

**MR2506360 (2010i:58009)** 58C20 (26B05 46E30 46E35)

**Ranjbar-Motlagh, Alireza (IR-SHAR)**

**Generalized Rademacher-Stepanov type theorem and applications. (English summary)**

*Z. Anal. Anwend.* **28** (2009), no. 3, 249–275.

A functional  $A: \mathbb{R}^n \rightarrow \mathbb{R}$  is said to be an  $L^p$ -differential of  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  at a point  $x$  if

$$\lim_{r \rightarrow 0} r^{-n-p} \int_{B(x,r)} |f(y) - f(x) - A \cdot (y - x)|^p dx = 0.$$

The author proves that a sufficient condition for  $L^p$ -differentiability a.e. is that

$$\limsup_{r \rightarrow 0} r^{-n-p} \int_{B(x,r)} |f(y) - f(x)|^p dy < \infty$$

for a.e.  $x \in \mathbb{R}^n$ . A similar result is proved for mappings on open domains in Lipschitz manifolds, with values in a Banach space or in a metric space. As a consequence, it is shown that the Sobolev spaces of Korevaar-Schoen and of Reshetnyak are equivalent. Further,  $L^\Phi$ -differentiability related to Orlicz spaces is introduced and a result of Rademacher-Stepanov type is applied to obtain (Trudinger type) exponential differentiability of  $W^{1,n}$ -functions.

Reviewed by *Jan Malý*

## References

1. Adams, R. A., *Sobolev Spaces*. New York: Academic Press 1975. [MR0450957 \(56 #9247\)](#)
2. Benyamini, Y. and Lindenstrauss, J., *Geometric Nonlinear Functional Analysis*. Vol. I. AMS Coll. Pub. 48. Providence (RI): Amer. Math. Soc. 2000. [MR1727673 \(2001b:46001\)](#)
3. Evans, L. C. and Gariepy, R. F., *Measure Theory and Fine Properties of Functions*. Studies Adv. Math.. Boca Raton (FL): CRC Press 1992. [MR1158660 \(93f:28001\)](#)
4. Federer, H., *Geometric Measure Theory*. Grundlehren math. Wiss. 153. New York: Springer 1969. [MR0257325 \(41 #1976\)](#)
5. Gilbarg, D. and Trudinger, N. S., *Elliptic Partial Differential Equations of Second Order*. Classics Math.. Berlin: Springer 2001. [MR1814364 \(2001k:35004\)](#)
6. Gregori, G., Sobolev spaces and harmonic maps between singular spaces. *Calc. Var.* 7 (1998), 1–18. [MR1624422 \(99e:58048\)](#)
7. Heinonen, J., Koskela, P., Shanmugalingam, N. and Tyson, J. T., Sobolev classes of Banach space-valued functions and quasiconformal mappings. *J. Anal. Math.* 85 (2001), 87–139. [MR1869604 \(2002k:46090\)](#)
8. Kirchheim, B., Rectifiable metric spaces: local structure and regularity of the Hausdorff measure. *Proc. Amer. Math. Soc.* 121 (1994), 113–123. [MR1189747 \(94g:28013\)](#)
9. Korevaar, N. and Schoen, R., Sobolev spaces and harmonic maps for metric space targets. *Comm. Anal. Geom.* 1 (1993)(4), 561–659. [MR1266480 \(95b:58043\)](#)
10. Malý, J. and Ziemer, W. P., *Fine Regularity of Solutions of Elliptic Partial Differential Equations*. New York: Springer 1982. [MR670156 \(84d:35001\)](#)

- tions. Math. Surveys Monogr. 51. Providence (RI): Amer. Math. Soc. 1997. [MR1461542 \(98h:35080\)](#)
11. Ranjbar-Motlagh, A., *Analysis on Metric-Measure Spaces*. Ph. D. Thesis. New York University 1998. [MR2697437](#)
  12. Rao, M. M. and Ren, Z. D., *Applications of Orlicz Spaces*. Pure Appl. Math. 250. New York: Marcel Dekker 2002. [MR1890178 \(2003e:46041\)](#)
  13. Reshetnyak, Yu. G., Sobolev-type classes of functions with values in a metric space (in Russian). Transl. in: *Siberian Math. J.* 38 (1997)(3), 567–583. [MR1457485 \(98h:46031\)](#)
  14. Reshetnyak, Yu. G., Sobolev-type classes of functions with values in a metric space II (in Russian). Transl. in: *Siberian Math. J.* 45 (2004)(4), 709–721. [MR2091651 \(2005e:46055\)](#)
  15. Saks, S., *Theory of the Integral* (sec. ed.). New York: Dover 1964. [MR0167578 \(29 #4850\)](#)
  16. Stein, E. M., *Singular Integrals and Differentiability Properties of Functions*. Princeton Math. Ser. 20. Princeton (NJ): Princeton Univ. Press 1970. [MR0290095 \(44 #7280\)](#)
  17. Väisälä, J., *Lectures on  $n$ -Dimensional Quasiconformal Mappings*. Lecture Notes Math. 229. Berlin: Springer 1971. [MR0454009 \(56 #12260\)](#)
  18. Ziemer, W. P., *Weakly Differentiable Functions*. Graduate Texts Math. 120. New York: Springer 1989. [MR1014685 \(91e:46046\)](#)

*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

© Copyright American Mathematical Society 2010, 2011