جمله دوم

$$-\frac{i}{2}\int d^4x\, d^4y\; J(x)\,\Delta_F(x-y)\,J(y)$$

$$= -\frac{i}{2}\int_{p,k,k'} d^4x\, d^4y\;\; \tilde{J}(k)\,\tilde{\Delta}_F(p)\,\tilde{J}(k')\; e^{-ikx-ik'y-ip(x-y)}$$

$$\int_p \equiv \int \frac{d^4p}{(2\pi)^4} \equiv \int d\bar{p}$$

$$= -\frac{i}{2}\underbrace{\int d^4x\, e^{-ix(k+p)}}_{(2\pi)^4\,\delta^4(k+p)}\;\underbrace{\int d^4y\, e^{-iy(k'-p)}}_{(2\pi)^4\,\delta(k'-p)}\;\tilde{J}(k)\,\tilde{\Delta}_F(p)\,\tilde{J}(k')\,d\bar{p}\,d\bar{k}\,d\bar{k}'$$

$$= -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\;\tilde{J}(-p)\,\tilde{\Delta}_F(p)\,\tilde{J}(p) = -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\;\frac{\tilde{J}(-p)\,\tilde{J}(p)}{p^2-m^2+i\epsilon}$$

$$\tilde{\Delta}_F(p) = \frac{1}{p^2-m^2+i\epsilon}$$

$$= \frac{1}{2}\;\times\!\!\text{———}\!\!\times$$

Remember:

$$(\Box + m^2 - i\epsilon)\,\Delta_F(x) = -\delta^4(x)$$

با زبان نموداری

———	$\dfrac{i}{p^2-m^2+i\epsilon}$
×——	$i\,\tilde{J}(p)$
——×	$i\,\tilde{J}(-p)$

جمله سوم:

$$\frac{1}{2!}\left(\frac{-i}{2}\right)^2\left(\int d^4x\, d^4y\;\; J(x)\,\Delta_F(x-y)\,J(y)\right)^2$$

$$\frac{1}{2!}\left(\frac{1}{2}\right)^2\;\begin{matrix}\times\text{——}\times\\ \times\text{——}\times\end{matrix}$$

$$\longrightarrow\; Z_0[J] = \left(1 + \frac{1}{2}\;\times\text{——}\times + \frac{1}{2!}\left(\frac{1}{2}\right)^2\begin{matrix}\times\text{——}\times\\ \times\text{——}\times\end{matrix} + \frac{1}{3!}\left(\frac{1}{2}\right)^3\begin{matrix}\times\text{——}\times\\ \times\text{——}\times\\ \times\text{——}\times\end{matrix} + \cdots\right)$$

propagation of one particle between two sources

propagation of 2 particles between 4 sources

etc.

a)

حالا می‌توان نشان داد:

$$\left(\frac{1}{i}\right)^2 \frac{\delta^2 Z_0[J]}{\delta J(x_1)\,\delta J(x_2)}\bigg|_{J=0} = i\,\Delta_F(x_1-x_2) = \underset{x_1}{\bullet}\text{———}\underset{x_2}{\bullet}$$

$$\frac{\delta^2 Z_0[J]}{\delta J(x_1)\,\delta J(x_2)}\bigg|_{J=0} = \frac{\delta}{\delta J(x_2)}\left\{-\frac{i}{2}\int d^4x\, d^4y\;\delta^4(x-x_1)\,\Delta_F(x-y)\,J(y) - \frac{i}{2}\int d^4x\, d^4y\; J(x)\,\Delta_F(x-y)\,\delta^4(y-x_1)\right\}$$

$$= \frac{\delta}{\delta J(x_2)}\left\{-\frac{i}{2}\int d^4y\;\;\Delta_F(x_1-y)\,J(y) - \frac{i}{2}\int d^4x\;\; J(x)\,\Delta_F(x-x_1)\right\}$$

$$= -\frac{i}{2}\int d^4y\,\Delta_F(x_1-y)\;\delta(x_2-y)$$

$$-\frac{i}{2}\int d^4x\;\;\delta(x-x_2)\,\Delta_F(x-x_1) = -\frac{i}{2}\,\Delta_F(x_1-x_2)$$

$$-\frac{i}{2}\,\Delta_F(x_2-x_1) = -i\,\Delta_F(x_1-x_2)$$

$$\Rightarrow \left(\frac{1}{i}\right)^2 \frac{\delta^2 Z_0[J]}{\delta J(x_1)\,\delta J(x_2)}\bigg|_{J=0} = i\Delta_F(x_1 - x_2) \quad \checkmark$$

b) $$\left(\frac{1}{i}\right)^4 \frac{\delta^4 Z_0[J]}{\delta J(x_1)\cdots\delta J(x_4)}\bigg|_{J=0} = \big(i\Delta_F(x_3-x_4)\big)\big(i\Delta_F(x_1-x_2)\big) + \big(i\Delta_F(x_1-x_3)\big)\big(i\Delta_F(x_2-x_4)\big) + \big(i\Delta_F(x_1-x_4)\big)\big(i\Delta_F(x_2-x_3)\big)$$

$$= \langle 0|T(\varphi(x_1)\cdots\varphi(x_4))|0\rangle$$

تابع چهار نقطه‌ای برای تئوری آزاد

تابع مولد در تئوری برهمکنشی :

تئوری $\lambda\varphi^4$:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{m^2}{2}\varphi^2 - \frac{g}{4!}\varphi^4$$

تئوری آزاد

$$Z_0[J] = \frac{\int \mathcal{D}\varphi\, e^{i\int d^4x(\mathcal{L}_0 + J\varphi)}}{\int \mathcal{D}\varphi\, e^{i\int d^4x\,\mathcal{L}_0}} \overset{OR}{=} \frac{\int \mathcal{D}\varphi\, e^{iS_0 + \int J\varphi d^4x}}{\int \mathcal{D}\varphi\, e^{iS_0}}$$

تئوری برهمکنشی

$$Z[J] = \frac{\int \mathcal{D}\varphi \exp\left(i\int d^4x(\mathcal{L} + J\varphi)\right)}{\int \mathcal{D}\varphi \exp(i\int d^4x\,\mathcal{L})} \overset{OR}{=} \frac{\int \mathcal{D}\varphi\, e^{iS + \int J\varphi d^4x}}{\int \mathcal{D}\varphi\, e^{iS}}$$

این روابط صرفاً تعریف توابع مولد هستند. هدف این است که ابتدا $Z[J]$ را به عنوان تابعی از J بنویسیم (درست مانند $Z_0[J]$ که در رابطهٔ $Z_0[J] = \exp\left(-\frac{i}{2}\int d^4x\,d^4y\, J(x)\Delta_F(x-y)J(y)\right)$ فقط تابعی از J است) پس باید انتگرال‌گیری تابعی روی φ را برای Z هم انجام دهیم (← بعداً) در اینجا فقط نتیجه و کاربرد آن را می‌نویسیم و اثبات فرمول زیر را بعداً می‌آوریم!

$$\boxed{\begin{aligned} Z[J] &= \exp\left(i\int d^4z\ \mathcal{L}_{int}\left[\frac{\delta}{\delta\varphi(z)}\right]\right) Z_0[J] \\ \text{with } Z_0[J] &= \exp\left(-\frac{i}{2}\int d^4x\,d^4y\ J(x)\,\Delta_F(x-y)\,J(y)\right)\end{aligned}}$$

مثال:

$$\mathcal{L}_{int} = -\frac{\lambda\varphi^4}{4!} = \mathcal{L}_{int}[\varphi]$$

$$\mathcal{L}_{int}\left[\frac{1}{i}\frac{\delta}{\delta J(z)}\right] = -\frac{\lambda}{4!}\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4$$

$$Z[J] = \frac{\exp\left(i\int \mathcal{L}_{int}\left[\frac{1}{i}\frac{\delta}{\delta J(z)}\right]d^4z\right)\exp\left(-\frac{i}{2}\int d^4x\,d^4y\ J(x)\,\Delta_F(x-y)\,J(y)\right)}{''\ \Big|_{J=0}}$$

که ضریب نرمالیزاسیون

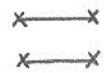
صورت کسر (A)

$$\exp\left(\frac{-i\lambda}{4!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4\right)Z_0[J]=$$

به ترتیب توان‌های λ

$$=\left\{1-\frac{i\lambda}{4!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4+\frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2\int d^4z_1 d^4z_2\left(\frac{1}{i}\frac{\delta}{\delta J(z_1)}\right)^4\left(\frac{1}{i}\frac{\delta}{\delta J(z_2)}\right)^4+O(\lambda^2)\right\}Z_0[J]$$

(A) جمله اول : $Z_0[J]=$ tree level contribution to the full generating functional

$$=1+\frac{1}{2}\;\times\!\!-\!\!\!-\!\!\!-\!\!\times\;+\frac{1}{2!}\left(\frac{1}{2}\right)^2$$  $+\cdots$

(A) جمله دوم :

$$\left[\frac{-i\lambda}{4!}\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4\right]Z_0[J]=$$

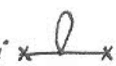
میتوان نشان داد

$$=\frac{-i\lambda}{4!}\int d^4z\left\{-3\left(\Delta_F(0)\right)^2+6i\Delta_F(0)\left(\int d^4x\,\Delta_F(x-z)J(x)\right)^2+\left(\int d^4x\,J(x)\Delta_F(x-z)\right)^4\right\}\underbrace{Z_0[J]}_{=\exp\left(\frac{-i}{2}\int J\Delta_F J\right)}$$

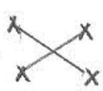
تمام مراحلی که به دست آمده است در نتیجه مشتق‌گیری‌ها است که پشت سر هم انجام داده‌ام.

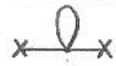
سهم صورت کسر $=\frac{-i\lambda}{4!}\int d^4z\Big\{-3\,\infty+6i$  $+$  $\Big\}Z_0[J]$

همین ترتیب می‌توان مخرج را محاسبه کرد و بدست آورد:

$Z[J]=\dfrac{\left[1-\frac{i\lambda}{4!}\int d^4z\left\{-3\,\infty+6i\right.\right.}{}$  $+$  $\left.\left.\right\}+O(\lambda^2)\right]Z_0[J]$

$\overline{\left(1-\frac{i\lambda}{4!}\int d^4z\,(-3\,\infty)\right)}$

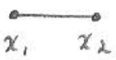
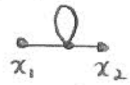
$\approx\left(1-\frac{i\lambda}{4!}\int d^4z\left\{-3\,\infty+6i\right.\right.$  $+$  $\left.\left.\right\}\right)\left(1+\frac{i\lambda}{4!}\int d^4z\,(-3\,\infty)\right)\times Z_0[J]+\cdots$

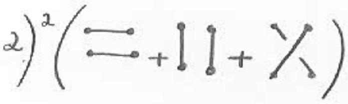
$\rightarrow\;\boxed{Z[J]=\left(1-\frac{i\lambda}{4!}\int d^4z\left(-6i\right.\right.}$  $+$  $\left.\left.\right)\right)Z_0[J]+O(\lambda^2)$

نمودارهای bubble حذف می‌شوند

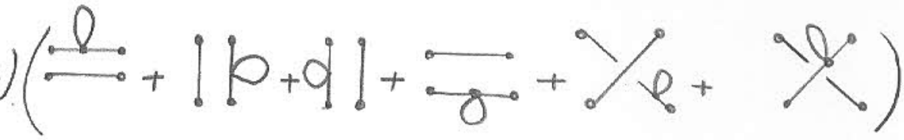
a) $\tau_2(x_1,x_2) = \langle \Omega | T(\varphi(x_1)\varphi(x_2)) | \Omega \rangle = \left(\frac{1}{i}\right)^2 \frac{\delta^2 Z[J]}{\delta J(x_1)\,\delta J(x_2)}\Big|_{J=0}$

$= i\Delta_F(x_1-x_2) - \frac{\lambda}{2}\int d^4z\, \Delta_F(0)\, \Delta_F(z-x_1)\,\Delta_F(z-x_2) + O(\lambda^2)$

$= i$  $- \frac{\lambda}{2}$  $+ O(\lambda^2)$

b) $\tau_4(x_1,x_2,x_3,x_4) = \left(\frac{1}{i}\right)^4 \frac{\delta^4 Z[J]}{\delta J(x_1)\ldots\delta J(x_4)}\Big|_{J=0} = \left(\frac{-i}{2}\times 2\right)^2$ 

تابع همبستگی ۴ نقطه‌ای شامل نمودارهای connected, disconnected.

$+ \frac{2\lambda}{4}\left(\frac{-i}{2}\times 2\right)$ 

$\frac{-i\lambda}{4!}\; 4!$ 

تابع مولد مربوط به تولید نمودارهای connected :

$W[J] = -i \ln Z[J] \quad \text{or} \quad Z[J] = e^{iW[J]}$

$$\frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)}\Big|_{J=0} = \frac{\delta^2}{\delta J(x_1)\delta J(x_2)}\left(-i \ln Z[J]\right) = \frac{\delta}{\delta J(x_1)}\left(\frac{-i}{Z[J]}\frac{\delta Z[J]}{\delta J(x_2)}\right)\Big|_{J=0}$$

$$= \left(-i\frac{\delta}{\delta J(x_1)}\frac{1}{Z[J]}\right)\frac{\delta Z[J]}{\delta J(x_2)}\Big|_{J=0} - \frac{i}{Z[J]}\frac{\delta^2 Z[J]}{\delta J(x_1)\delta J(x_2)}\Big|_{J=0}$$

جمله اول $= \frac{\delta}{\delta J(x_1)}\frac{1}{Z[J]} = \frac{-1}{Z[J]}\frac{\delta Z[J]}{\delta J(x_1)}$

$$= \cancel{+\frac{i}{Z[J]}\underbrace{\frac{\delta Z[J]}{\delta J(x_1)}\frac{\delta Z[J]}{\delta J(x_2)}}_{\langle\phi(x_1)\rangle\,\langle\phi(x_2)\rangle}\Big|_{J=0}} - \frac{i}{Z[0]}\frac{\delta^2 Z[J]}{\delta J(x_1)\delta J(x_2)}\Big|_{J=0}$$

Note: a) $Z[0]=1$

b) $\frac{\delta Z[J]}{\delta J(x_1)}\Big|_{J=0} = \langle\phi(x_1)\rangle = 0$ بنا به فرض

نتیجه: $$\boxed{\frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)}\Big|_{J=0} = -i\,\frac{\delta^2 Z[J]}{\delta J(x_1)\delta J(x_2)}\Big|_{J=0}}$$

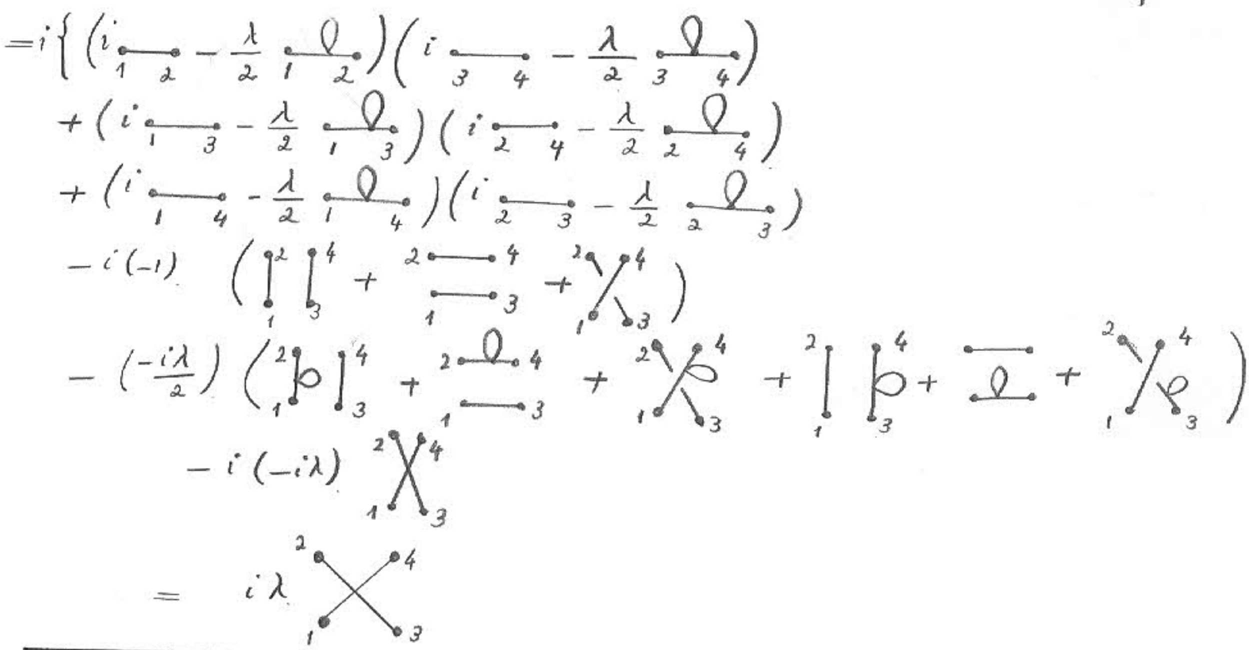
$\tau_2(x_1,x_2) = -\frac{\delta Z[J]}{\delta J(x_1)\delta J(x_2)}\Big|_{J=0} = i$  $+ \left(\frac{-\lambda}{2}\right)$ 

$= i\,\frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)}\Big|_{J=0}$

Four - point function:

$$\frac{\delta^4 W[J]}{\delta J(x_1)\dots\delta J(x_4)}\bigg|_{J=0} = i\left\{ \frac{1}{Z^2[J]} \frac{\delta^2 Z[J]}{\delta J_1\,\delta J_2}\frac{\delta^2 Z[J]}{\delta J_3\,\delta J_4} + \frac{1}{Z^2[J]} \frac{\delta^2 Z[J]}{\delta J_1\,\delta J_3}\frac{\delta^2 Z[J]}{\delta J_2\,\delta J_4} + \frac{1}{Z^2[J]} \frac{\delta^2 Z[J]}{\delta J_1\,\delta J_4}\frac{\delta^2 Z[J]}{\delta J_2\,\delta J_3} - \frac{i\,\delta^4 Z[J]}{\delta J_1\dots\delta J_4}\right\}\bigg|_{J=0}$$

$$= i\left\{\tau(x_1-x_2)\,\tau(x_3-x_4) + \tau(x_1-x_3)\,\tau(x_2-x_4) + \tau(x_1-x_4)\,\tau(x_2-x_3) - \tau(x_1,x_2,x_3,x_4)\right\}$$



یادآوری:
$$\tau(x_1,\dots,x_n) = \left(\frac{1}{i}\right)^{N-1} \frac{\delta^N Z[J]}{\delta J_1\dots\delta J_N}\bigg|_{J=0}$$
connected + disconnected diagrams

$$\phi(x_1\dots x_N) = \left(\frac{1}{i}\right)^{N} \frac{\delta^N W[J]}{\delta J_1\dots\delta J_N}\bigg|_{J=0}$$
irreducible (connected) diagrams

$$\tau(x_1,x_2) = \phi(x_1,x_2)$$

$$\tau(x_1,\dots x_4) = i\,\phi(x_1,\dots x_4) - \sum_P \phi(x_{i_1}-x_{i_2})\,\phi(x_{i_3}-x_{i_4})$$
all partitions of $x_1\dots x_4$

• اثبات رابطه

$$Z[J] = \exp\left(i \int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right) d^4z \right) Z_0[J]$$

لم ۱ — *Lemma 1*

ابتدا معادله دیفرانسیلی را که $Z_0[J]$ در آن صدق می‌کند را بدست می‌آوریم؛

$$(\Box + m^2)\, \Delta_F(x-y) = -\delta^4(x-y)$$

$$Z_0[J] = \exp\left(\frac{-i}{2} \int d^4x\, d^4y \;\; J(x)\, \Delta_F(x-y)\, J(y)\right)$$

$$\frac{1}{i}\frac{\delta}{\delta J(x)} Z_0[J] = \frac{1}{i}\left(\frac{-i}{2}\right) 2 \left(\int d^4y \;\; \Delta_F(x-y)\, J(y)\right) Z_0[J]$$

$$= - \left(\int d^4y \;\; \Delta_F(x-y)\, J(y)\right) Z_0[J]$$

$$(\Box_x + m^2)\left(\frac{1}{i}\frac{\delta}{\delta J(x)} Z_0[J]\right) = -\left(\int d^4y \;\; \underbrace{(\Box_x + m^2)\,\Delta_F(x-y)}_{= -\delta^4(x-y)}\, J(y)\right) Z_0[J]$$

$$= J(x)\, Z_0[J]$$

$$\longrightarrow \boxed{(\Box_x + m^2)\left(\frac{1}{i}\frac{\delta}{\delta J(x)} Z_0[J]\right) = J(x)\, Z_0[J]}$$

Lemma 2 $\qquad \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right) = ?$

$$\mathcal{L}_{int}[\varphi] = \mathcal{L}_{int}[0] + \varphi\, \mathcal{L}'_{int}[0] + \dots = \sum_{n=0}^{\infty} \frac{\varphi^n}{n!} \mathcal{L}^{(n)}_{int}[0]$$

$$\longrightarrow \boxed{\mathcal{L}_{int}\left[\frac{1}{i}\frac{\delta}{\delta J(y)}\right] = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)^n \mathcal{L}^{(n)}_{int}[0]}$$

ادعا

$$\left[J(x), \exp\left(i\int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right) d^4y\right)\right] = -\exp\left(i\int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right) d^4y\right) \mathcal{L}'_{int}\left[\frac{1}{i}\frac{\delta}{\delta J(x)}\right]$$

اثبات

$$J(x) = B$$

$$\exp\left(-i \int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right) \equiv e^A \curvearrowright A = -i\int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right) d^4y$$

now use: $\quad e^A [B, e^{-A}] = ?$

$$e^A [B, e^{-A}] = e^A (B e^{-A} - e^{-A} B) = B + [A,B] + \overset{=0}{\cancel{\dots}} - B = [A,B] = -[B,A]$$

Hausdorff - Baker - Campbell formula

$$[B,A] = -i\left[J(x), \int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right) d^4y\right]$$

$$= -i\left[J(x), \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)^n \mathcal{L}^{(n)}_{int}(0)\right]$$

$$= -i \sum_{n=0}^{\infty} \frac{1}{n!}\left[J(x), \left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)^n\right] \mathcal{L}^{(n)}_{int}(0)$$

Use: $\quad \left[J(x), \frac{1}{i}\frac{\delta}{\delta J(y)}\right] = i\,\delta^4(x-y)$

$$\boxed{\left[J(x),\ \left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)^n\right] \overset{!}{=} i n\, \delta^4(x-y)\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)^{n-1}}$$

و

$$[B,A] = -i\sum_{n=0}^{\infty}\frac{1}{n!}\, i\delta^4(x-y)\, n\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)^{n-1}\mathcal{L}_{int}^{(n)}[0] =$$

$$= \sum_{n=0}^{\infty}\frac{1}{(n-1)!}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)^{n-1}\mathcal{L}_{int}^{(n)}[0] = \mathcal{L}'_{int}\left[\frac{1}{i}\frac{\delta}{\delta J(y)}\right]$$

$$e^{A}[B, e^{-A}] = -[B,A] \curvearrowright [B, e^{-A}] = -e^{-A}[B,A] \quad \checkmark$$

و این با توجه به تعریف A, B اثبات ادعای ماست.

قدم ۲ حالا معادله دیفرانسیلی را می‌توان بدست آورد که $Z[J]$ در آن صدق می‌کند

تعریف

$$\hat{Z}[\varphi] = \frac{e^{iS}}{\int \mathcal{D}\varphi\, e^{iS}} \longrightarrow Z[J] = \int\mathcal{D}\varphi\ \hat{Z}[\varphi]\, e^{i\int J\varphi}$$

Here: $S = -\int d^4z\left\{\frac{1}{2}\varphi(\Box+m^2)\varphi - \mathcal{L}_{int}\right\}$

$$\bullet\ i\frac{\delta\hat{Z}[\varphi]}{\delta\varphi(x)} = i\frac{\delta}{\delta\varphi(x)}(iS)\frac{e^{iS}}{\int\mathcal{D}\varphi\, e^{iS}}$$

$$= i\frac{\delta}{\delta\varphi(x)}\left(-i\int d^4z\left\{\frac{1}{2}\varphi(\Box+m^2)\varphi - \mathcal{L}_{int}\right\}\right)\underbrace{\frac{e^{iS}}{\int\mathcal{D}\varphi\, e^{iS}}}_{\hat{Z}[\varphi]}$$

$$(A)\quad \boxed{i\frac{\delta\hat{Z}[\varphi]}{\delta\varphi(x)} = i(-i)\left\{(\Box+m^2)\varphi(x) - \frac{\delta\mathcal{L}_{int}[\varphi]}{\delta\varphi(x)}\right\}\hat{Z}[\varphi]}$$

multiply both sides with $e^{i\int J\varphi}$ & integrate over φ

(A طرف چپ)

$$\int\mathcal{D}\varphi\left(i\frac{\delta\hat{Z}[\varphi]}{\delta\varphi(x)}e^{i\int J\varphi}\right) \underset{P.I.}{=} \int\mathcal{D}\varphi\frac{\delta}{\delta\varphi}\left(i\hat{Z}[\varphi]e^{i\int J\varphi}\right) - \int\mathcal{D}\varphi\ \hat{Z}[\varphi]\left(\frac{i\delta}{\delta\varphi}e^{i\int J\varphi}\right)$$

$$= (-i)\, i\int\mathcal{D}\varphi\ \hat{Z}[\varphi]\, J(x)\, e^{i\int J\varphi} = J(x)\, Z[\varphi]$$

بنابه تعریف

(A طرف راست)

$$1)\ \int\mathcal{D}\varphi\ \{(\Box+m^2)\underbrace{\varphi(x)}_{=\frac{1}{i}\frac{\delta}{\delta J}}\}\hat{Z}[\varphi]e^{i\int J\varphi} = (\Box+m^2)\frac{1}{i}\frac{\delta}{\delta J(x)}\underbrace{\left(\int\mathcal{D}\varphi\ \hat{Z}[\varphi]e^{i\int J\varphi}\right)}_{Z[J]}$$

$$= (\Box_x+m^2)\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)}$$

$$2)\ -\int\mathcal{D}\varphi\ \mathcal{L}'_{int}[\varphi]\hat{Z}[\varphi]e^{i\int J\varphi} = -\mathcal{L}'_{int}\left[\frac{1}{i}\frac{\delta}{\delta J}\right]Z[J]$$

$$A\ \text{طرف راست} = \left((\Box+m^2)\frac{1}{i}\frac{\delta}{\delta J} - \mathcal{L}'_{int}\left[\frac{1}{i}\frac{\delta}{\delta J}\right]\right)Z[J]$$

$\longrightarrow$ (B)

$$\Longrightarrow \boxed{\begin{gathered}\left((\Box+m^2)\frac{1}{i}\frac{\delta}{\delta J}-\mathcal{L}'_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)Z[J]=J(x)\,Z[J]\\ \text{with}\qquad Z[J]=\frac{\int \mathcal{D}\varphi\, e^{iS}\, e^{i\int J\varphi}}{\int\mathcal{D}\varphi\, e^{iS}}\end{gathered}}$$

<u>قدم ۳</u>

ادعا کرده بودیم که $Z[J]=\exp\left(\frac{1}{i}\int\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)Z_0[J]$

ثابت می‌کنیم که این $Z[J]$ در معادله دیفرانسیل (B) صدق می‌کند.

<u>اثبات</u>

Use: $$J(x)\,Z[J]=\underbrace{J(x)\;\exp}_{\text{Lemma 2}}\left(\frac{1}{i}\int\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)Z_0[J]$$

$$=\exp\left(\frac{1}{i}\int\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)\left(J(x)-\mathcal{L}'_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)Z_0[J]$$

Lemma 1

$$=\exp\left(i\int\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)(\Box+m^2)\left(\frac{1}{i}\frac{\delta}{\delta J}Z_0[J]\right)$$
$$-\exp\left(i\int\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)\mathcal{L}'_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)Z_0[J]$$
$$=\left\{(\Box+m^2)\frac{1}{i}\frac{\delta}{\delta J}-\mathcal{L}'_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right\}\left(e^{i\int\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)}\right)Z_0[J]$$

همان معادله (B) است. در نتیجه ادعای ما $Z[J]=\exp\left(\frac{1}{i}\int\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right)Z_0$ صحت دارد.