

On the Psychology of Teaching Mathematics

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It is evident to any able mathematics teacher that the process of mathematical education of students should be organized according to their particular capacities, interests, abilities, culture and the particular nature of their mind and creativity. In this paper, we shall focus on the personality of the educators in making educational decisions for students and their scientific perspectives in mathematical education. Psychology of mathematics teachers is an important component in teachers' education and also in curriculum planning and the process of implementing the curriculum. We try to introduce mathematical models for different personalities of mathematics teachers and discuss how these models can be used in classroom and in curriculum planning.

Introduction

The complicity of teaching mathematics in classroom is due not only to teacher's personality and belief system, but also to social aspects of teacher's mathematical communication. Internal psychological aspects are discussed in an earlier paper where mathematical models for mathematical communication are introduced [2]. A good source for finding important factors taking part in the social aspects of teaching mathematics can be borrowed from sociology of science. The amount of similarity between the scientific community and the process of mathematical education of students in classroom depends on how natural the process of learning is implemented by teacher.

In this paper, we focus on the psychology of mathematics teachers, with particular attention to social aspects of learning. We try to give mathematical models for teaching mathematics and try to understand the behavior of teachers using these models.

Different models for a mathematics classroom

Field study has indicated that the meaning of doing, teaching and learning mathematics varies in different classes. This variation is partially due to the perspectives of teachers towards mathematics and towards the role of teacher in education and these perspectives are affected by their mathematical knowledge and their professional abilities as a teacher. Both of these could be strengthened by teachers' education programs, but promotion of the level of abstractness of teacher's knowledge and his/her professional abilities takes time and is not easily achieved. Therefore, not every teacher could teach every style. We have to introduce different models for a mathematics classroom, so that different teachers with different levels of sophistication in teaching could find a model appropriate for their professional capacities.

Mathematics teacher as a source of information

Knowledge in such a classroom is nothing but raw information and teaching and learning are nothing but transformation and communication of information. Teacher either orally explains the information to the class or writes the information on board or shows students demonstrations on computer or on data show. Some other information are made available to students by handing them books and papers. Student's access to information is not always in full detail. Most of the time teacher chooses the key information and makes it known to his/her pupils. Teacher also gives some explanations on the meaning and the relations between the packages of information. This role of such a teacher could be easily played by computers. Information technology could provide students with more detailed, accurate, advanced, and accessible information.

The role of student in such a class is taking notes and trying to memorize them. Repetition helps students to keep information in mind for a longer period of time.

Decisions that a teacher should make in such a classroom are very formal and can be modeled to be performed by a computer. Teacher decides about the language and tools which each piece of information shall be demonstrated to the class, and also the exercises which help students internalize the information. Teacher has to choose the key information and decide about appropriate references to introduce students for further study. Teacher has to decide about how to assess a piece of information too. We shall introduce models for different personalities of teachers in making such decisions.

Mathematics teacher as a physician

In such a classroom, teacher is trying to give special treatments to the minds of his/her students. For this kind of teacher, knowledge is only used as a means of communication in order to get information on how students think. A teacher tries to replace an unorganized mind by an ordered healthy mind. This is why he/she demonstrates some examples of truthful deductions and accurate reasonings to the class. The main occupation of the teacher in this classroom, is discovering the mental weaknesses of students and trying to bring them to a normal position in which all of their different abilities demonstrate a coherent progress during the process of mathematical education. Only a very well-trained experienced teacher can play this role in the classroom. There is some chance that some day a detailed classification of learner's behavior plus a well developed theory of information engineering could partially replace the role of these teachers in the educational system.

In this classroom, students try to awaken the abilities of their mind, by learning and imitating examples of deductions and reasonings

presented in class. They try to cooperate with their teacher in discovering their mental weaknesses.

Teacher has to have a classification of human mathematical mind so that he/she can decide each student belongs to which class and then comparing student's abilities with the particular class he/she has chosen, decide about weaknesses and strengths of the student. Classification of weaknesses is an abstract procedure, because unusual situations are exceptions; but teacher has to put similar ones in the same class, so that he/she be able to study them by their symptoms. Making such decisions is very abstract and complicated.

Mathematics teacher as a coach

In this classroom, teacher is trying to discover his/her students' personal talents and train them in direction of perfection of these talents. The particular characteristics of the mathematical knowledge being discussed in class has an important role in this training. Teacher makes determining decisions in choosing the material to be discussed in class. Different examples demonstrated in class are aimed to train different groups of students of different talents. Teacher shall be able to teach students the particular practices and exercises which help them in perfection of their talents. Teacher supports and protects his/her students in the storm of their mental contradictions. He/she has to provide an appropriate atmosphere in classroom for treatment of students' mental contradictions.

The role of student in this classroom is doing mathematics. Student personally gets actively involved in enrichment of his/her knowledge upto a level in which he/she be able to overcome mental contradictions leaning on his/her own personal abilities. Students also cooperate with teacher in the process of discovery of their talents.

Teacher has to introduce a classification of mathematical talents and decide about the manners in which these talents show off so that he/she can decide about talents of students and exercises which can lead students to perfection of their talents. Teacher has to decide about different levels of student access to each of these talents.

Mathematics teacher as a role model

In this classroom various forms of mathematical knowledge are discussed. Teacher tries to openly demonstrate the process of thinking to the class. Students could learn how to solve problems, only if they can openly watch their teacher wrestle with hard problems. Teacher must be able to think loudly so that students could imitate his/her skills of problem solving. Teacher could serve as a good role model, only if students can watch his/her thoughts inside out. In this classroom, teacher shall do mathematics which is not familiar with, in front of the students. Teacher tries to make students involved in his/her process of

problem solving. If this role could be successfully played in classroom, students have been given the best possible education for group thinking. A teacher for this class should be very experienced and knowledgeable in mathematics.

The role of students in such a classroom is doing mathematics while keeping the teacher in mind as a role model. Students are able to construct their mathematical abilities personally and guide the direction of progress in their mental skills.

Most of the decisions that such a teacher should make are personal decisions which he/she makes in problem solving and the class is supposed to immitate him in making personal decisions.

Mathematics teacher as a life companion

In this classroom, teacher tries to apply mathematics to students' everyday life situations. To this teacher, knowledge is what is practically useful. Teacher tries to introduce mathematics as a human endeavor. The aim of teacher is to translate problems of everyday life situations to the abstract language of mathematics and solve them using mathematical machinery and then translate the solution back to the language of everyday life. This ability should be internalized in students so that they use their mathematical skills outside school. In such a class, the human history of evolution of mathematical concepts is under focus. Also, the process of discovery in problem solving should be recorded carefully by students. Strategies of problem solving and skills of decision making in course of discovery are taught carefully in this classroom.

Students engage in groups in solving everyday life problems using mathematics. Learning is a group activity but each student has a particular interest and perspective in doing mathematics. Every student helps his/her colleagues during the learning process and takes advantage of the helps offered by other colleagues.

Teacher has to decide about different skills that students should learn and about different levels of suffistication in these skills. Teacher has to decide about the skills which each of the students could learn better by having a classification of students' learning strategies.

Mathematics teacher as a scientist

In this classroom, teacher unwails the mathematical order of the universe we live in. For this teacher, mathematical knowledge is based on ideas borrowed from nature. Teacher makes sure his/her students learn to observe nature, discover mathematical ideas from it and use them in development of mathematics. History of science is under focus. Students should learn how mathematical ideas taken from nature have affected human civilization. Proposing reasonable conjectures based on scientific methodology is an important skill which students should learn. The aim of teacher is not only fammiliarizing students with applications of

mathematics to other branches of science, but also teaching them skills of discovering new applications of mathematics to other areas of science. These applications do influence the practice of scientists in other areas.

Students in such a classroom search in groups for mathematical ideas in nature. They recognize mathematics as the language of nature and try to get a better understanding of nature through this language. Student also tries to apply mathematics to other branches of science.

Teacher has to decide about how he/she can motivate students to become interested in nature and its order. Teacher has to decide about the branches of science that a student is fascinated by, so that he/she could insert mathematical ideas to teach the student how to apply mathematics in basic different branches of science.

Mathematics teacher as a philosopher

In this classroom, mathematics is an abstract knowledge independent of nature which is governed by the mathematical essence of the world of creation. Mathematical ideas could be borrowed not only from nature, but also from beyond. Teacher tries to discover the mathematical order with the aim of demonstrating his/her students unity in the essence of creation. Teacher tries to find similar ideas in different parts of mathematics and use them to unveil the unity beyond these ideas. In such a class, the abstract science of mathematics is formulated independent of nature. Mathematics has many abstract layers and teacher tries to connect these layers by unifying different mathematical theories in each layer. This is a new perspective to learning mathematics in which each student does mathematics in a particular layer of abstractness.

Student tries to structure the abstract mathematical nature of his/her mind and is interested in knowing the essence beyond the mathematical ideas. Relating different branches of mathematics and performing computations and comparing them in different branches are tools which students use to extend the abstract structure of their mind. Learning and doing mathematics is no longer possible in groups because of the high level of abstractness of this kind of mathematical activity. Student is learning mathematics for reasons beyond mathematics. So, again the particular mathematical content of mathematics does not play any important role in such a classroom.

In such a classroom, teacher has to decide about the particular level of abstractness each student could handle. Teacher also has to make sure if unification of a few different examples and creation of a more abstract generalization has actually occurred for a particular student. Teacher has to have a personal knowledge of different layers of abstractness and be able to decide each particular mathematical content belongs to which of these abstract layers. Teacher chooses a particular mathematical content which he/she is most familiar with its abstract layers.

Personality of teachers in decision making

Many different decisions has to be made by teachers and the examples we have already mentioned is by no means a complete list. Nor is the different models we introduced for a mathematics classroom. Therefore, we shall find a way in which mathematical modeling of the personality of teachers in decision making be independent of the particular decisions that they have to make. Otherwise classification of their behavior in class would be a complicated task. To do this, we shall focus on the social aspects of learning. This will help us to understand the personality of the teacher independent of the particular meaning of learning in different classes.

The social aspects of learning

Learning mathematics and problem solving in classroom has a completely different nature from individual attempts in doing mathematics. When working in groups, not only decision making in different stages of problem solving is influenced by group discussions, but also diversity of perspectives towards the solution of the problem smoothens the process of individual learning. Human forms of communication makes the process of learning and its content to be considered in more human perspectives. For example, mathematical arguments instead of being considered as abstract human independent truth, seem like an address which is directing our mind to a particular place. Different addresses could lead to the same conclusions and at the end, the new places discovered are of main importance, not the particular nature of the addresses leading to that place. In better words, different minds could be satisfied with arguments of different natures. Students eventually develop different personalities in accepting arguments and the teacher will not be able to force them to think the same about the truth of a mathematical proposition. Teacher can only focus on skills of students in applying the mathematics they learn to different aspects of their life.

The Management of Group Learning

Management of learning in a classroom is very similar to management of development of science in a scientific society. Different social aspects of science [4], could be regarded as different aspects of group learning in classroom, and the teacher has to make decisions about each of these aspects for the good of his/her students' learning. Teacher's decision making is affected by many external factors. Here we mention a few of them:

The mathematical subject which is to be discussed could have numerical, geometric, algebraic or analytic nature. This could cause different decisions of teacher in his/her particular approach in teaching the subject matter.

Students' motivation in learning mathematics could be of many different levels of abstractness. Students with different motivations could not be treated the same and this affects the process of teacher's decision making.

Different learning strategies of students, and their particular mental process of thinking calls for different educational treatments and different styles of teaching mathematics. Efficient decision making of teachers could cause a more fruitful education for each of student's minds.

The goals and points of view of the educational system towards mathematical education of students force limitations in the teachers' personality in educating students. They have to follow a predetermined path in developing students' mathematical skills.

The particular shape and direction of historical movement of the educational system which is not personal, and is headed forward, reducing the value of previous systems, forces the teachers to keep themselves up to date in their teaching style and personality in decision making in classroom.

The particular mother tongue of students and the scientific language in which the subject matter is being discussed, affects teachers' decision making as a means of communication. The scientific language is usually internationalized but has to be made compatible with the mother tongue of students. Students learning mathematics in second language has to get particular educational treatment which affects the teaching personality of educators.

The social hierarchy of the mathematically educated, for example engineers, affects the perspective of students towards mathematics and directly affect the teaching personalities of mathematics teachers.

The social role of mathematics in the society which is changed by time does affect the practical meaning of mathematics in classroom and in term could cause affections in mathematics teachers' decision making.

The actual sites of learning and practicing mathematics like schools, universities, research institutes, government organizations, and their scientific structure defines a mathematical culture which affects the ways teachers decide in classroom.

The social human personality of a mathematics teacher or a mathematician also puts limitations on mathematics teachers' behavior in classroom, which in turn affects their educational decision making.

Development of mathematical knowledge in classroom

Under all the abovementioned limitations and governing rules, the process of teachers' decision making is aimed towards development of mathematical knowledge in classroom. In order to gain a better knowledge of the social structure of students' scientific progress, the essential factors of development of science in society could be modeled

in a mathematics classroom. The history of development of science contains detailed information about the social structure of scientific progress [1]. Here we list a few important factors which teachers could regard as the subject of decision making in mathematics classroom.

Distinction of social and mental roles of students

Teachers shall try to distinguish the role of each student in group learning. Also the particular mental habits and learning schemes of all students shall be made clear to them. The philosophical and scientific aspects of the mathematical subject matter shall also be separated. Each of these distinctions supports the development of mathematical knowledge in classroom.

Structural values

Many of the modern values in classroom plays an efficient role in progress of group learning. Valuing rationalism against traditionalism, liberationism against authoritarianism, getting actively involved against inactive indifference, equity against injustice, supports development of different components of knowledge in classroom.

Instrumental needs

Many students have instrumental perspectives towards learning and gaining knowledge. The environment around them, and opportunities available to them makes them have opportunistic motivations for learning. Teacher can use these instrumental motivations to guide the process of group learning in classroom.

Economical factors

One may think that economical motivations for development of science contradicts the innocence and purity of science. But modern history of science has shown that if a modest budget be available to the teachers, they can use the money for the well-being of the education of their students. Making right decisions about economical issues could help the process of learning.

Political structures and needs

Financial matters and issues of school politics usually have the tendency to combine in affecting the education of students. School has political needs and would be happy to get support from its students in this direction. Matters of politics could be directed by teacher to motivate improvement of mathematical knowledge in classroom.

Religion

In every society, values and religion have a very close relationship. A religion that its values supports the progress of science could be used in guiding the classroom. It shall be pointed out that religion and science have a complicated relationship which could be contradictory in many societies. Using religion as a motivation is a complicated and delicate art that only few teachers are capable of.

The social structure of classroom

In case development of group thinking in classroom be emphasized in an educational system, and particular social structures could be engineered in classroom to support development of mathematical knowledge, teacher can guide students' education by making correct decisions in engineering the working groups and the ways in which they scientifically communicate.

Mathematical modeling of social decision making

All of the above factors are ways in which a teacher can motivate a social behavior. What the teacher has to decide about is if an action will have the effect he/she expects and if his/her decision will find social ground. The easiest way is trial and error and gaining experience about the social behavior of students in a society. But this is not an efficient way, since in this period of time, social structures are under constant construction and development. Experiences that a teacher gains soon will be useless because of social evolutions. So every teacher has a mental model which he/she uses for prediction of social behavior of students. If we could approximate teachers' mental models with mathematical models, such mathematical models not only could replace the experience of teachers, but also would give information about decision making of teachers for different generations. One could predict the social behavior of students via these mathematical models and make decisions according to one's predictions.

Teachers have different mental models, because they have different philosophies for a social phenomena taking shape. It is these different philosophies which shall be the subject of our field study. Here we list a few perspectives which is by no means a complete list:

Philosophy of influential individuals

Many teachers believe that classroom is socially following a few influential individuals. These individuals could be the teacher of a few students. According to this philosophy, a group behavior in classroom is accepted by students, only if a few influential individuals follow it. Teachers who believe this philosophy try to be influensive in classroom, or at least influensive of the social leaders of students among them. Sometimes they try to choose individuals and make them role models, so that they become influential in time.

Mathematical modeling of this system is very easy. It is enough to record the actions of students and discover which of the students show influential social roles in classroom.

Philosophy of majority

This philosophy suggests that classroom follows the majority. If the majority of the students are prepared to accept a social behavior in classroom, the minority will follow them. In order to predict the social

behavior in such a classroom, teacher has to be familiar with many of the students so that he/she could predict individual reactions of students to the social behavior.

For mathematical modelling of this system, one can decide if the majority accepts a social change, by asking students to fill a questionnaire which shows students' particular opinions in this regard.

Philosophy of coherence of social structures

According to this philosophy, the classroom follows the dominant global social structures outside the classroom. Teachers following this philosophy try to model the social behavior they want to suggest to the classroom, in the global society and try to verify if it will find social ground or not.

Mathematical modeling of this kind of decision making is going to be very difficult. Because a natural dictionary between the social phenomena in classroom and social equivalents in the outside society does not exist. One shall make the task of decision making very abstract in order to find an equivalent form of it, in the outside society.

Philosophy of social evolution in time

Some teachers believe that if one is supposed to guide the students towards predetermined goals, very rarely the classroom is naturally prepared to move in the direction suggested by the teacher. In fact, a social change should occur so that the teacher be able to guide the classroom to new social order. But social change is a continuous slow process which takes time and constant supervision. In this philosophy decision making is a long term process.

Mathematical modeling of this system of decision making is possible if the teacher can list a few behavioral goals for the changes he/she suggests. Then constant recording of instances that these behaviors are shown in classroom, can give us information on the extent to which the classroom has had progress in achieving the behavioral goals.

Philosophy of social character

Following this philosophy is an abstract practice. Some teachers believe that one can regard the classroom as a whole and define characters for the classroom as one defines a character for an individual. In fact one is modeling the society by an individual. Teacher in the process of making decisions, considers the abstract individual in his/her mind and tries to predict this individual's reaction towards the social suggestions. Then he/she translates back this behavior to the social language of classroom.

This intuitive abstract process is very hard to be modeled by a mathematical system. Because it is not at all mathematically clear in which ways a society is similar to an individual. Also it is not possible to predict individual's behavior by a mathematical model.

Suggestions for curriculum planning

As one can see from the above different personalities of teachers, there is no way that a curriculum could plan for a single model of mathematics teacher. But what can be done, is to implement a social structure in classroom for which every teacher could easily communicate socially with this structure and guide the process of development of mathematical knowledge in classroom.

On the other hand, the implemented social structure on classroom should not cause limitations on the social character of classroom. Because such un-natural limitations make the structure unacceptable to students.

We suggest that the curriculum should stress on coherence of the social structure of the classroom, in a way that the social system could be approximated by the character of an individual as close as possible. This approach seems natural because character of an individual is in its own right, a superposition of internal personalities which communicate inside the individual's personality.

Curriculum planning and educational system can support the coherence of the social structure of the classroom by insisting on the growth and complicacy of the social structure of the classroom. More precisely, the more the social structure of the classroom becomes advanced and mature, the more coherence between the subsystems of the classroom is implemented.

The easy way, is to try to model the whole social structure of the global society inside the classroom. But this way, educational system loses ground in influencing the global social structure, which is one of the main roles that a pioneer educational system should play in the society. The correlation of educational systems and the social scientific structure is studied in an earlier paper [3]. We have tried to give a mathematical model for an educational system and for scientific research and study their correlation using these mathematical models.

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