!\!/ - \sigma) = \det(i\gamma^0\partial_0 + i\gamma^i\partial_i - \sigma)$$

$$= \det\left\{ i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\partial_0 + i\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\partial_1 - \sigma\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right\}$$

$$= \det\begin{pmatrix} -\sigma & \partial_0 - \partial_1 \\ -\partial_0 - \partial_1 & -\sigma \end{pmatrix} = \sigma^2 + (\partial_0 - \partial_1)(\partial_0 + \partial_1)$$

$$= \sigma^2 + (\partial_0^2 - \partial_1^2) = (\partial^2 + \sigma^2)$$

$$\rightarrow \det(i\partial\!\!\!/ - \sigma) \overset{(b)}{=} (\partial^2 + \sigma^2)$$

(طبق a) $\det\big((i\partial\!\!\!/ + \sigma)(i\partial\!\!\!/ - \sigma)\big) = \det(i\partial\!\!\!/ + \sigma)\underbrace{\det(i\partial\!\!\!/ - \sigma)}_{(\partial^2+\sigma^2)} = (\partial^2+\sigma^2)^2$

$$\rightarrow \det(i\partial\!\!\!/ + \sigma) = (\partial^2 + \sigma^2)$$

<u>In other words:</u> $\det(i\partial\!\!\!/ - \sigma) = \det(i\partial\!\!\!/ + \sigma) = (\partial^2+\sigma^2) = \big(\det(\partial^2+\sigma^2)\big)^{\frac{1}{2}}$

$$\rightarrow N_f \ln\det(i\partial\!\!\!/ - \sigma_{cl}) = N_f \ln\big(\det(\partial^2+\sigma^2)\big)^{1/2} = \frac{N_f}{2}\ln\det(\partial^2+\sigma^2)$$

$$\longrightarrow \boxed{\Gamma[\sigma] = -i\frac{N_f}{2}\ln\det(\partial^2+\sigma^2) - \Omega\frac{\sigma^2}{2g^2}}$$

$$\rightarrow \ln\det(\partial^2+m^2) = \mathrm{Tr}\ln(\partial^2+\sigma^2) = \underbrace{\int d^dx}_{=\Omega_d}\int\frac{d^dp}{(2\pi)^d}\ln(\sigma^2 - p^2)$$

<u>Method (See - Peskin Chapter 11)</u>:

- $\displaystyle\int\frac{d^dp}{(2\pi)^d}\ln(\sigma^2 - p^2) \underset{\substack{p_0 = ip_4 \\ p^2 = p_0^2 - \vec{P}^2 = -p_4^2 - \vec{P}^2 = -P_E^2}}{=\!=\!=}$

$$= i\int\frac{d^dp_E}{(2\pi)^d}\ln(\sigma^2 + p_E^2) = ?$$

Use: $$\frac{\partial}{\partial\alpha} x^{-\alpha} = \frac{\partial}{\partial\alpha}\left(e^{\ln x^{-\alpha}}\right) = \frac{\partial}{\partial\alpha} e^{-\alpha \ln x} = -\ln x \; e^{-\alpha \ln x} = -x^{-\alpha}\ln x$$

$$-\frac{\partial}{\partial\alpha} x^{-\alpha}\Big|_{\alpha=0} = \left(x^{-\alpha}\ln x\right)\Big|_{\alpha=0} = \ln x$$

$$\ln x = -\frac{\partial}{\partial\alpha} x^{-\alpha}\Big|_{\alpha=0}$$

$$\rightarrow \quad i\ln(\sigma^2+P_E^2) = -i\frac{\partial}{\partial\alpha}\frac{1}{(\sigma^2+P_E^2)^\alpha}\Big|_{\alpha=0}$$

$$\rightarrow \quad i\int\frac{d^dp}{(2\pi)^d}\ln(\sigma^2+P_E^2) = -i\int\frac{d^dp}{(2\pi)^d}\frac{\partial}{\partial\alpha}\frac{1}{(\sigma^2+P_E^2)^\alpha}\Big|_{\alpha=0}$$

$$= -i\frac{\partial}{\partial\alpha}\left\{\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(\alpha-\frac{d}{2})}{\Gamma(\alpha)}\frac{1}{(\sigma^2)^{\alpha-\frac{d}{2}}}\right\}\Big|_{\alpha=0}$$

$$= -\frac{i\sigma^d}{(4\pi)^d}\Gamma\left(-\frac{d}{2}\right)$$

$$= -i\frac{\Gamma(1-\frac{d}{2})}{(1-\frac{d}{2})}\frac{\sigma^2}{4\pi}\left(\frac{4\pi}{\sigma^2}\right)^{1-\frac{d}{2}}$$

$$= +i\frac{\sigma^2}{4\pi}\left(\frac{2}{\epsilon}-\gamma_E\right)\left(1-\frac{\epsilon}{2}\ln\frac{\sigma^2}{4\pi}\right) \qquad \epsilon = 2-d$$

$$= \frac{i\sigma^2}{4\pi}\left(\frac{2}{\epsilon}-\gamma_E-\ln\frac{\sigma^2}{4\pi}\right) \underset{\overline{MS}\text{-Scheme}}{=\!=} \frac{i\sigma^2}{4\pi}\left(-\ln\frac{\sigma^2}{\mu^2}\right) \; \star$$

$$\text{OR} \quad \frac{i\sigma^2}{4\pi}\left(1-\ln\frac{\sigma^2}{\mu^2}\right)$$

$$\Gamma[\sigma] = -i\frac{N_f}{2}\ln\det(\partial^2+\sigma^2) - \Omega\frac{\sigma^2}{2g^2} =$$

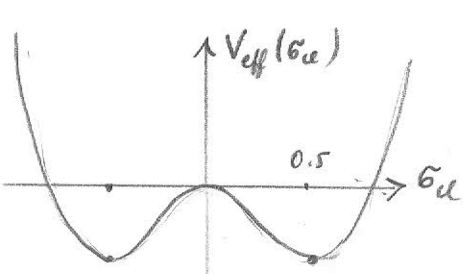
$$= -i\frac{N_f}{2}\Omega\int\frac{d^dp}{(2\pi)^d}\ln(\sigma^2-p^2) - \Omega\frac{\sigma^2}{2g^2}$$

$$= \left[-\frac{\sigma^2}{2g^2} - \frac{\sigma^2}{8\pi}N_f\ln\frac{\sigma^2}{\mu^2}\right]\Omega = -\Omega V_{eff} \qquad \text{for } \sigma=\text{const}.$$

$$\longrightarrow \quad V_{eff}(\sigma_{cl}) = \frac{\sigma_{cl}^2}{2g^2} + \frac{N_f}{8\pi}\sigma^2\ln\frac{\sigma^2}{\mu^2}$$

$$\overset{\text{or}}{=} \frac{\sigma_{cl}^2}{2g^2} + \frac{N_f}{8\pi}\sigma^2\ln\left(\frac{\sigma^2}{\mu^2}-1\right)$$

For $2g^2 = 20$ $N_f = 1$ $\mu^2 = 1$, I got



Mexican Hat potential $\longrightarrow$ Spontaneous Symmetry Breaking!