مقدمه‌ای در شکست خودبخودی تقارن:

مثال: شکست خودبخودی تقارن $U(1)$

$$\mathcal{L} = (\partial_\mu \varphi^*)\partial^\mu \varphi - m^2 \varphi^* \varphi - \frac{\lambda}{4}(\varphi^*\varphi)^2$$

$$= (\partial_\mu \varphi^*)(\partial^\mu \varphi) - V(\varphi, \varphi^*) \quad \text{with} \quad V(\varphi,\varphi^*) = m^2 \varphi^* \varphi + \frac{\lambda}{4}(\varphi^*\varphi)^2$$

Global $U(1)$ – Transformation:

$$\varphi \to e^{i\alpha}\varphi \quad \& \quad \varphi^* \to e^{-i\alpha}\varphi \;\curvearrowright\; \delta\mathcal{L} = 0$$

Ground State: $\dfrac{\partial V}{\partial \varphi} = m^2 \varphi^* + \varphi^* \dfrac{\lambda}{2}(\varphi^*\varphi) = 0$

a) $\varphi^* = 0$

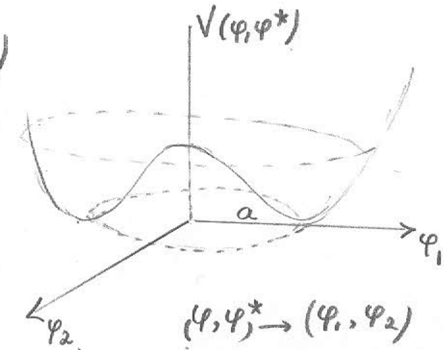
b) and/or $\varphi^*\varphi = |\varphi|^2 = \dfrac{-2m^2}{\lambda} \equiv a^2$ تعریف

If $m^2 > 0 \longrightarrow \varphi^* = 0$ (is the only solution)

If $m^2 < 0 \longrightarrow \varphi^* = 0$ (maximum)

$|\varphi| = \pm a$ (minimum)

In QFT: $|\langle 0|\varphi|0\rangle|^2 = a^2$



$(\varphi,\varphi^*) \longrightarrow (\varphi_1, \varphi_2)$

$\varphi_1 = Re\,\varphi$

$\varphi_2 = Im\,\varphi$

✓ در این مثال برای $m^2<0$ ، بی‌نهایت مینیمم ممکن وجود دارد که با تبدیل $U(1)$ به یکدیگر تبدیل می‌شوند. همه این min ها روی دایره‌ای به شعاع a قرار دارند.

✓ ما انتظار داریم که $\langle 0|\varphi|0\rangle = 0$ به این معنی که خلاء "واقعی" تئوری در $\varphi=0$ باشد

در حالی که پتانسیل را min می‌کند.

← برای اینکه این اتفاق بیفتد باید در مسئله بالا φ را shift بدهیم:

1) ابتدا $\varphi(x) = \rho(x) e^{i\theta(x)}$: $\langle 0|\varphi(x)|0\rangle = a$

انتخاب: $\rho(x) \curvearrowright \langle 0|\rho(x)|0\rangle = a$

but $\langle 0|\theta(x)|0\rangle = 0$

ما به این ترتیب خلاء $|0\rangle$ را نحوی انتخاب کردیم که مقدار چشمداشتی θ در مجاورت آن صفر شود ولی مقدار چشمداشتی ρ در مجاورت آن همان a باشد.

2) $\rho(x) - a = \rho'(x)$ Shift of variable

$$\langle 0|\rho'(x)|0\rangle = \langle 0|\rho(x) - a|0\rangle = a - a = 0$$

حالا می‌توان لاگرانژی را حول ρ' بسط داد یعنی با $\rho(x) = \rho' + a$ نوشت

a) $\partial_\mu \varphi^* \partial^\mu \varphi = \partial_\mu \rho' \partial^\mu \rho' + (\rho' + a)^2 \partial_\mu \theta \partial^\mu \theta$

b) $\varphi^* \varphi = (\rho' + a)^2$

$$\rightarrow V(\varphi, \varphi^*) = m^2 (\varphi^* \varphi) + \frac{\lambda}{4} (\varphi^* \varphi)^2$$

$$= m^2 (\rho' + a)^2 + \frac{\lambda}{4} (\rho' + a)^4$$

$$\underset{a^2 = -\frac{2m^2}{\lambda}}{=} \frac{\lambda}{4} \left[(\rho' + a)^2 - a^2 \right]^2 - \frac{\lambda}{4} a^4$$

$$\longrightarrow \mathcal{L}' = \partial_\mu \rho' \partial^\mu \rho' + (\rho' + a)^2 \partial_\mu \theta \partial^\mu \theta - \frac{\lambda}{4} \left[(\rho' + a)^2 - a^2 \right]^2 + \frac{\lambda}{4} a^4$$

a) ρ' is massive

$$\frac{1}{2} m_{\rho'}^2 \rho'^2 = \frac{1}{2} \lambda a^2 \rho'^2 \rightarrow m_{\rho'}^2 = \lambda a^2 \longrightarrow m_\rho = a\sqrt{\lambda}$$

b) θ is massless $\longrightarrow$ Goldstone boson.

c) $\mathcal{L}'$ is not symmetric (invariant) under $U(1)$ transformation

$\longrightarrow$ The symmetry is spontaneously broken (SSB)

As a result: φ_1, φ_2 were massless before SSB.

After SSB $\rightarrow$ $\rho(x)$ is massive

$\theta(x)$ is massless

بقیه مطالب مربوط به SSB ← بعداً

پتانسیل مؤثر در مدل سیگمای خطی: (Linear Sigma-Model)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_i)^2 + \frac{1}{2}\mu^2 (\varphi_i)^2 - \frac{\lambda}{4}(\varphi_i^2)^2 \qquad i = 1, \dots, N_f$$

$$\varphi_i(x) = \varphi_i^{cl} + \eta_i(x) \qquad N_f = \#\text{ of flavors.}$$

$$\varphi_i^{cl} = \text{const.}$$

We expand $\mathcal{L}$ around φ_i^{cl}:

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left(\partial_\mu (\varphi_i^{cl} + \eta_i(x))^2\right) + \frac{1}{2}\mu^2 (\varphi_i^{cl} + \eta_i(x))^2 - \frac{\lambda}{4}\left((\varphi_i^{cl} + \eta_i(x))^2\right)^2 \\
&= \frac{1}{2}(\partial_\mu \eta_i)^2 + \frac{1}{2}\mu^2 (\varphi_i^{cl})^2 + \frac{1}{2}\mu^2 (\eta_i(x))^2 + \mu^2 \varphi_i^{cl} \eta_i \\
&\quad - \frac{\lambda}{4}\left((\varphi_i^{cl})^2 + (\eta_i(x))^2 + 2\varphi_i^{cl}\eta_i(x)\right)^2 \\
&= \frac{1}{2}(\partial_\mu \vec{\eta})^2 + \frac{\mu^2}{2}(\vec{\varphi}_{cl})^2 + \frac{1}{2}\mu^2 (\vec{\eta}(x))^2 + \mu^2\, \vec{\varphi}_{cl}\cdot\vec{\eta}(x) \\
&\quad - \frac{\lambda}{4}(\vec{\varphi}_{cl}^{\,2})^2 - \frac{\lambda}{4}(\vec{\eta}^{\,2})^2 - \lambda(\vec{\varphi}_{cl}\cdot\vec{\eta})^2 \\
&\quad - \frac{\lambda}{2}\vec{\varphi}_{cl}^{\,2}\,\vec{\eta}^{\,2} - \lambda\, \vec{\varphi}_{cl}^{\,2}(\vec{\varphi}_{cl}\cdot\vec{\eta}) - \lambda\,\vec{\eta}^{\,2}(\vec{\varphi}_{cl}\cdot\vec{\eta}(x)) \\
&= \frac{\mu^2}{2}\vec{\varphi}_{cl}^{\,2} - \frac{\lambda}{4}(\vec{\varphi}_{cl}^{\,2})^2 + (\mu^2 - \lambda\vec{\varphi}_{cl}^{\,2})(\vec{\varphi}_{cl}\cdot\vec{\eta}) \\
&\quad + \frac{1}{2}(\partial_\mu\vec{\eta})^2 + \frac{\mu^2}{2}\vec{\eta}^{\,2}(x) \\
&\quad - \frac{\lambda}{2}\left(\vec{\varphi}_{cl}^{\,2}\vec{\eta}^{\,2} + 2(\vec{\varphi}_{cl}\cdot\vec{\eta})^2\right) - \lambda\vec{\eta}^{\,2}(x)(\vec{\varphi}_{cl}\cdot\vec{\eta}(x)) \\
&\quad - \frac{\lambda}{4}(\vec{\eta}^{\,2})^2
\end{aligned}$$

Remember:

$$e^{i\int \mathcal{L}[\varphi]\, d^dx} = e^{i\int \mathcal{L}[\varphi_{cl}]\, d^dx + i\int \cancel{\frac{\delta\mathcal{L}}{\delta\eta_i(x)}}^{=0}\Big|_{\vec{\varphi}_{cl}} \eta_i(x)\, d^dx}$$

$$\times \exp\left(\frac{i}{2}\int d^dx\, d^dy\; \eta_i(x)\,\eta_j(y)\, \frac{\delta^2\mathcal{L}[\varphi]}{\delta\eta_i(x)\,\delta\eta_j(y)}\Big|_{\vec{\varphi}_{cl}} + \cdots\right)$$

$$= e^{i\Gamma[\vec{\varphi}_{cl}]}$$

$$\boxed{\Gamma[\vec{\varphi}_{cl}] = \int d^dx\, \mathcal{L}[\vec{\varphi}_{cl}] + \frac{i}{2}\log\det\left(\frac{-\delta^2\mathcal{L}}{\delta\eta_i(x)\,\delta\eta_j(y)}\Big|_{\vec{\varphi}_{cl}}\right) + \cdots}$$

Zeroth order: $\mathcal{L}_0[\vec{\varphi}_{cl}] = \frac{\mu^2}{2}\vec{\varphi}_{cl}^{\,2} - \frac{\lambda}{4}(\vec{\varphi}_{cl}^{\,2})^2 \qquad \vec{\varphi}_{cl} = \text{const.}$

First Order: $\frac{\delta\mathcal{L}}{\delta\eta_i(x)}\Big|_{\vec{\varphi}_{cl}} = 0$ بنا به فرض

جملهٔ خطی در η را می‌توان فراموش کرد زیرا $\vec{\varphi}_{cl}$ بنا به فرض پتانسیل کلاسیک را minimize می‌کرد $\longleftarrow$

2nd Order: $\quad \dfrac{\delta^2 \mathcal{L}}{\delta\eta_i(x)\,\delta\eta_j(y)}\Big|_{\vec\varphi_{cl}} = ?$

$$\mathcal{L}_{quadr.} = \frac{1}{2}(\partial_\mu\vec\eta)^2 + \frac{1}{2}\mu^2\vec\eta^{\,2}(x) - \frac{\lambda}{2}\left[\vec\varphi_{cl}^{\,2}\,\vec\eta^{\,2}(x) + 2(\vec\varphi_{cl}\cdot\vec\eta)^2\right]$$

$$= -\frac{1}{2}\eta_i(z)\,\partial^2\eta_j(z)\delta_{ij} + \frac{1}{2}\mu^2\eta_i\eta_j(z)\,\delta_{ij}$$

$$-\frac{\lambda}{2}\left[(\varphi_{cl}^k)^2\eta_i\eta_j\delta_{ij} + 2\varphi_{cl}^i\varphi_{cl}^j\eta_i\eta_j\right]$$

$$= \frac{1}{2}\eta_i(z)\left\{-\partial^2\delta_{ij} + \mu^2\delta_{ij} - \lambda\left[(\varphi_{cl}^k)^2\delta_{ij} + 2\varphi_{cl}^i\varphi_{cl}^j\right]\eta_j(z')\right.$$

$$\times\,\delta^d(z-z')$$

$$-\frac{\delta^2\mathcal{L}}{\delta\eta_l(x)\,\delta\eta_m(y)}\Big|_{\vec\varphi_{cl}} = -\left\{-\partial^2\delta_{lm} + \mu^2\delta_{lm} - \lambda\left[(\varphi_{cl}^k)^2\delta_{lm} + 2\varphi_{cl}^l\varphi_{cl}^m\right]\right\}\delta^d(x-y)$$

$\longrightarrow$ One loop effective action:

$$\frac{i}{2}\log\det\left(-\frac{\delta^2\mathcal{L}}{\delta\eta_i(x)\delta\eta_j(y)}\Big|_{\vec\varphi_{cl}}\right) =$$

$$= \frac{i}{2}\log\det\left(\left\{\partial^2\delta_{ij} - \mu^2\delta_{ij} - \lambda\left[(\varphi_k^{cl})^2\delta_{ij} + 2\varphi_{cl}^i\varphi_{cl}^j\right]\right\}\delta^d(x-y)\right)$$

✓ برای اینکه بتوانیم logdet را بدست بیاوریم برای $\vec\varphi_{cl}$ یک جهت مشخص انتخاب می‌کنیم:

$$\vec\varphi_{cl} = (0, 0, \ldots, \varphi_{cl})$$

عدد!

$\longrightarrow$ (شکست تقارن، این انتخاب جهت کاملاً دلخواه است
(به بحث شکست خودبخودی تقارن رجوع شود.)

$$\vec\varphi_{cl}^{\,2} = \varphi_{cl}^2$$

$$\varphi_{cl}^i\varphi_{cl}^j = \begin{cases} i=j=N_f \rightarrow \varphi_{cl}^2 \\ i\neq j \rightarrow 0 \end{cases}$$

• حال می‌توان ماتریس $\left(-\dfrac{\delta^2\mathcal{L}}{\delta\eta_i(x)\delta\eta_j(y)}\Big|_{\vec\varphi_{cl}}\right)$ را بصورت زیر نوشت:

$$\left(-\frac{\delta^2\mathcal{L}}{\delta\eta_i(x)\delta\eta_j(y)}\Big|_{\vec\varphi_{cl}}\right) = \begin{pmatrix} \partial^2-\mu^2+\lambda\varphi_{cl}^2 & 0 & \cdots & 0 \\ 0 & \partial^2-\mu^2+\lambda\varphi_{cl}^2 & \cdots & 0 \\ 0 & & \ddots & \vdots \\ \vdots & & & \\ 0 & & \partial^2-\mu^2+\lambda\varphi_{cl}^2 & 0 \\ 0 & 0 & \cdots & \partial^2-\mu^2+3\lambda\varphi_{cl}^2 \end{pmatrix}$$

می‌توان det این ماتریس را بصورت زیر نوشت:

$$\boxed{\det\left(\frac{-\delta^2 \mathcal{L}}{\delta\eta_i(x)\,\delta\eta_j(y)}\right)\Bigg|_{\vec{\varphi}_{cl}} = \left[\det\left(\partial^2-\mu^2+\lambda\varphi_{cl}^2\right)\right]^{N_f-1}\det\left(\partial^2-\mu^2+3\lambda\varphi_{cl}^2\right)}$$

در حالت کلی با دو نوع دترمینان سروکار داریم:

$$\det(\partial^2+m_i^2):\begin{cases} m_i^2=-\mu^2+\lambda\varphi_{cl}^2 & (i=1,\dots,N_f-1)\\ m_i^2=-\mu^2+3\lambda\varphi_{cl}^2 & i=N_f\end{cases}$$

$$\rightarrow \log\det(\partial^2+m^2) = \mathrm{Tr}\log(\partial^2+m^2) = \underbrace{\int d^dx}_{\Omega_d}\int\frac{d^dp}{(2\pi)^d}\log(-p^2+m^2)$$

$*\quad p_0 = ip_4 \qquad p^2=-p_E^2 \qquad \& \qquad \ln x = -\frac{\partial}{\partial\alpha}x^{-\alpha}\Big|_{\alpha=0}$

$$\rightarrow i\int\frac{d^dp_E}{(2\pi)^d}\log(p_E^2+m^2) = -i\frac{\partial}{\partial\alpha}\int\frac{d^dp_E}{(2\pi)^d}\frac{1}{(p_E^2+m^2)^\alpha}\Big|_{\alpha=0}$$

$$= -\frac{i\,(m^2)^{d/2}}{(4\pi)^{d/2}}\,\Gamma\left(-\frac{d}{2}\right)$$

قبل از اینکه قسمت ∞ این جواب را بدست آوریم،

جواب نهائی دترمینان را می‌نویسیم با استفاده از همین عبارت. بعداً منظم‌سازی را در $\overline{MS}$-scheme انجام می‌دهیم و

Counter Term

های مربوطه را تعریف می‌کنیم.

تا به اینجا داریم:

$$\log\det\left(\frac{-\delta^2\mathcal{L}}{\delta\eta_i(x)\,\delta\eta_j(y)}\Big|_{\vec{\varphi}_{cl}}\right) = \log\left[\det\left(\partial^2-\mu^2+\lambda\varphi_{cl}^2\right)\right]^{N_f-1} + \log\left[\det\left(\partial^2-\mu^2+3\lambda\varphi_{cl}^2\right)\right]$$

$$= (N_f-1)\log\det\left(\partial^2-\mu^2+\lambda\varphi_{cl}^2\right) + \log\det\left(\partial^2-\mu^2+3\lambda\varphi_{cl}^2\right)$$

$$= -i\Omega_d\left\{\frac{(N_f-1)}{(4\pi)^{d/2}}\Gamma\left(-\frac{d}{2}\right)\left(-\mu^2+\lambda\varphi_{cl}^2\right)^{d/2} + \frac{1}{(4\pi)^{d/2}}\Gamma\left(-\frac{d}{2}\right)\left(-\mu^2+3\lambda\varphi_{cl}^2\right)^{d/2}\right\}$$

$$\rightarrow V_{eff}(\varphi_{cl}) = -\frac{1}{\Omega_d}\Gamma[\varphi_{cl}] = -\mathcal{L}_0[\varphi_{cl}] - \frac{i}{2}\frac{1}{\Omega_d}\log\det\left(\frac{-\delta^2\mathcal{L}}{\delta\eta_i\,\delta\eta_j}\Big|_{\vec{\varphi}_{cl}}\right)+\cdots$$

قبلاً داشتیم $\quad \mathcal{L}_0[\vec{\varphi}_{cl}] = \frac{\mu^2}{2}\vec{\varphi}_{cl}^{\,2} - \frac{\lambda}{4}(\vec{\varphi}_{cl}^{\,2})^2 \xrightarrow{\vec{\varphi}_{cl}=(0,\dots,\varphi_{cl})} \frac{\mu^2}{2}\varphi_{cl}^2 - \frac{\lambda}{4}\varphi_{cl}^4$

one-loop effective potential

$$\boxed{\begin{aligned} V_{eff}(\varphi_{cl}) = &-\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 \\ &-\frac{i}{2}\left\{\frac{-i(N_f-1)}{(4\pi)^{d/2}}\Gamma\left(-\frac{d}{2}\right)\left(-\mu^2+\lambda\varphi_{cl}^2\right)^{d/2} - \frac{i}{(4\pi)^{d/2}}\Gamma\left(-\frac{d}{2}\right)\left(-\mu^2+3\lambda\varphi_{cl}^2\right)^{d/2}\right\} \\ &+ \underbrace{\frac{1}{2}\delta_\mu\,\varphi_{cl}^2 + \frac{1}{4}\delta_\lambda\,\varphi_{cl}^4}_{\text{Counterterms (to be determined) in } d=2\ \&\ d=4\ \text{dim}}\end{aligned}}$$

In $d=2$ dim :

Use: $\quad z\Gamma(z) = \Gamma(z+1) \qquad \Gamma\left(-\frac{d}{2}\right) = \dfrac{\Gamma\left(1-\frac{d}{2}\right)}{-\frac{d}{2}}$

For $d=2$

$$V_{eff}(\varphi_{cl}) = -\frac{1}{2}\mu^2\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 - \frac{1}{2}\frac{\Gamma\left(1-\frac{d}{2}\right)}{\left(-\frac{d}{2}\right)(4\pi)^{d/2}}\left\{(N_f-1)\left(-\mu^2+\lambda\varphi_{cl}^2\right) + \left(-\mu^2+3\lambda\varphi_{cl}^2\right)\right\}$$

$$+\frac{1}{2}\delta_\mu\varphi_{cl}^2 + \frac{1}{4}\delta_\lambda\varphi_{cl}^4$$

$\delta_\lambda = 0 \longleftarrow$ از آنجایی که هیچ جمله‌ای از نوع φ_{cl}^4 با ضریب ∞ وجود ندارد ✓

جملات متناسب با φ_{cl}^2 : $\quad \varphi_{cl}^2\left[-\frac{1}{2}\mu^2 + \frac{1}{2}\delta_\mu - \frac{\lambda}{2}\frac{\Gamma\left(1-\frac{d}{2}\right)}{\left(-\frac{d}{2}\right)(4\pi)^{d/2}}\underbrace{(N_f-1+3)}_{N_f+2}\right]$

$$\boxed{\begin{aligned}\delta_\mu &= -(N_f+2)\frac{\lambda\Gamma\left(1-\frac{d}{2}\right)}{\left(\frac{d}{2}\right)(4\pi)^{d/2}}\\ \delta_\lambda &= 0\end{aligned}}$$

For $d=4$

$$V_{eff} = -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 - \frac{1}{2}\frac{\Gamma\left(2-\frac{d}{2}\right)}{\left(-\frac{d}{2}\right)\left(1-\frac{d}{2}\right)(4\pi)^{\frac{d}{2}}}\left\{(N_f-1)\left(-\mu^2+\lambda\varphi_{cl}^2\right)^2 + \left(-\mu^2+3\lambda\varphi_{cl}^2\right)^2\right\}$$

$$+\frac{1}{2}\delta_\mu\varphi_{cl}^2 + \frac{1}{4}\delta_\lambda\varphi_{cl}^4$$

$$= -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 + \frac{\delta_\mu}{2}\varphi_{cl}^2 + \frac{\delta_\lambda}{4}\varphi_{cl}^4 - \frac{1}{2}\frac{\Gamma(2-d/2)}{\left(-\frac{d}{2}\right)\left(1-\frac{d}{2}\right)(4\pi)^{d/2}}\left\{(N_f-1)\left(\mu^4+\lambda^2\varphi_{cl}^4-2\mu^2\lambda\varphi_{cl}^2\right) + \left(\mu^4+9\lambda^2\varphi_{cl}^4-6\mu^2\lambda\varphi_{cl}^2\right)\right\}$$

$$= -\frac{\mu^2}{2}\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4 + \frac{\delta_\mu}{2}\varphi_{cl}^2 + \frac{\delta_\lambda}{4}\varphi_{cl}^4 - \frac{1}{2}\frac{\Gamma(2-d/2)}{\left(-\frac{d}{2}\right)\left(1-\frac{d}{2}\right)(4\pi)^{d/2}}\left\{\mu^4 N_f + \varphi_{cl}^4\left(\lambda^2 N_f + 8\lambda^2\right) + \varphi_{cl}^2\left(-2\mu^2\lambda\right)(N_f+2)\right\}$$

$$= \varphi_{cl}^2\cdot\left\{-\frac{1}{2}\mu^2 + \frac{1}{2}\delta_\mu - \frac{1}{2}\frac{\Gamma(2-d/2)}{\left(-\frac{d}{2}\right)\left(1-\frac{d}{2}\right)(4\pi)^{d/2}}\left(-2\lambda\mu^2\right)(N_f+2)\right\}$$

$$+\varphi_{cl}^4\left\{\frac{\lambda}{4} + \frac{1}{4}\delta_\lambda - \frac{1}{2}\frac{\Gamma(2-d/2)}{(-d/2)(1-d/2)(4\pi)^{d/2}}\lambda^2\cdot(N_f+8)\right\}$$

$$\boxed{\begin{aligned} \delta_\mu &= -\frac{\Gamma(2-d/2)}{(-\frac{d}{2})(1-\frac{d}{2})(4\pi)^{d/2}}\,(2\lambda\mu^2)(N_f+2) \\ \delta_\lambda &= +\frac{\Gamma(2-\frac{d}{2})}{(-\frac{d}{2})(1-\frac{d}{2})(4\pi)^{d/2}}\,\lambda^2(N_f+8) \end{aligned}} \longrightarrow V_{eff} = \frac{-1}{2}\mu\varphi_{cl}^2 + \frac{\lambda}{4}\varphi_{cl}^4$$

اما $\overline{MS}$-scheme ← شکل کلی جمله‌های واگرا به صورت $\dfrac{\Gamma(-\frac{d}{2})\,(m^2)^{d/2}}{(4\pi)^{d/2}}$ بود

برای $d=4$ می‌توان از عملیات زیر برای منظم‌سازی استفاده کرد:

$$\epsilon = 4-d\,; \quad \frac{\Gamma(2-\frac{d}{2})\,(m^2)^2\,(m^2)^{\frac{d}{2}-2}}{(-\frac{d}{2})(1-\frac{d}{2})\,(4\pi)^{\frac{d}{2}-2}\,(4\pi)^2} = \frac{\Gamma(\frac{\epsilon}{2})}{2-\frac{3}{2}\epsilon+O(\epsilon^2)}\left(\frac{4\pi}{m^2}\right)^{\frac{\epsilon}{2}}\frac{m^4}{16\pi^2}$$

$$= \left(\frac{2}{\epsilon}-\gamma_E\right)\left(1-\frac{\epsilon}{2}\ln\frac{m^2}{4\pi}\right)\left(1+\frac{3}{4}\epsilon\right)\frac{m^4}{32\pi^2}+O(\epsilon^2)$$

$$= \frac{m^4}{32\pi^2}\left(\frac{2}{\epsilon}-\ln\frac{m^2}{4\pi}-\gamma_E+\frac{3}{2}\right)$$

$$\xrightarrow[\overline{MS}\text{-scheme}]{} \frac{m^4}{32\pi^2}\left(-\ln\frac{m^2}{M^2}+\frac{3}{2}\right)$$

→ Coleman-Weinberg potential: بدون در نظر گرفتن counter Terms

$$V_{eff} = -\frac{1}{2}\mu^2\varphi_{cl}^2+\frac{\lambda}{4}\varphi_{cl}^4 - \frac{1}{2}\frac{\Gamma(-\frac{d}{2})}{(4\pi)^{d/2}}\left\{(N_f-1)(-\mu^2+\lambda\varphi_{cl}^2)^2+(-\mu^2+3\lambda\varphi_{cl}^2)^2\right\}$$

$$= -\frac{1}{2}\mu^2\varphi_{cl}^2+\frac{\lambda}{4}\varphi_{cl}^4 - \frac{1}{2}\frac{1}{32\pi^2}(N_f-1)(-\mu^2+\lambda\varphi_{cl}^2)^2\left[-\ln\frac{(-\mu^2+\lambda\varphi_{cl}^2)}{M^2}+\frac{3}{2}\right] - \frac{1}{2}\frac{1}{32\pi^2}(-\mu^2+3\lambda\varphi_{cl}^2)^2\left[-\ln\frac{(-\mu^2+3\lambda\varphi_{cl}^2)}{M^2}+\frac{3}{2}\right]$$

→ Mathematica

Note: The effective potential gives an easily visualized function whose minimization defines the exact vacuum state of the quantum field theory, including all effects of quantum corrections.