

# Some Relations Among Lusternik-Schnirelmann Categories

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**Abstract.** This paper is concerned with well-known Lusternik-Schnirelmann categories. We desire to find some links and relations among them. This has been done by using the concepts of precategory, categorical collection and closure of a category.

**Keywords:** Categorical collection, Critical point theory, cuplength, Precategory, Lusternik-Schnirelmann category.

## 1. Introduction

The Lusternik-Schnirelmann category is a topological invariant that has proved to be a useful tool in critical point theory. This invariant helps us to find lower bounds for the number of critical points of smooth functions [12]. In Lusternik-Schnirelmann theory, there are several categories with different properties. First of all, Lusternik introduced a category by means of closed contractible sets which is known as (classical) Lusternik-Schnirelmann category [5]. Then Schnirelmann estimated this category by using cohomology theory [6]. This method is still the most convenient way to compute LS categories. For this reason, some authors use the same notations for cohomological category [8]. The cuplength category is also derived from this approach to LS theory. Besides these categories, there is another LS category which is similar to the classical one and it is defined by means of open contractible subsets [4]. For the excision property of open subsets, this category is preferable especially in computation.

In the literature, all of the above categories are known as Lusternik-Schnirelmann category and it is customary to use the notation *cat* for them. In order to distinguish these categories, we call them: (classical) Lusternik-Schnirelmann (shortly LS) category, Cohomology Lusternik-Schnirelmann (CLS) category, Cup