Homework 5: Optimal Monetary Policy

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1. Suppose the economy is described by the following log-linearized system:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + E_t(z_{t+1} - z_t) + u_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

where u_t is a demand shock, z_t is a productivity shock, and e_t is a cost shock. Assume that:

$$u_t = \rho_u u_{t-1} + \xi_t \quad , \quad z_t = \rho_z z_{t-1} + \psi_t \quad , \quad e_t = \rho_e e_{t-1} + \varepsilon_t$$

where ξ, ψ, ε are white noice processes. The central bank sets the nominal interest rate i_t to minimize

$$0.5 E_t \left[\sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) \right]$$

   (a) Derive the optimal time-consistent policy for the discretionary central banker. Write the first order conditions and the reduced form solutions for x_t and π_t

   (b) Drive the interest rate feedback rule implied by the optimal discretionary policy

   (c) Show that under the optimal policy, nominal interest rates are increased enough to raise the real interest rate in response to a rise in expected inflation

   (d) How will x_t and π_t move in response to a demand shock? a productivity shock?

2. As shown in Woodford (2003), in the presence of real balances as a source of indirect utility in an otherwise standard New Keynesian model, a second-order approximation to the representative household's welfare is proportional to

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta_t (\pi_t^2 + \nu \tilde{y}_t^2 + \alpha_i i_t^2)$$

Consider the problem of choosing the state contingent policy $\{\tilde{y}_t, \pi_t\}_{t=0}^{\infty}$ that maximizes welfare subject to the sequence of constraints

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t$$

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\tilde{y}_{t+1}\}$$

for $t = 0, 1, \cdots$ where the natural rate r_t^n is assumed to follow an exogenous process.

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(a) Determine the optimality conditions for the problem described above

(b) Show that the implied optimal policy can be implemented by means of a interest rate rule of the form

$$i_t = (1 + \frac{\kappa}{\sigma\beta})i_{t-1} + \frac{1}{\beta}\Delta i_{t-1} + \frac{\kappa}{\alpha_i\sigma}\pi_t + \frac{\nu}{\alpha_i\sigma}\Delta\tilde{y}_t$$

that is independent of r_t^n and its properties.