General Physics I

chapter 8

Sharif University of Technology
Mehr 1401 (2022-2023)

M. Reza Rahimi Tabar

Chapter 8
Potential Energy and Conservation of Energy

- ✓ 8.01 Distinguish a conservative force from a nonconservative force.

- ✓ 8.02 For a particle moving between two points, identify that the work done by a conservative force does not depend on which path the particle takes.

- ✓ 8.03 Calculate the gravitational potential energy of a particle (or, more properly, a particle–Earth system).

- ✓ 8.04 Calculate the elastic potential energy of a block–spring system.

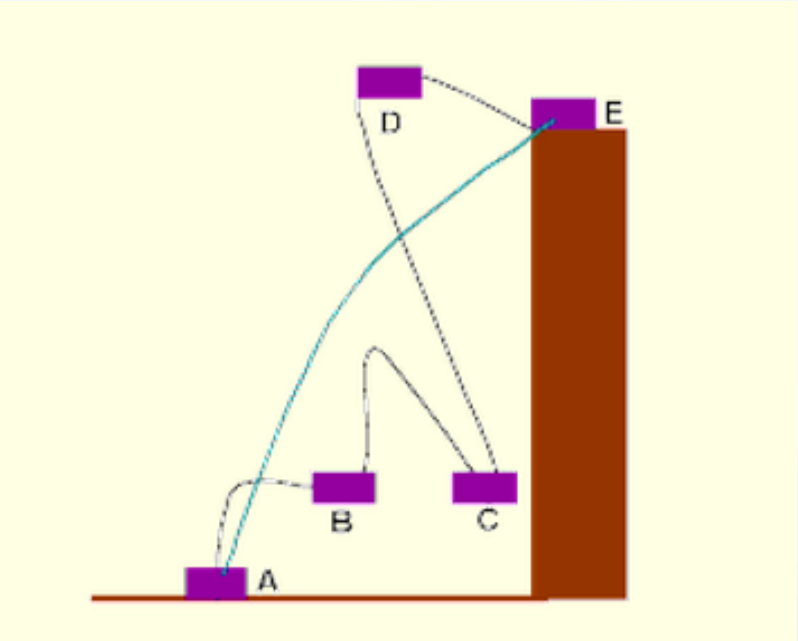
What Is Physics?

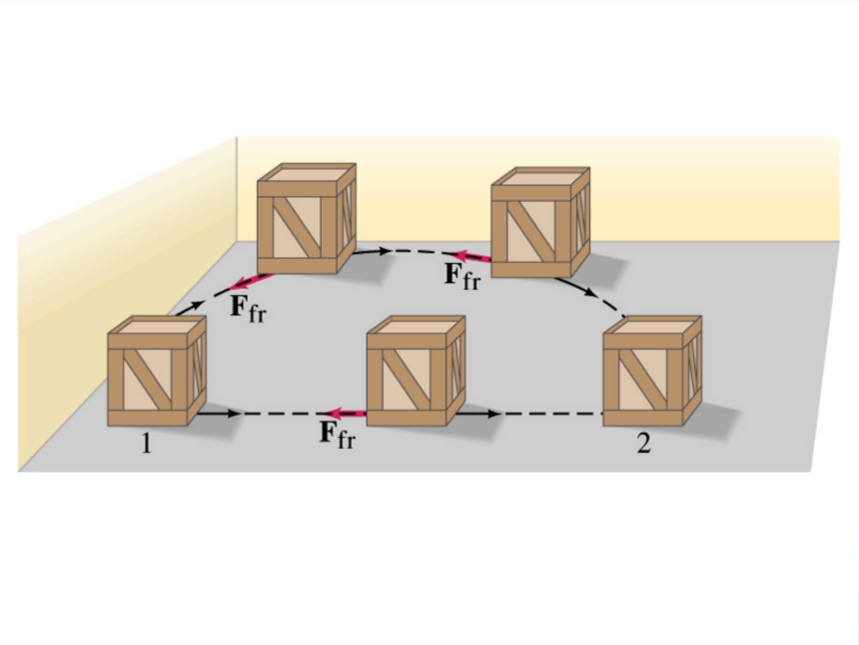
✓ Potential Energy U

Potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

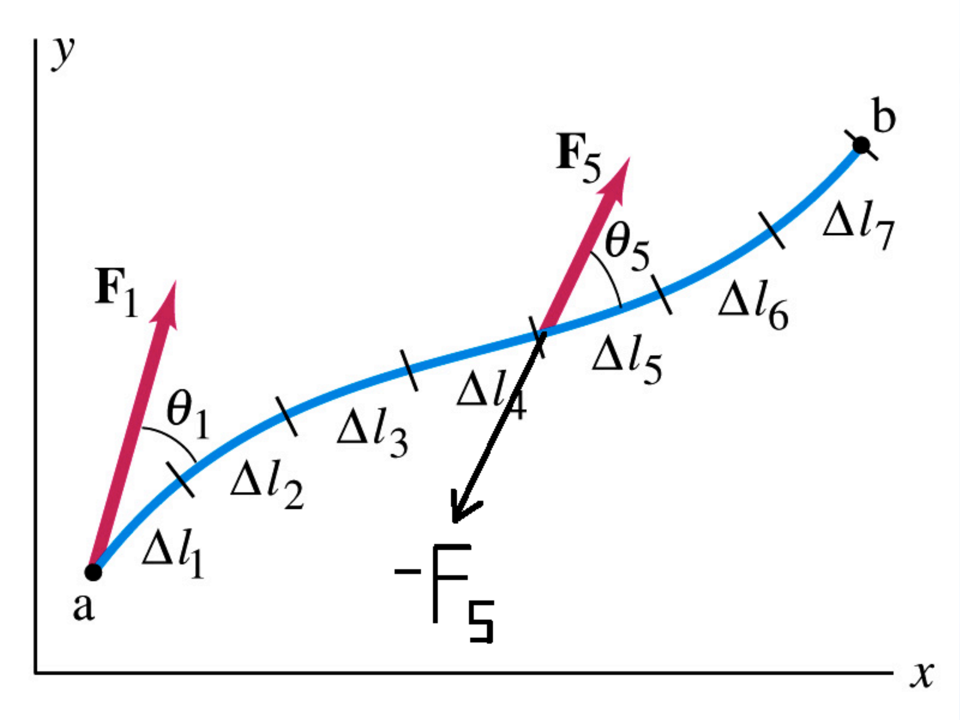
Conservative and non-conservative forces

Gravity and friction forces





Potential Energy



- The potential energy change is the work done against the conservative force

Work and Potential Energy

- Potential energy function:

$$\Delta U = U(\vec{r}_2) - U(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

$$\Delta U = -W.$$

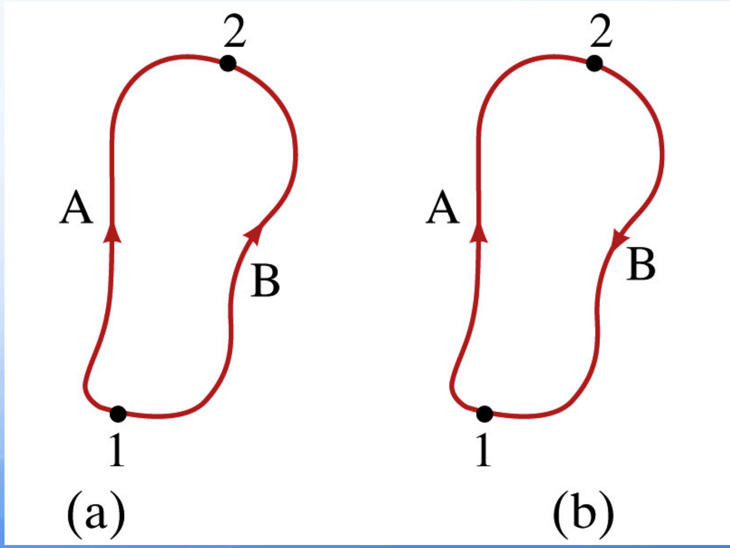
Work in closed loop

- Consider the work done in going from point r1 to point r2, W12. If we go, now, from point r2 to r1, we have W21 = −W12 since the total work

  W12 + W21 = (U2 − U1) + (U1 − U2) = 0.

$$W_{12} + W_{21} = \int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r} + \int_{r_2}^{r_1} \mathbf{F} \cdot d\mathbf{r}$$

$$W_{12} + W_{21} = \oint \mathbf{F} \cdot d\mathbf{r} = 0$$



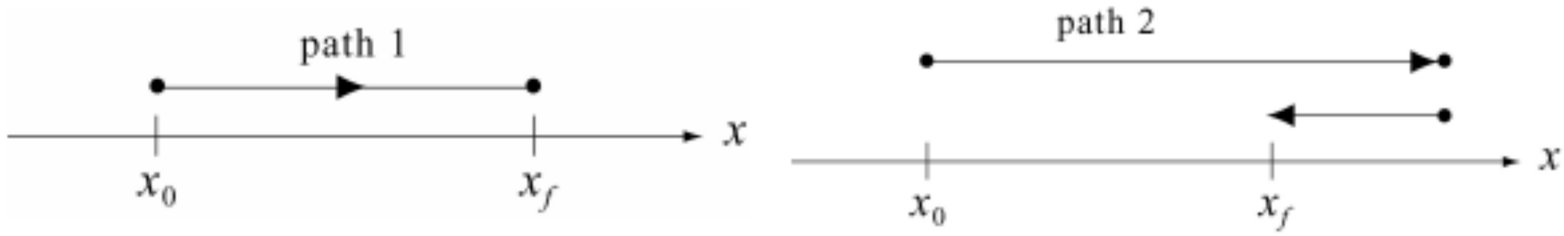
- ***F*** is a conservative force if its integral cover any closed path is zero.

Work in closed loop

- ***F*** is a conservative force if its integral cover any closed path is zero.

$$\oint \vec{F} \cdot d\vec{r} = 0 \qquad \text{(conservative force)}$$

Non-conservative Force



$$W_{\text{friction}} = \int_{\text{path 1}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{\text{path 1}} F_x\, dx = -\mu_k N s_1 = -\mu_k N \Delta x < 0$$

$$W_{\text{friction}} = \int_{\text{path 2}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{\text{path 2}} F_x\, dx = -\mu_k N s_2 < 0\,.$$

$s_2 > s_1$

Determining Potential Energy Values

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx.$$

$$dU = -F(x)dx$$

$$F = -\frac{dU}{dx}$$

- Freedom in choosing the origin

U ----→ U + C (constant)   F -----→ F

Gravitational Potential Energy

$$F = -mg\hat{k}$$

$$U(z) = -\int_0^z (-mg)dz = mgz$$

$$F_z = -\frac{dU(z)}{dz} = -mg$$

Elastic Potential Energy

$$F = -kx$$

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \tfrac{1}{2}k\left[x^2\right]_{x_i}^{x_f},$$

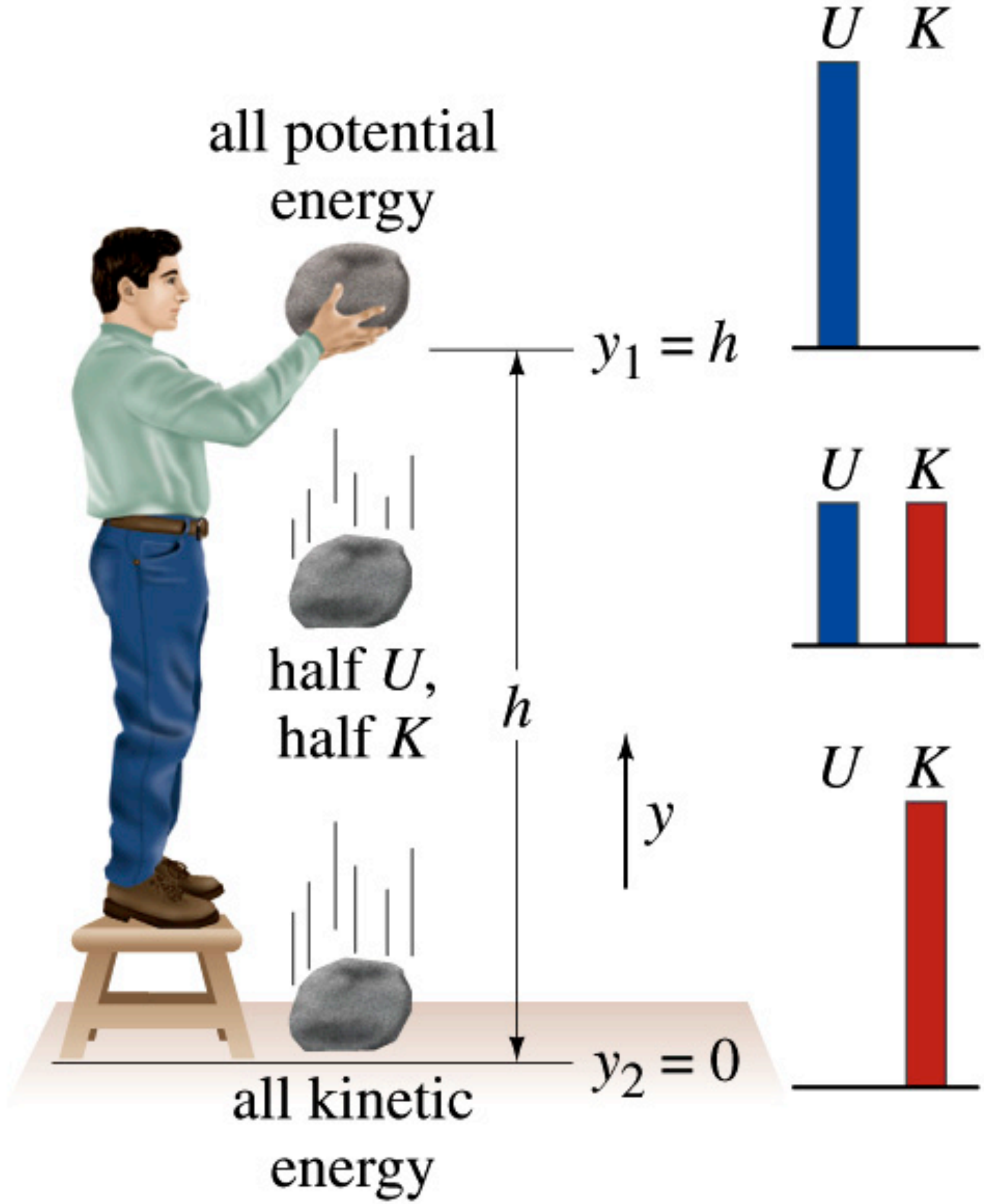
$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

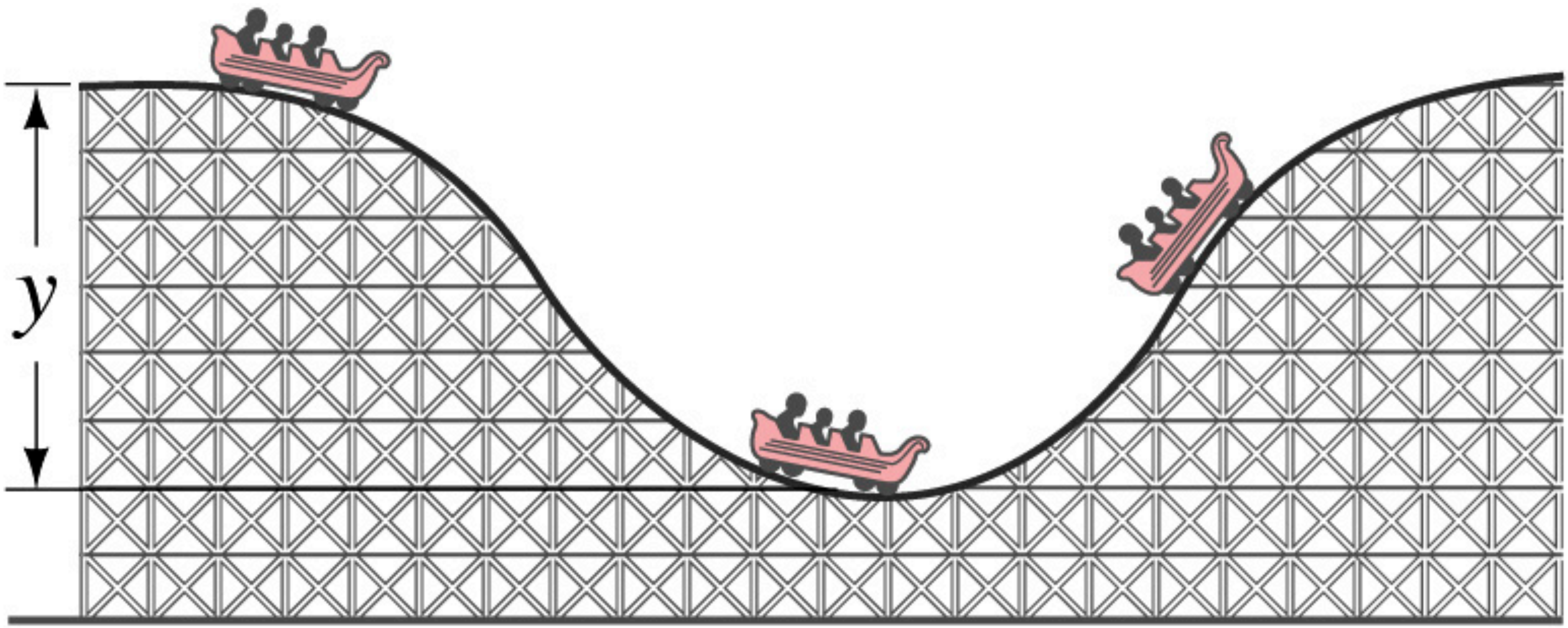
$$U(x) = \tfrac{1}{2}kx^2 \qquad \text{(elastic potential energy).}$$

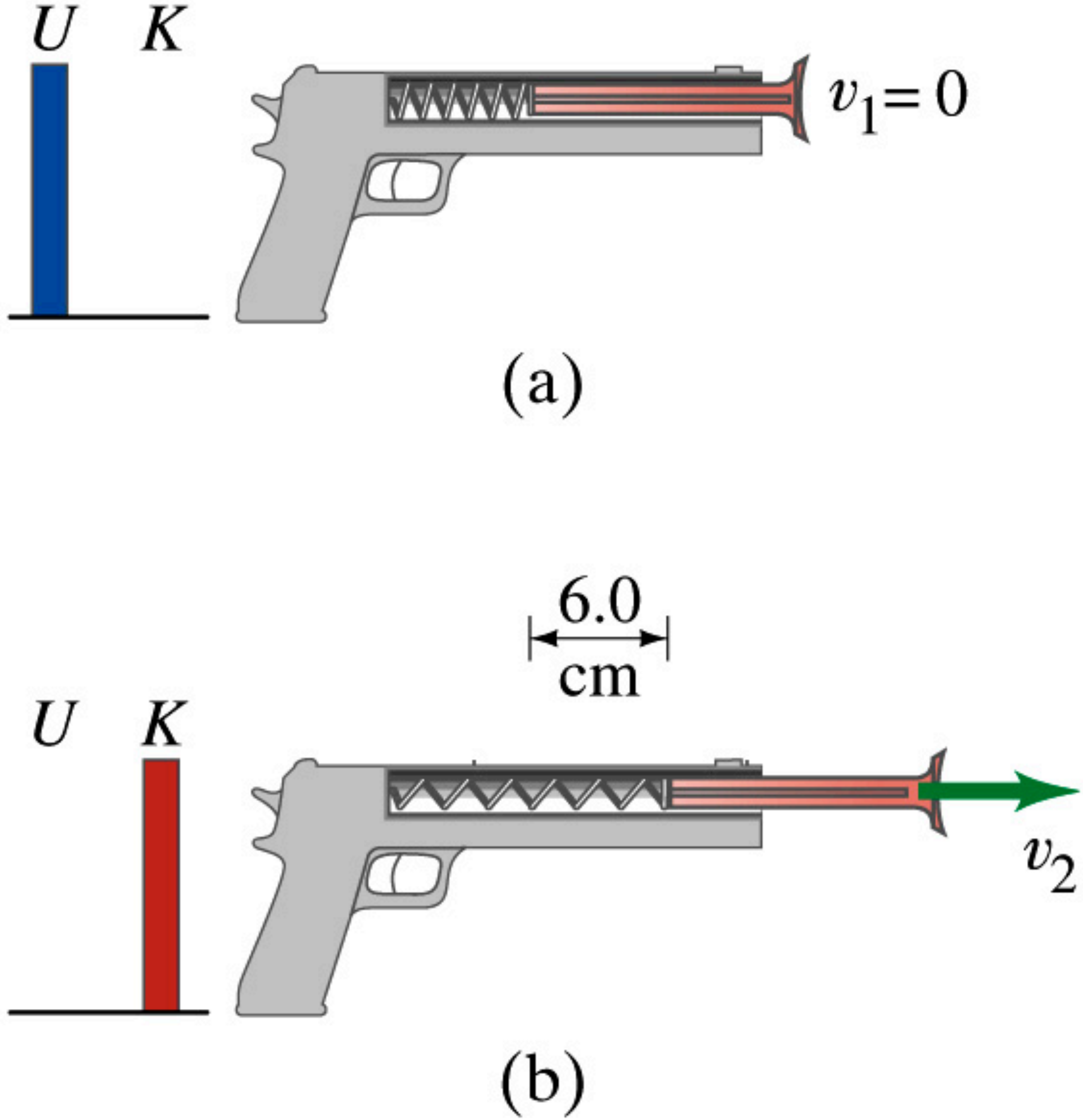
$$F_x = -\frac{dU(x)}{dx} = -kx$$

Conservation of Mechanical Energy

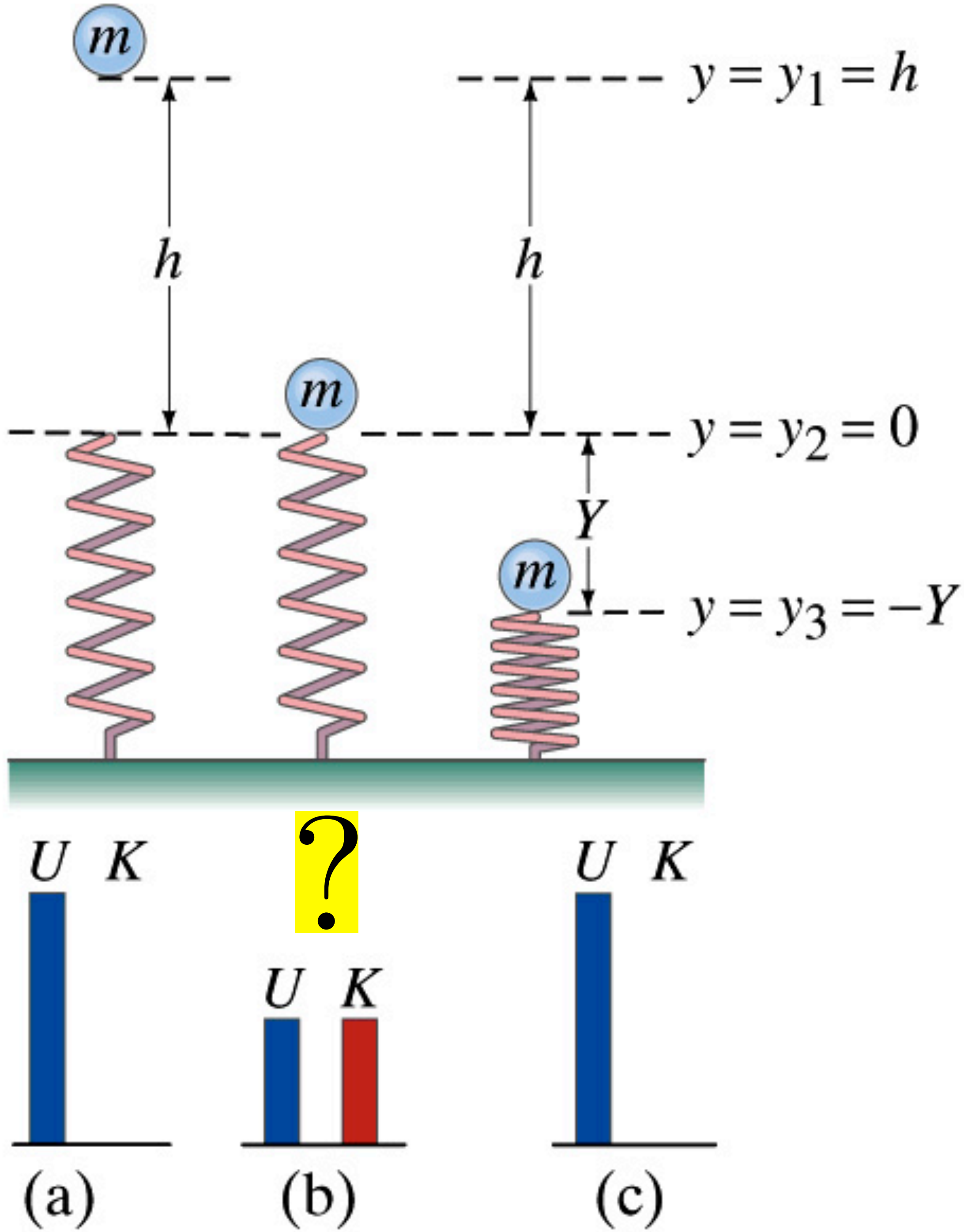
$$W = -\Delta U = K_f - K_i$$
$$-(U_f - U_i) = K_f - K_i$$
$$K_i + U_i = K_f + U_f = E_{mech} = Constant$$


U K
all potential
energy
y1 = h
U K
half U,
half K
h
y
U K
y2 = 0
all kinetic
energy

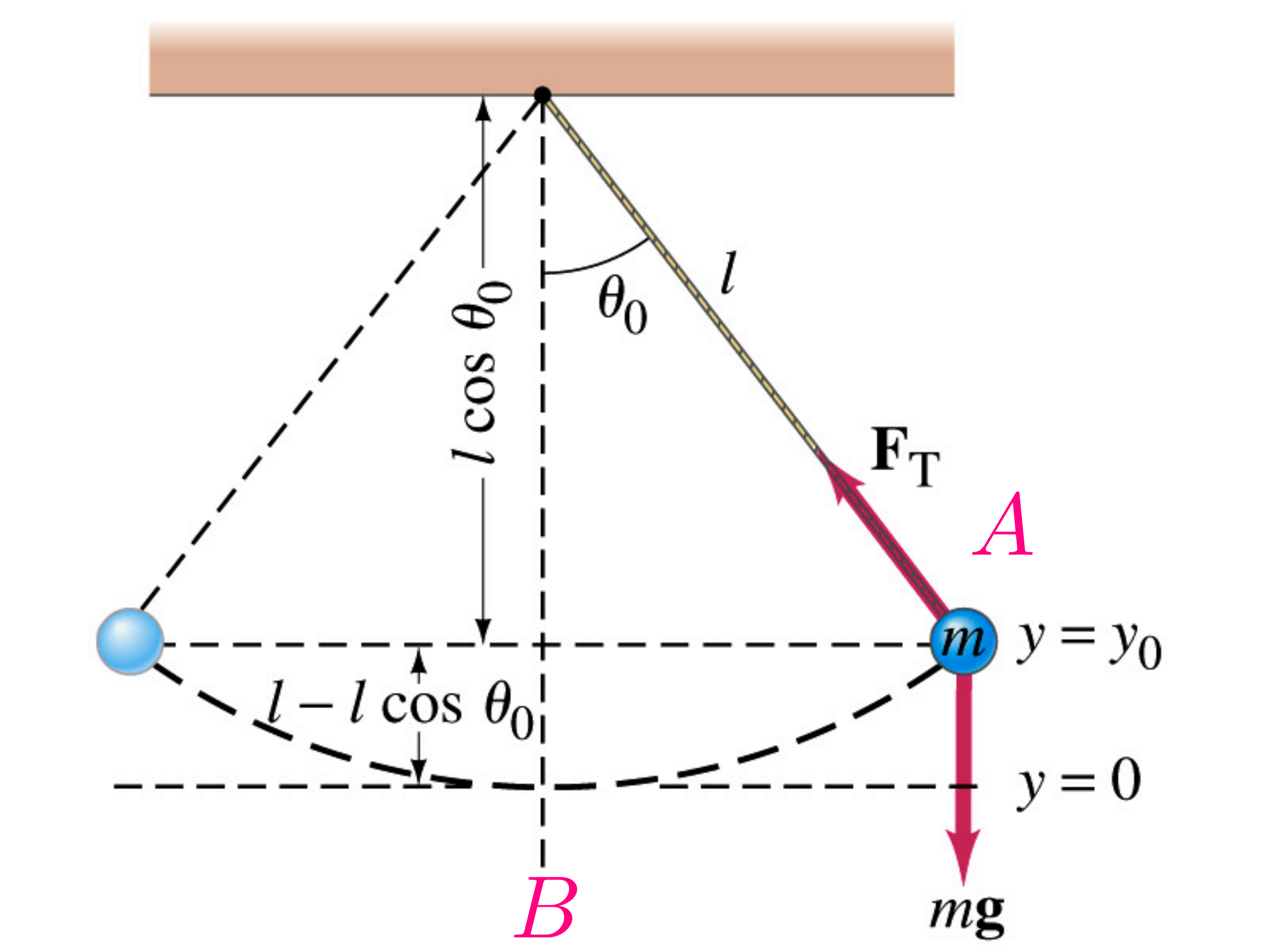

y



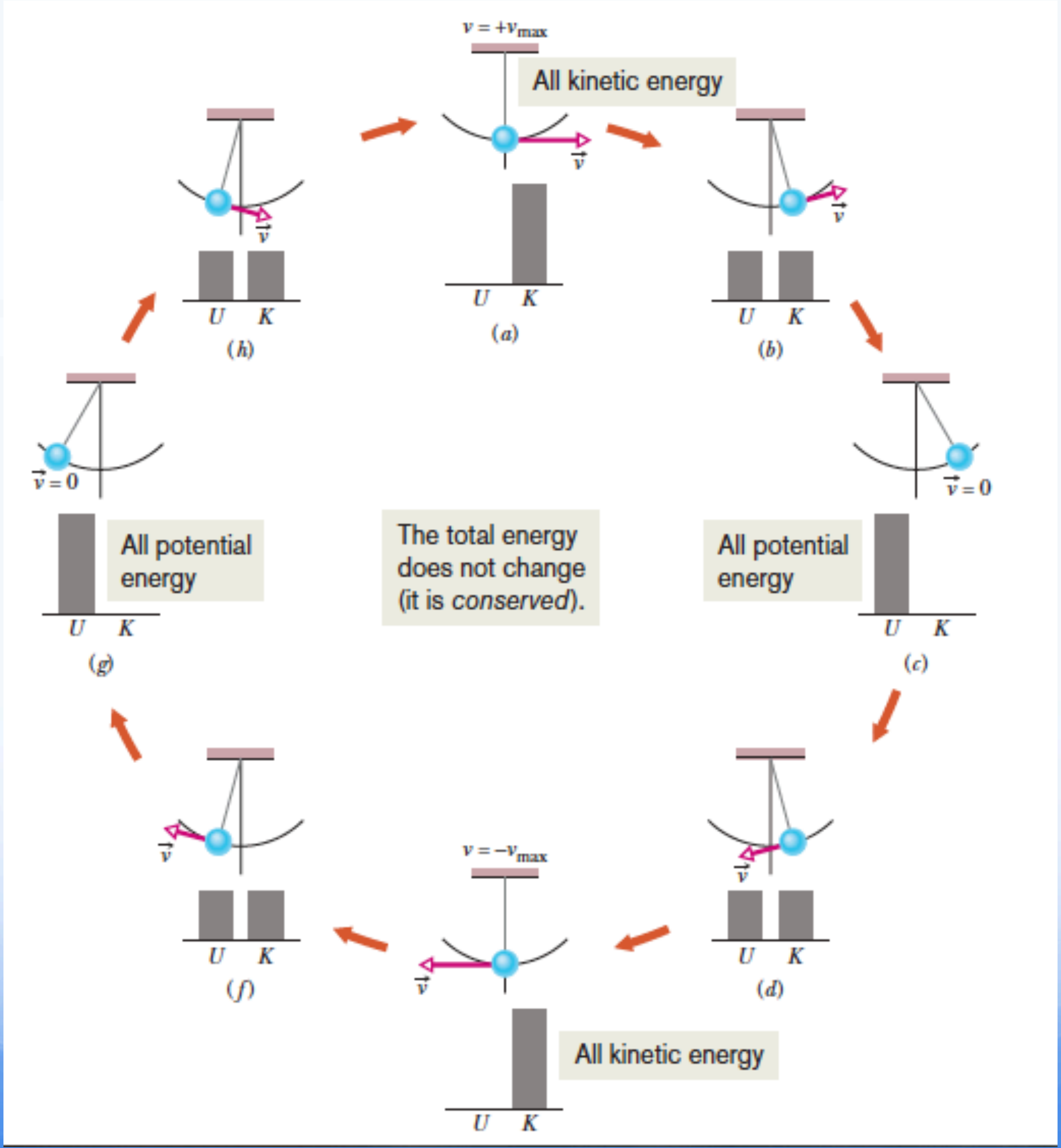
U
K
$v_1 = 0$
(a)
6.0
cm
U
K
v_2
(b)



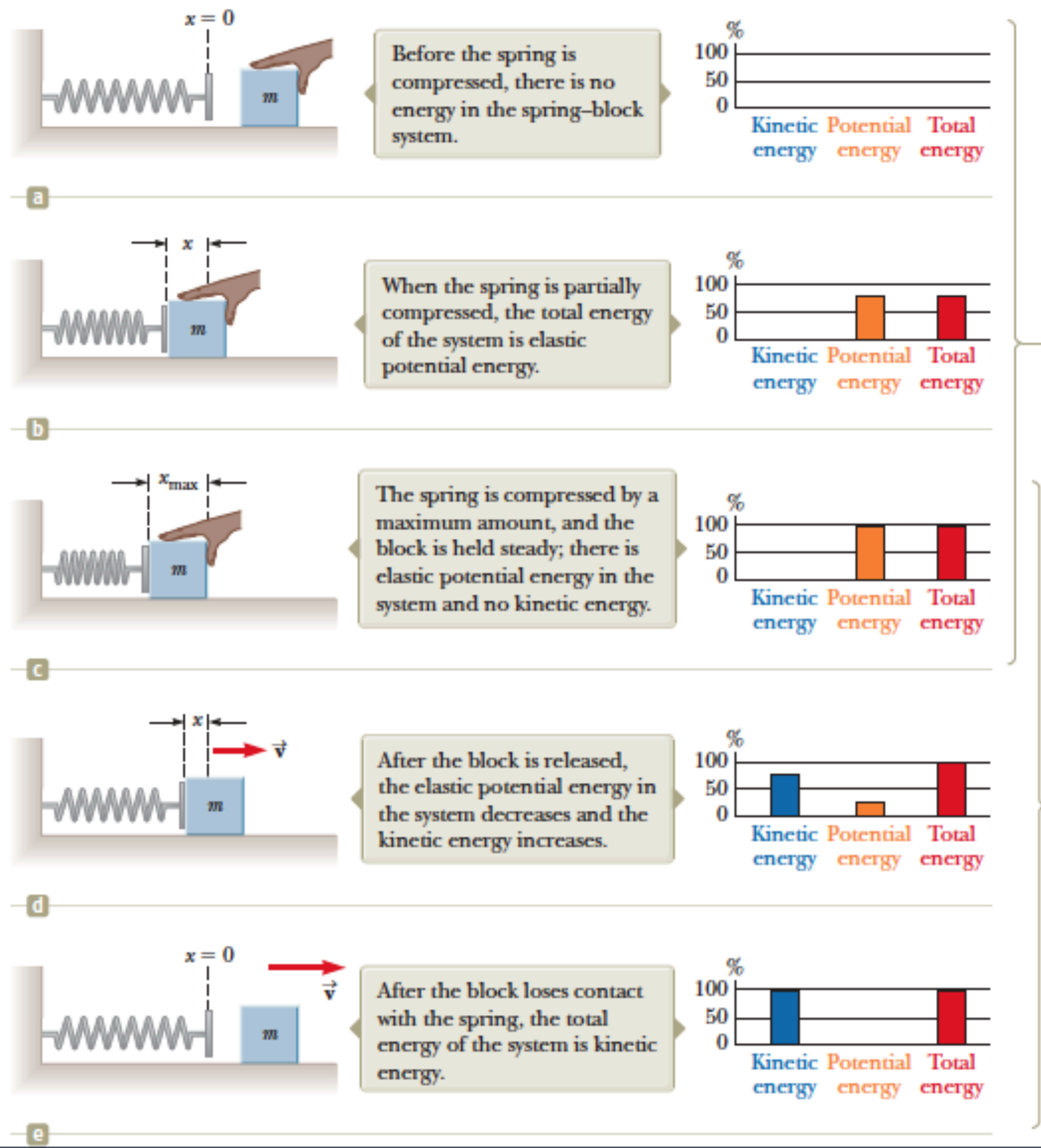
m
h
h
$y = y_1 = h$
m
$y = y_2 = 0$
Y
m
$y = y_3 = -Y$
?
U K
U K
U K
(a)
(b)
(c)



$$V_B = ?$$



$v = +v_{max}$
All kinetic energy
$\vec{v}$
U K
(a)
$\vec{v}$
U K
(b)
$\vec{v} = 0$
All potential energy
U K
(c)
$\vec{v}$
U K
(d)
$v = -v_{max}$
$\vec{v}$
All kinetic energy
U K
$\vec{v}$
U K
(f)
$\vec{v} = 0$
All potential energy
U K
(g)
$\vec{v}$
U K
(h)
The total energy does not change (it is *conserved*).


x = 0
m
Before the spring is compressed, there is no energy in the spring–block system.
%
100
50
0
Kinetic energy
Potential energy
Total energy
a
x
m
When the spring is partially compressed, the total energy of the system is elastic potential energy.
%
100
50
0
Kinetic energy
Potential energy
Total energy
b
x_max
m
The spring is compressed by a maximum amount, and the block is held steady; there is elastic potential energy in the system and no kinetic energy.
%
100
50
0
Kinetic energy
Potential energy
Total energy
c
x
v
m
After the block is released, the elastic potential energy in the system decreases and the kinetic energy increases.
%
100
50
0
Kinetic energy
Potential energy
Total energy
d
x = 0
v
m
After the block loses contact with the spring, the total energy of the system is kinetic energy.
%
100
50
0
Kinetic energy
Potential energy
Total energy
e

Reading a potential Energy Curve

$$W = -\Delta U = K_f - K_i$$
$$-(U_f - U_i) = K_f - K_i$$
$$K_i + U_i = K_f + U_f = E_{mech} = Constant$$

$$U(x_2) - U(x_1) = -\int_{x_1}^{x_2} F(x)\, dx$$

$$F(x) = -\frac{dU(x)}{dx} \qquad \text{(one-dimensional motion),}$$

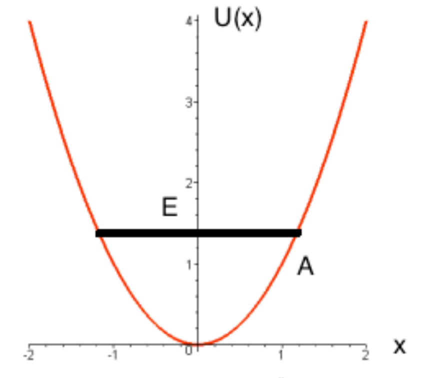
Elastic potential curve

- Turning Points: K=0 E=U(x), x_i' s = turning points

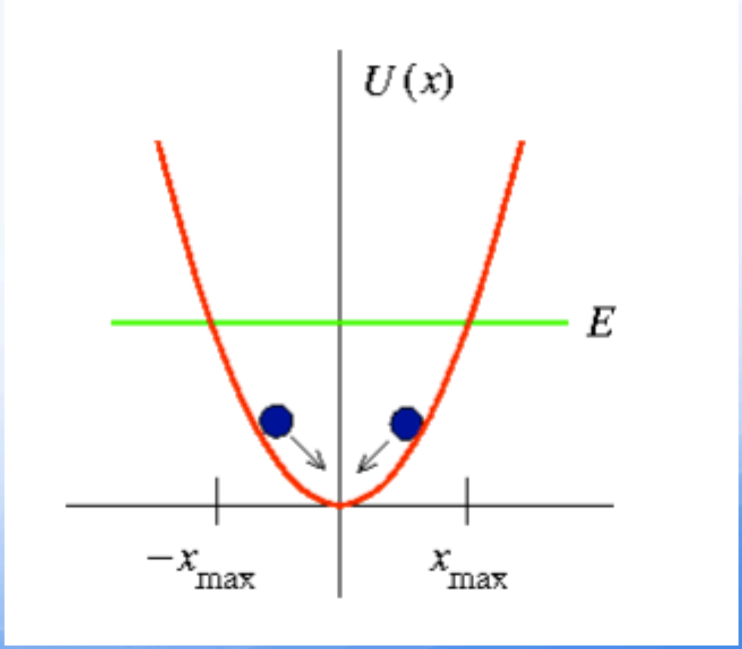
$$U(x) = \frac{1}{2} k x^2 . \qquad (13.6.4)$$

$$F_x = -\frac{dU(x)}{dx} = -\frac{d}{dx}\left(\frac{1}{2} k x^2\right) = -k x . \qquad (13.6.5)$$

the potential energy function for the spring force as function of x with $U(x_0 = 0) = 0$ (the units are arbitrary).



E= 1/2 k A^2



Potential Curve

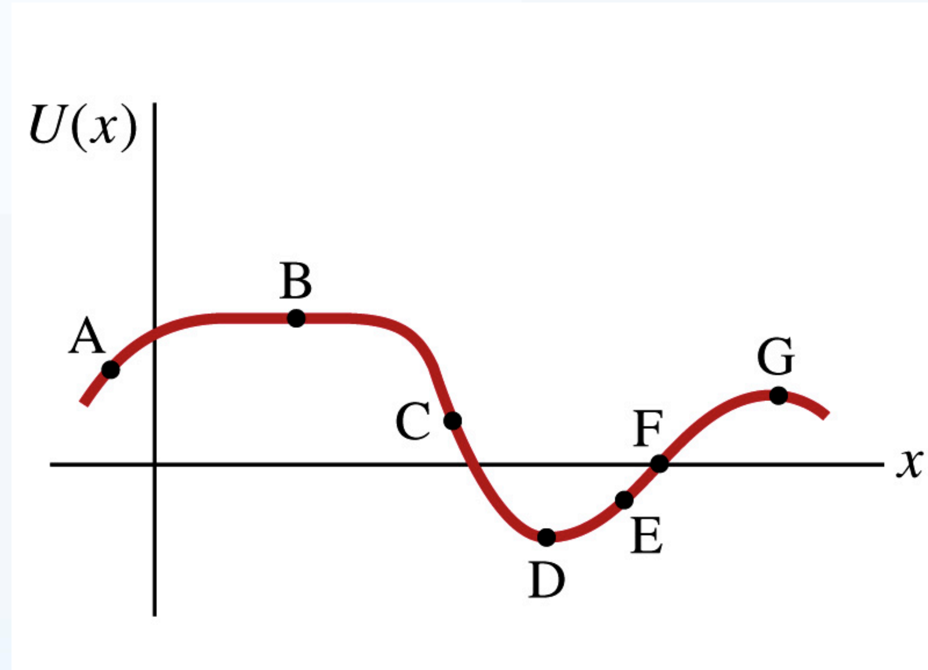
- Extremums of potentials:

$$\frac{dU(x)}{dx} = 0 .$$

$$0 = \frac{dU(x)}{dx} = k\,x .$$

Stability analysis

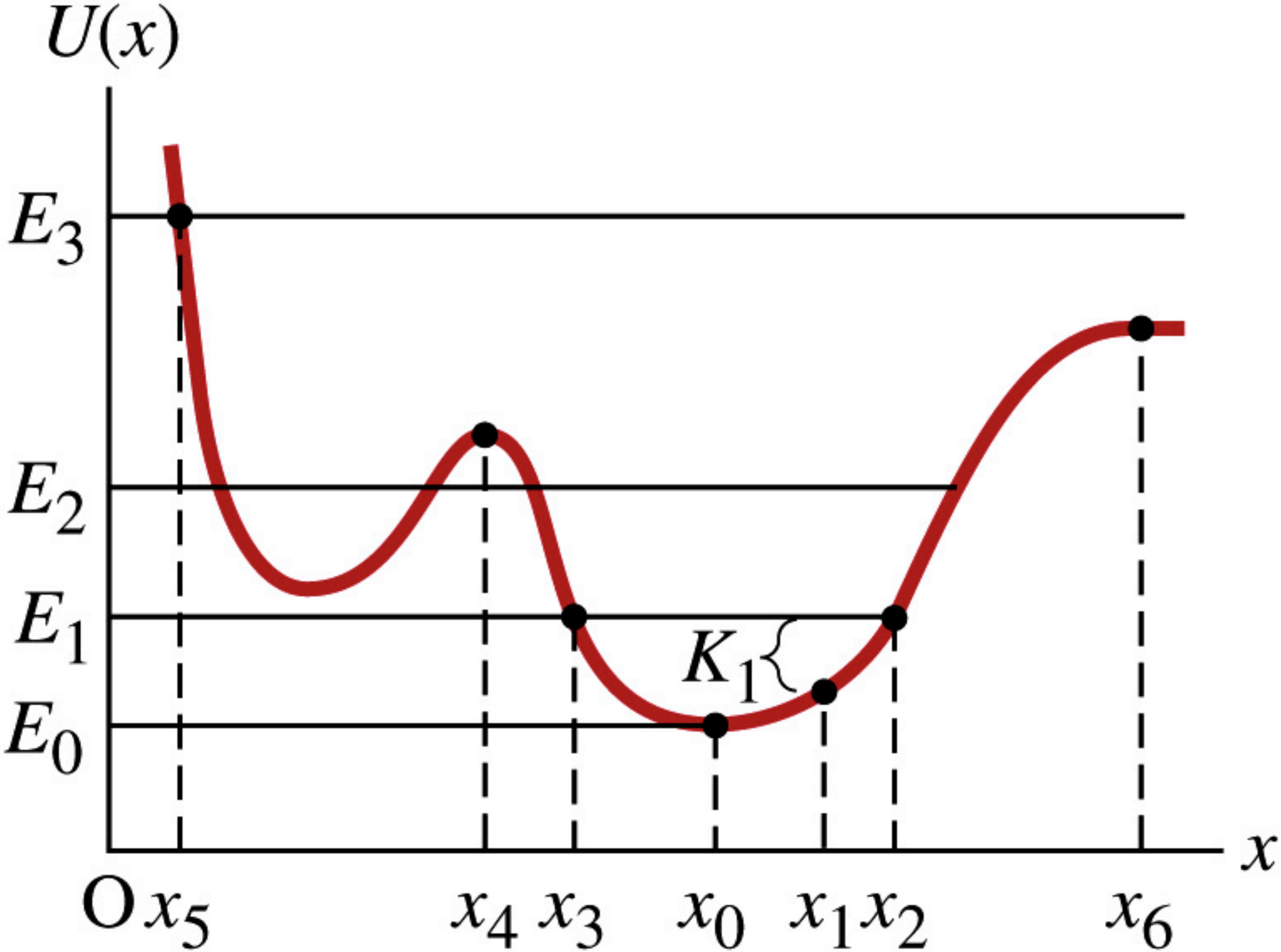
- neutral equilibrium
- unstable equilibrium
- stable equilibrium



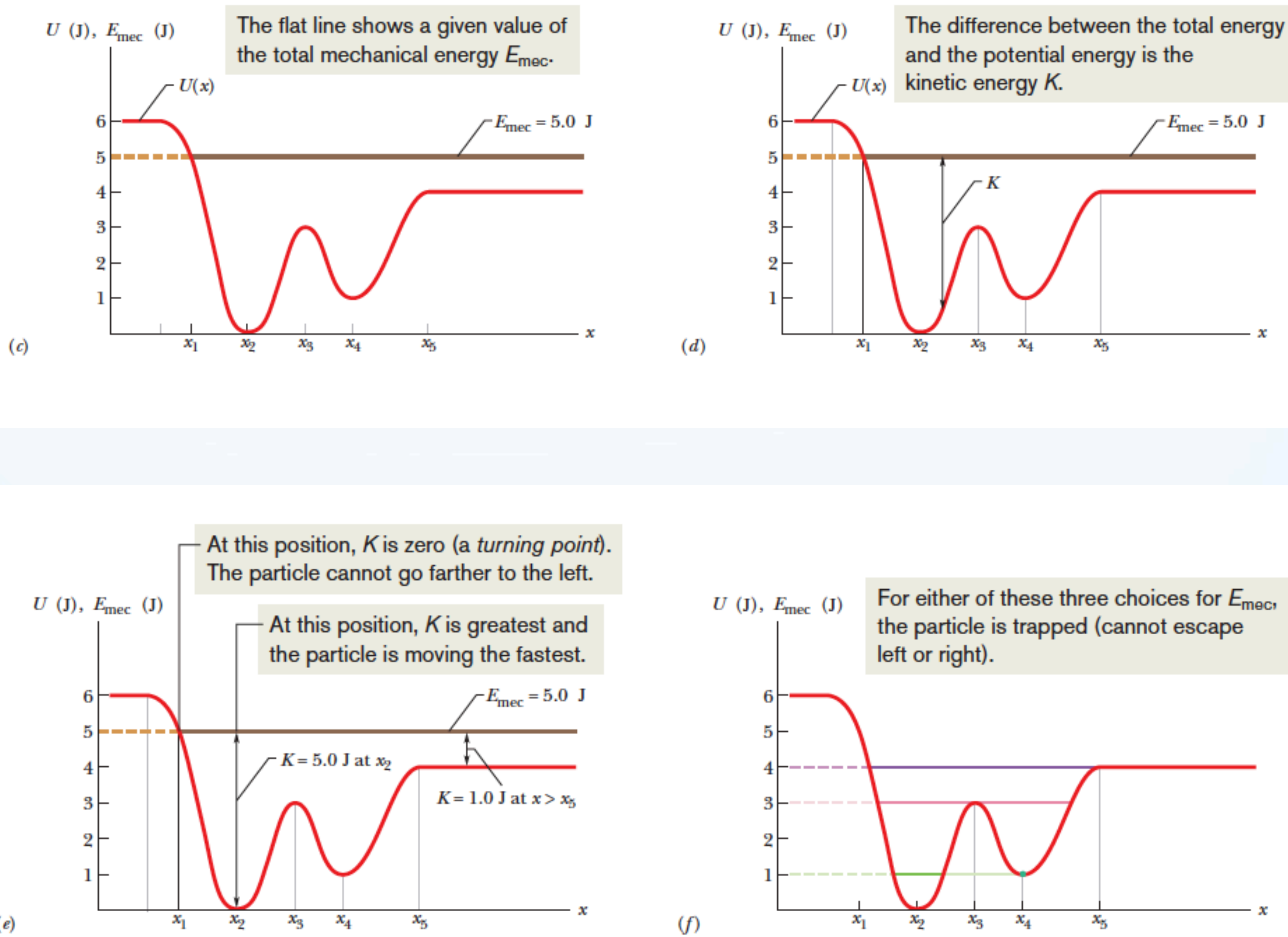
$$\frac{d^2U(x)}{dx^2}|x = x_{ext} = 0$$

$$\frac{d^2U(x)}{dx^2}|x = x_{ext} < 0$$

$$\frac{d^2U(x)}{dx^2}|x = x_{ext} > 0$$



U(x)
E3
E2
E1
E0
K1
x
O
x5
x4
x3
x0
x1
x2
x6



U (J), E_mec (J)
The flat line shows a given value of the total mechanical energy E_mec.
U(x)
E_mec = 5.0 J
6
5
4
3
2
1
x
x_1
x_2
x_3
x_4
x_5
(c)
U (J), E_mec (J)
The difference between the total energy and the potential energy is the kinetic energy K.
U(x)
E_mec = 5.0 J
K
6
5
4
3
2
1
x
x_1
x_2
x_3
x_4
x_5
(d)
At this position, K is zero (a turning point). The particle cannot go farther to the left.
U (J), E_mec (J)
At this position, K is greatest and the particle is moving the fastest.
E_mec = 5.0 J
K = 5.0 J at x_2
K = 1.0 J at x > x_5
6
5
4
3
2
1
x
x_1
x_2
x_3
x_4
x_5
(e)
U (J), E_mec (J)
For either of these three choices for E_mec, the particle is trapped (cannot escape left or right).
6
5
4
3
2
1
x
x_1
x_2
x_3
x_4
x_5
(f)

Example

- Potential vs. Force

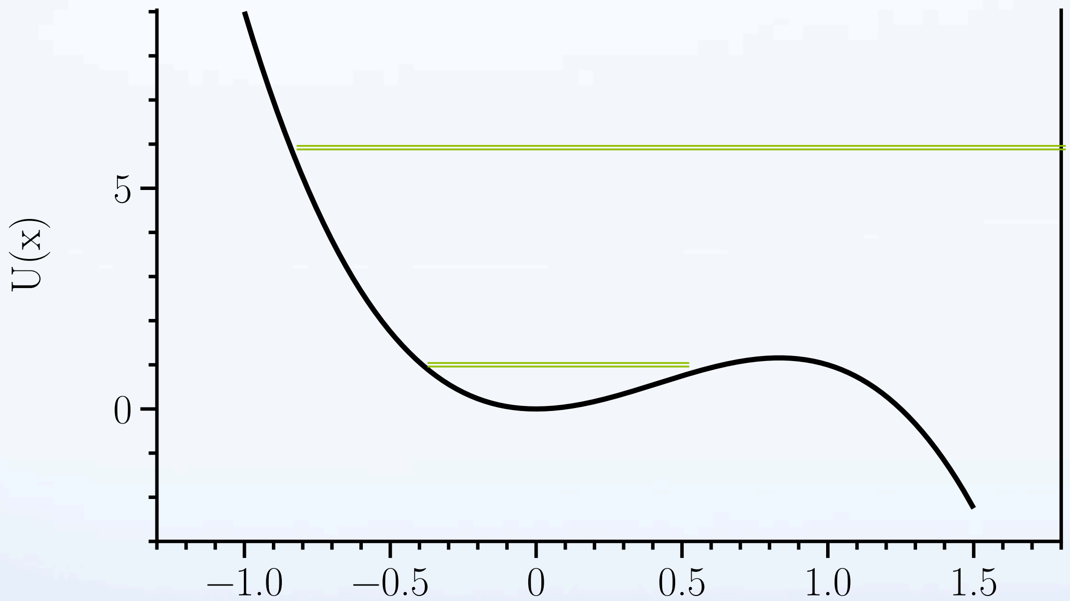
Example

The potential energy of an object is given by

$$U(x) = 5x^2 - 4x^3$$

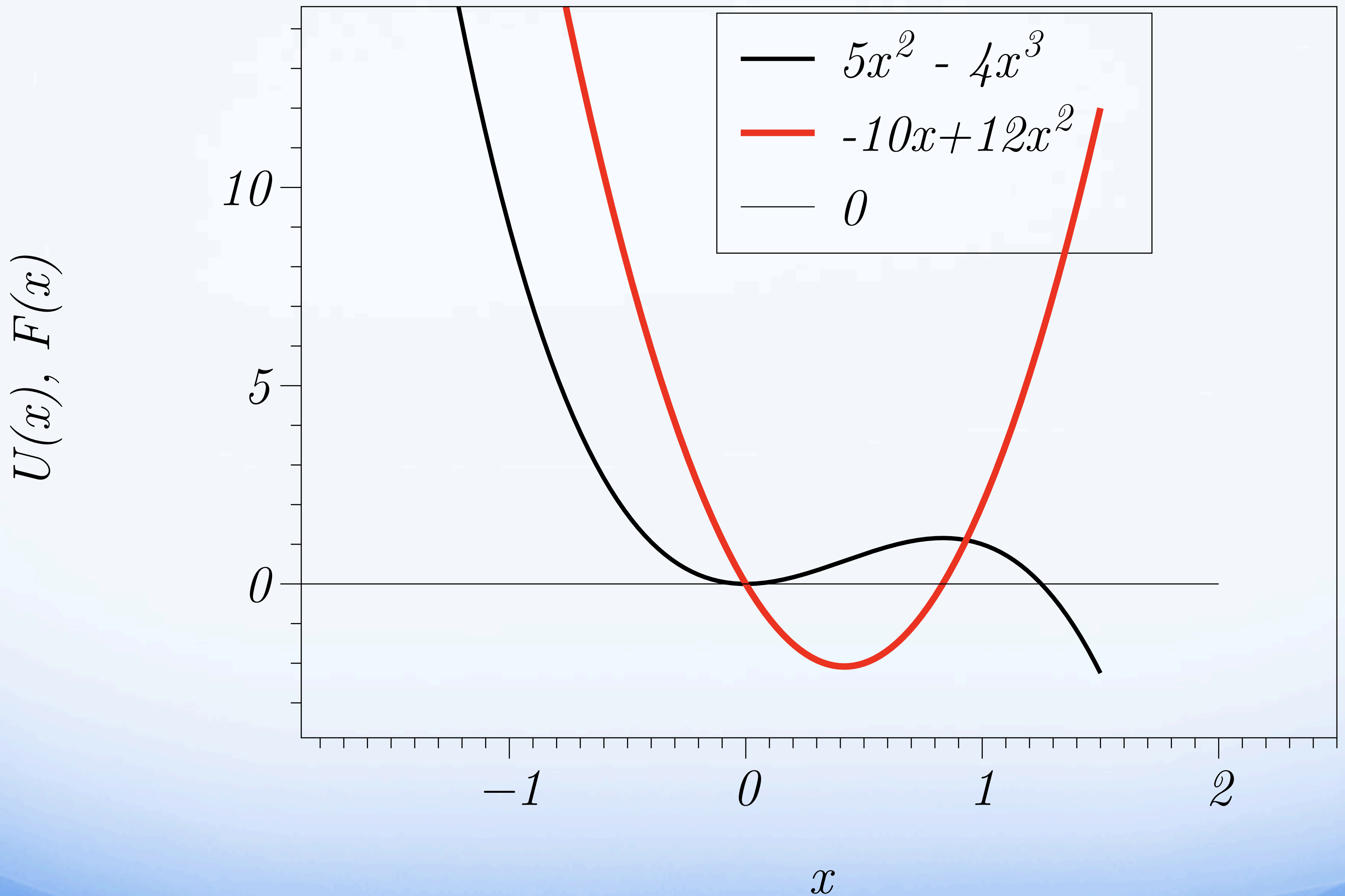
where U is in joules and x is in metres.

(i) What is the force, $F(x)$, acting on the object?
(ii) Determine the positions where the object is in equilibrium and state whether they are stable or unstable.



E_2

E_1



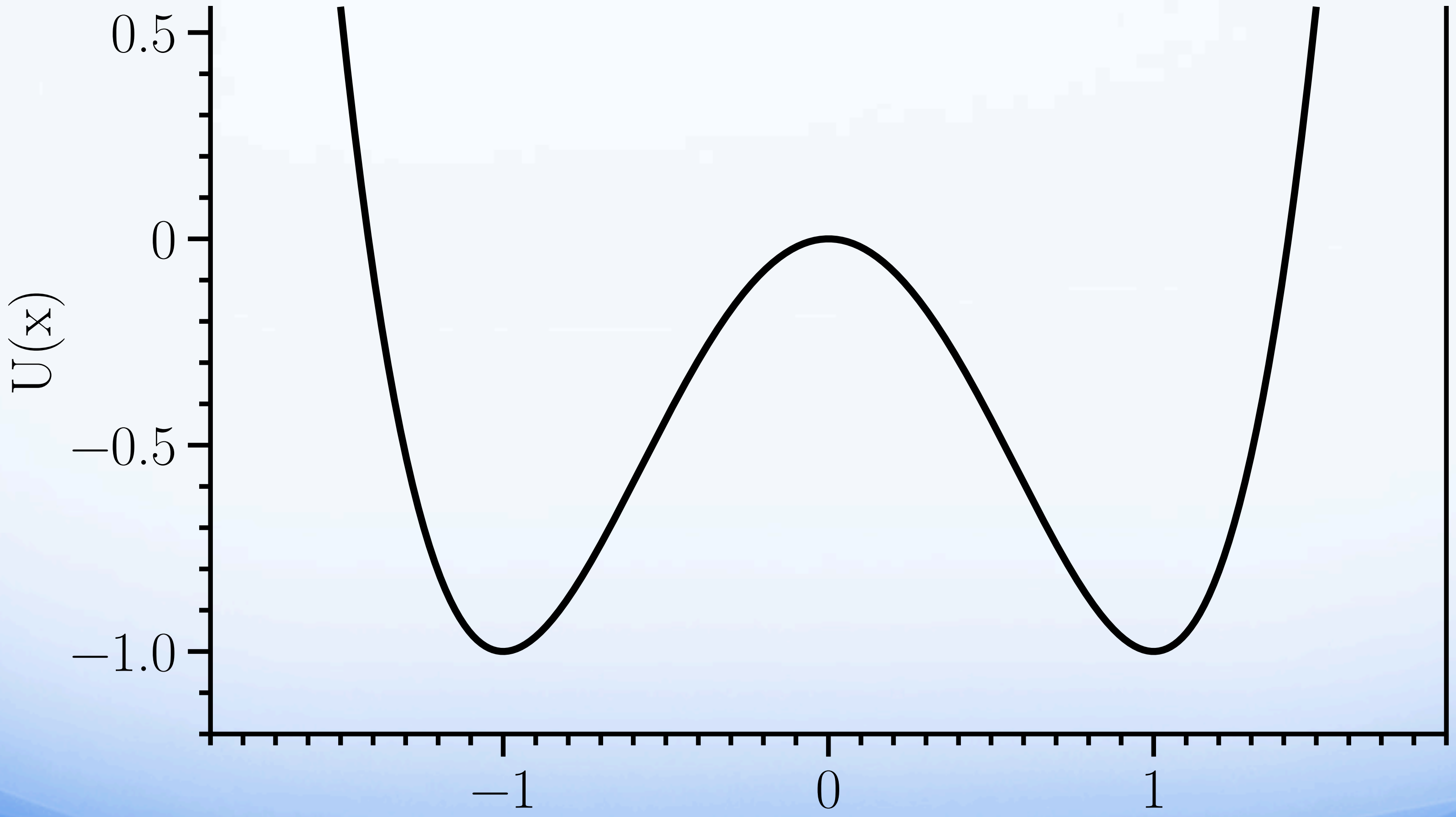
5x^2 - 4x^3
-10x+12x^2
0
10
5
0
−1
0
1
2
x
U(x), F(x)

Example

$$U(x) = -2x^2 + x^4$$

(i) What is the force, $F(x)$, acting on the object?

(ii) Determine the positions where the object is in equilibrium and state whether they are stable or unstable.



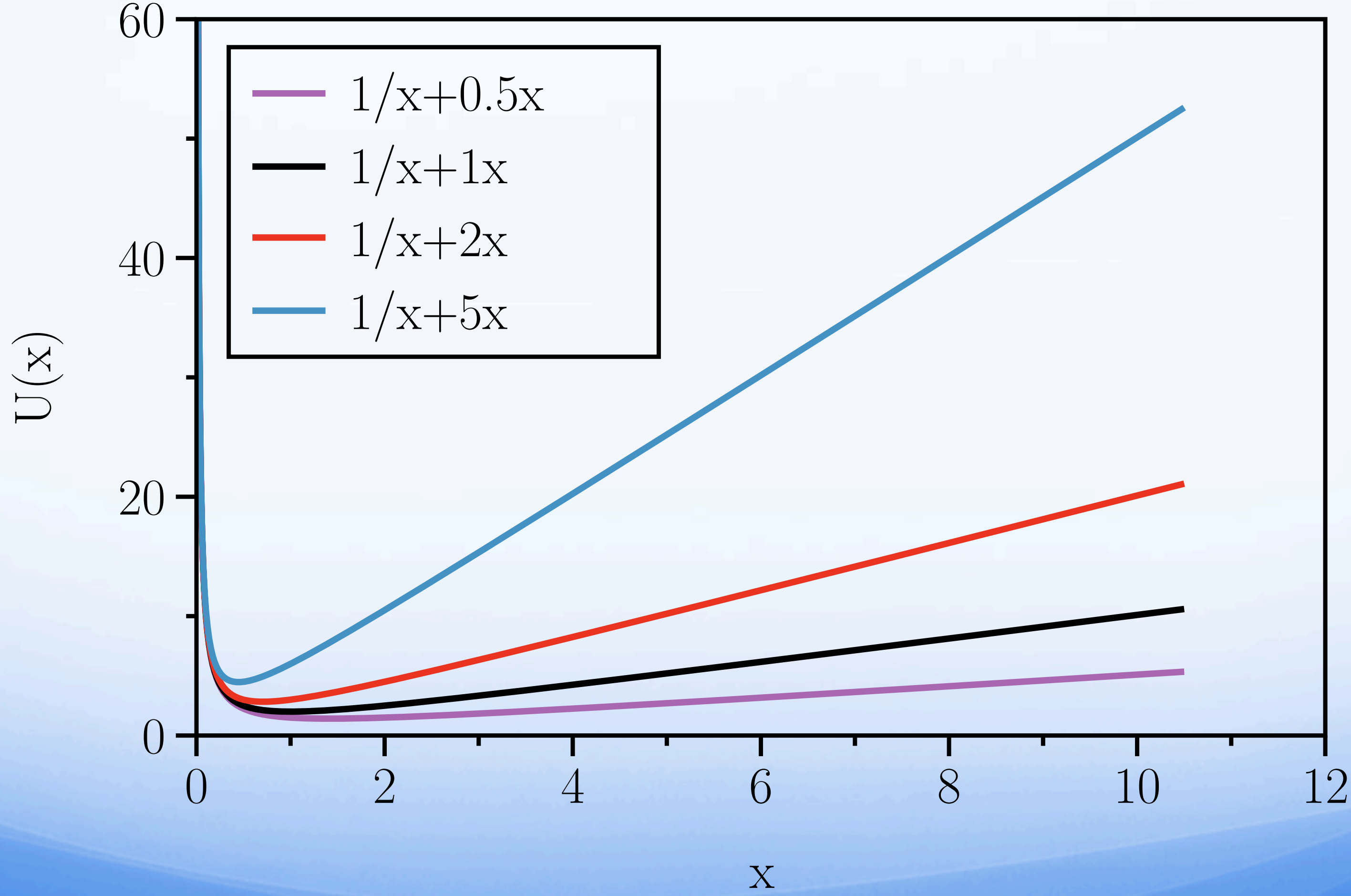
0.5
0
−0.5
−1.0
U(x)
−1
0
1

Example

A particle of mass m moves under the influence of a potential energy

$$U(x) = \frac{a}{x} + bx$$

where a and b are positive constants and the particle is restricted to the region $x > 0$. Find a point of equilibrium for the particle and demonstrate that it is stable.



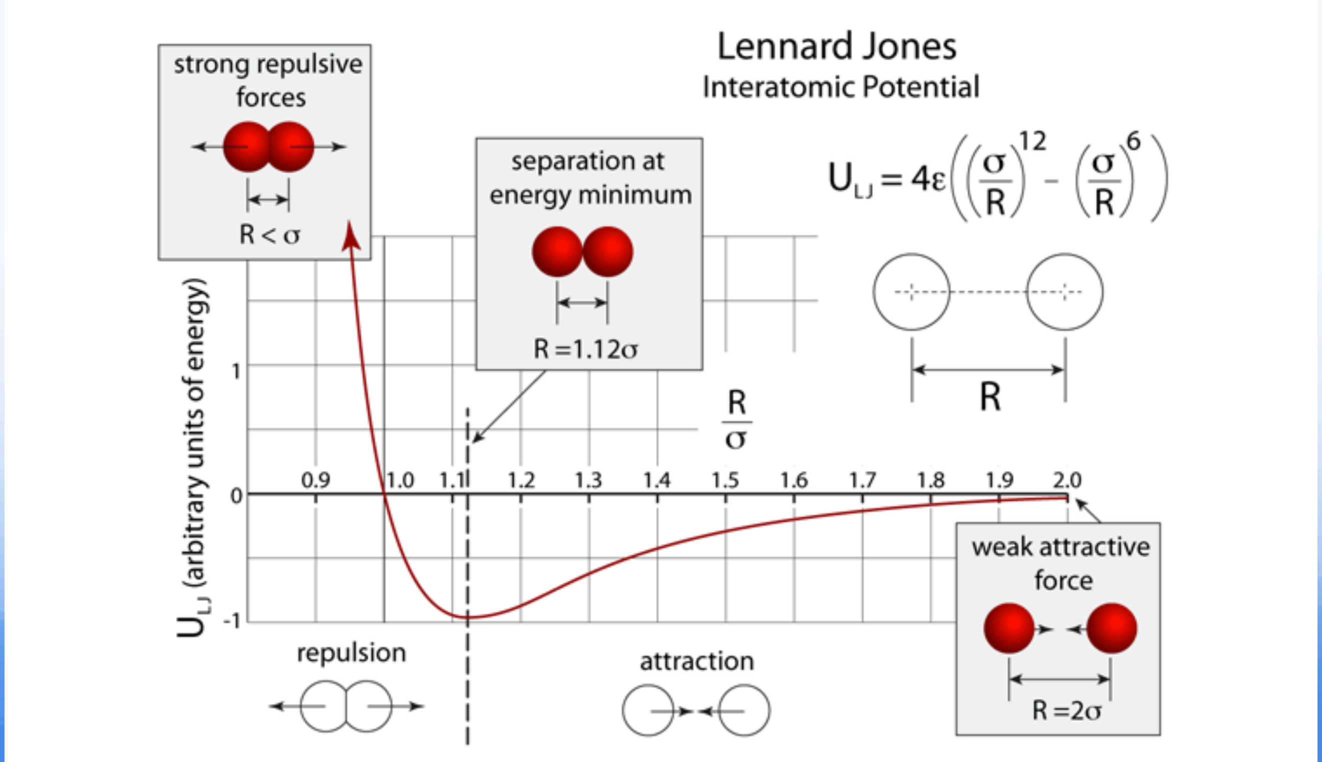
1/x+0.5x
1/x+1x
1/x+2x
1/x+5x
U(x)
x
0
20
40
60
0
2
4
6
8
10
12

Example

The potential energy associated with the force between two neutral atoms in a molecule can be modeled by the Lennard–Jones potential energy function:

$$U(x) = 4\epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

where x is the separation of the atoms. The function $U(x)$ contains two parameters σ and ϵ that are determined from experiments. Sample values for the interaction between two atoms in a molecule are $\sigma = 0.263$ nm and $\epsilon = 1.51 \times 10^{-22}$ J. Using a spreadsheet or similar tool, graph this function and find the most likely distance between the two atoms.



Analyze Stable equilibrium exists for a separation distance at which the potential energy of the system of two atoms (the molecule) is a minimum.

Take the derivative of the function $U(x)$:

$$\frac{dU(x)}{dx} = 4\epsilon \frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right] = 4\epsilon\left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^{6}}{x^{7}}\right]$$

Minimize the function $U(x)$ by setting its derivative equal to zero:

$$4\epsilon\left[\frac{-12\sigma^{12}}{x_{eq}{}^{13}} + \frac{6\sigma^{6}}{x_{eq}{}^{7}}\right] = 0 \quad \rightarrow \quad x_{eq} = (2)^{1/6}\sigma$$

Evaluate x_{eq}, the equilibrium separation of the two atoms in the molecule:

$$x_{eq} = (2)^{1/6}(0.263 \text{ nm}) = \boxed{2.95 \times 10^{-10} \text{ m}}$$

We graph the Lennard–Jones function on both sides of this critical value to create our energy diagram as shown in Figure 7.22.

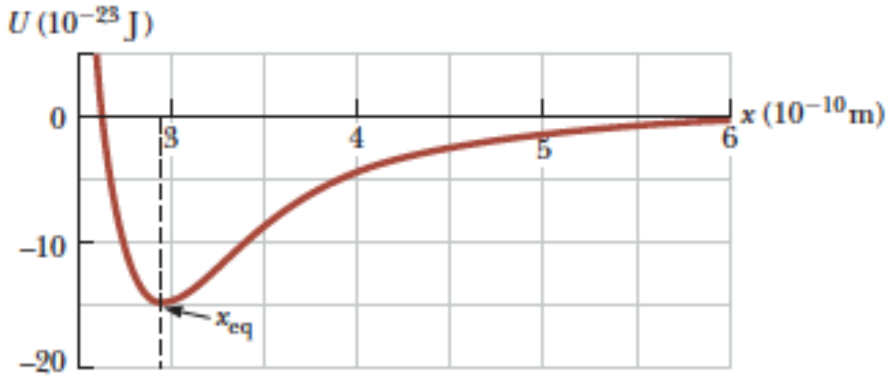


Figure 7.22 (Example 7.9) Potential energy curve associated with a molecule. The distance x is the separation between the two atoms making up the molecule.

Finalize Notice that $U(x)$ is extremely large when the atoms are very close together, is a minimum when the atoms are at their critical separation, and then increases again as the atoms move apart. When $U(x)$ is a minimum, the atoms are in stable equilibrium, indicating that the most likely separation between them occurs at this point.

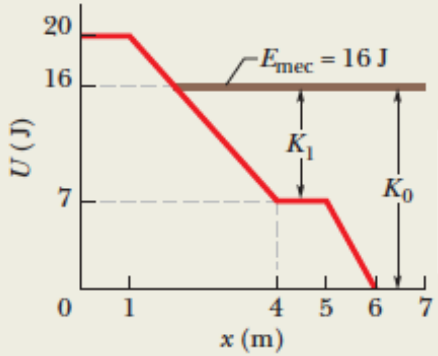
Example

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10*a*. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U. At $x = 6.5$ m, the particle has velocity $\vec{v}_0 = (-4.00 \text{ m/s})\hat{\text{i}}$.

(a) From Fig. 8-10*a*, determine the particle's speed at $x_1 = 4.5$ m.

(b) Where is the particle's turning point located?

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0 \text{ m}$.



(*a*)

Kinetic energy is the difference between the total energy and the potential energy.

Calculations: At $x = 6.5$ m, the particle has kinetic energy

$$K_0 = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(2.00 \text{ kg})(4.00 \text{ m/s})^2$$
$$= 16.0 \text{ J}.$$

Because the potential energy there is $U = 0$, the mechanical energy is

$$E_{mec} = K_0 + U_0 = 16.0 \text{ J} + 0 = 16.0 \text{ J}.$$

This value for E_{mec} is plotted as a horizontal line in Fig. 8-10*a*. From that figure we see that at $x = 4.5$ m, the potential energy is $U_1 = 7.0$ J. The kinetic energy K_1 is the difference between E_{mec} and U_1:

$$K_1 = E_{mec} - U_1 = 16.0 \text{ J} - 7.0 \text{ J} = 9.0 \text{ J}.$$

Because $K_1 = \tfrac{1}{2}mv_1^2$, we find

$$v_1 = -3.0 \text{ m/s}. \qquad \text{(Answer)}$$

The turning point is where the force momentarily stops and then reverses the particle's motion. That is, it is where the particle momentarily has $v = 0$ and thus $K = 0$.

Calculations: Because K is the difference between E_{mec} and U, we want the point in Fig. 8-10*a* where the plot of U rises to meet the horizontal line of E_{mec}, as shown in Fig. 8-10*b*. Because the plot of U is a straight line in Fig. 8-10*b*, we can draw nested right triangles as shown and then write the proportionality of distances

$$\frac{16 - 7.0}{d} = \frac{20 - 7.0}{4.0 - 1.0},$$

which gives us $d = 2.08$ m. Thus, the turning point is at

$$x = 4.0 \text{ m} - d = 1.9 \text{ m}. \qquad \text{(Answer)}$$

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0 \text{ m}$.

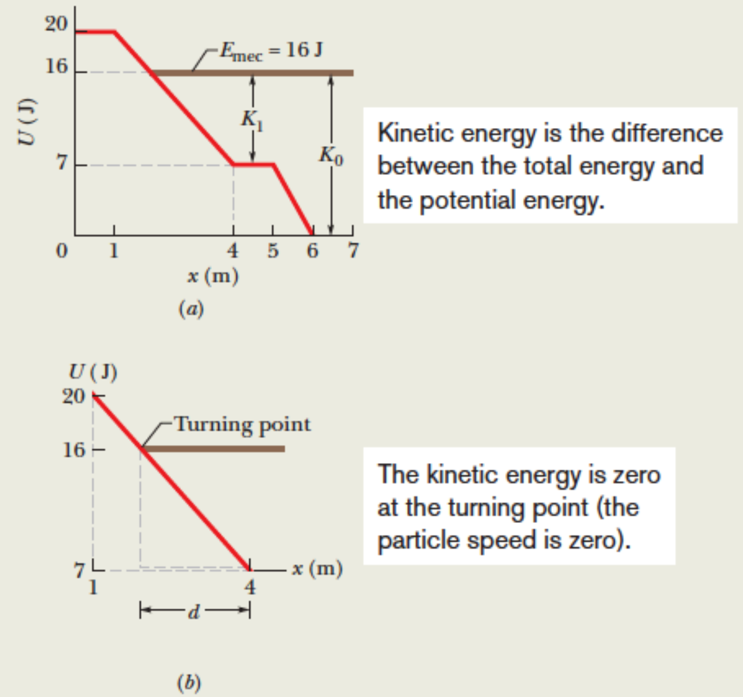


Figure 8-10 (*a*) A plot of potential energy U versus position x. (*b*) A section of the plot used to find where the particle turns around.

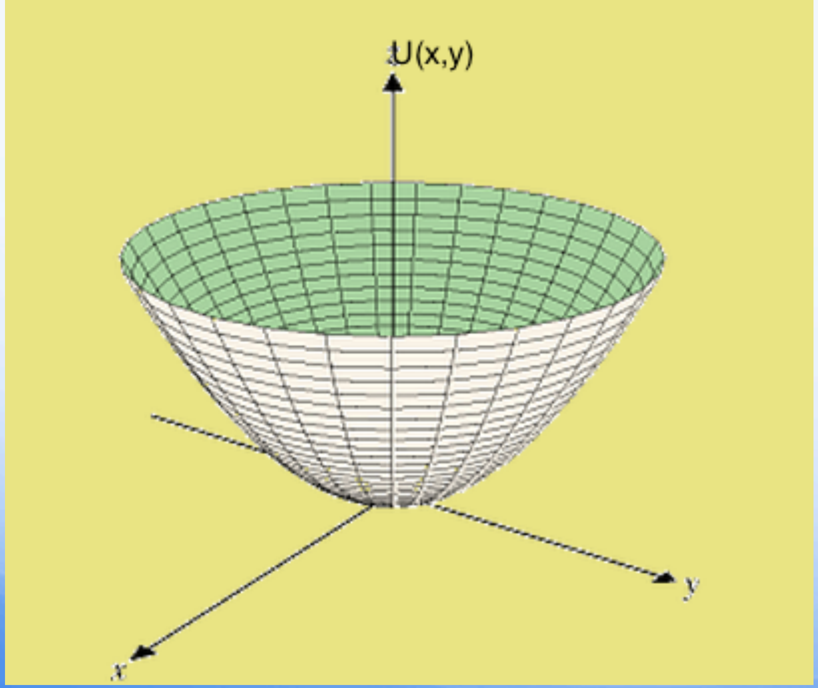
The force is given by Eq. 8-22 ($F(x) = -dU(x)/dx$): The force is equal to the negative of the slope on a graph of $U(x)$.

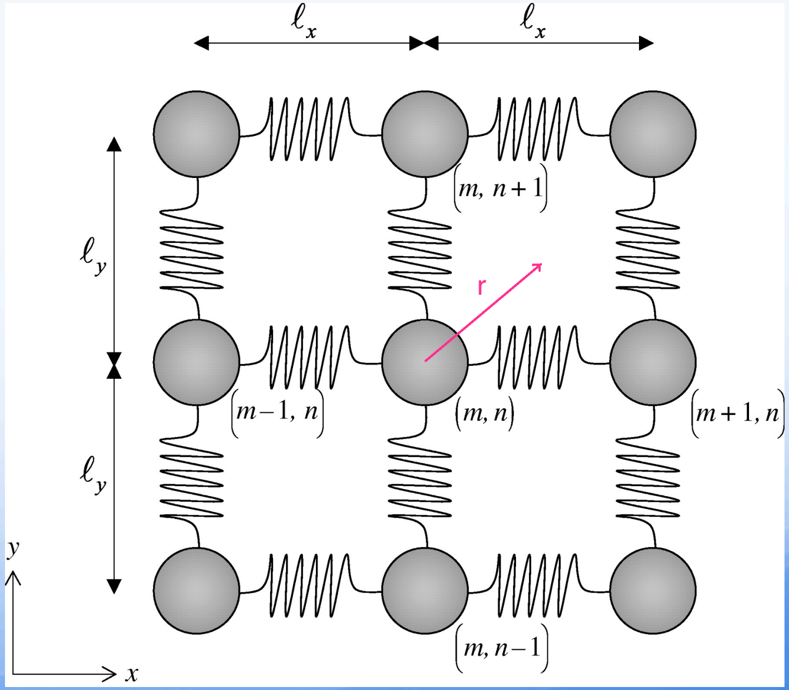
Calculations: For the graph of Fig. 8-10*b*, we see that for the range $1.0 \text{ m} < x < 4.0 \text{ m}$ the force is

$$F = -\frac{20 \text{ J} - 7.0 \text{ J}}{1.0 \text{ m} - 4.0 \text{ m}} = 4.3 \text{ N}. \qquad \text{(Answer)}$$

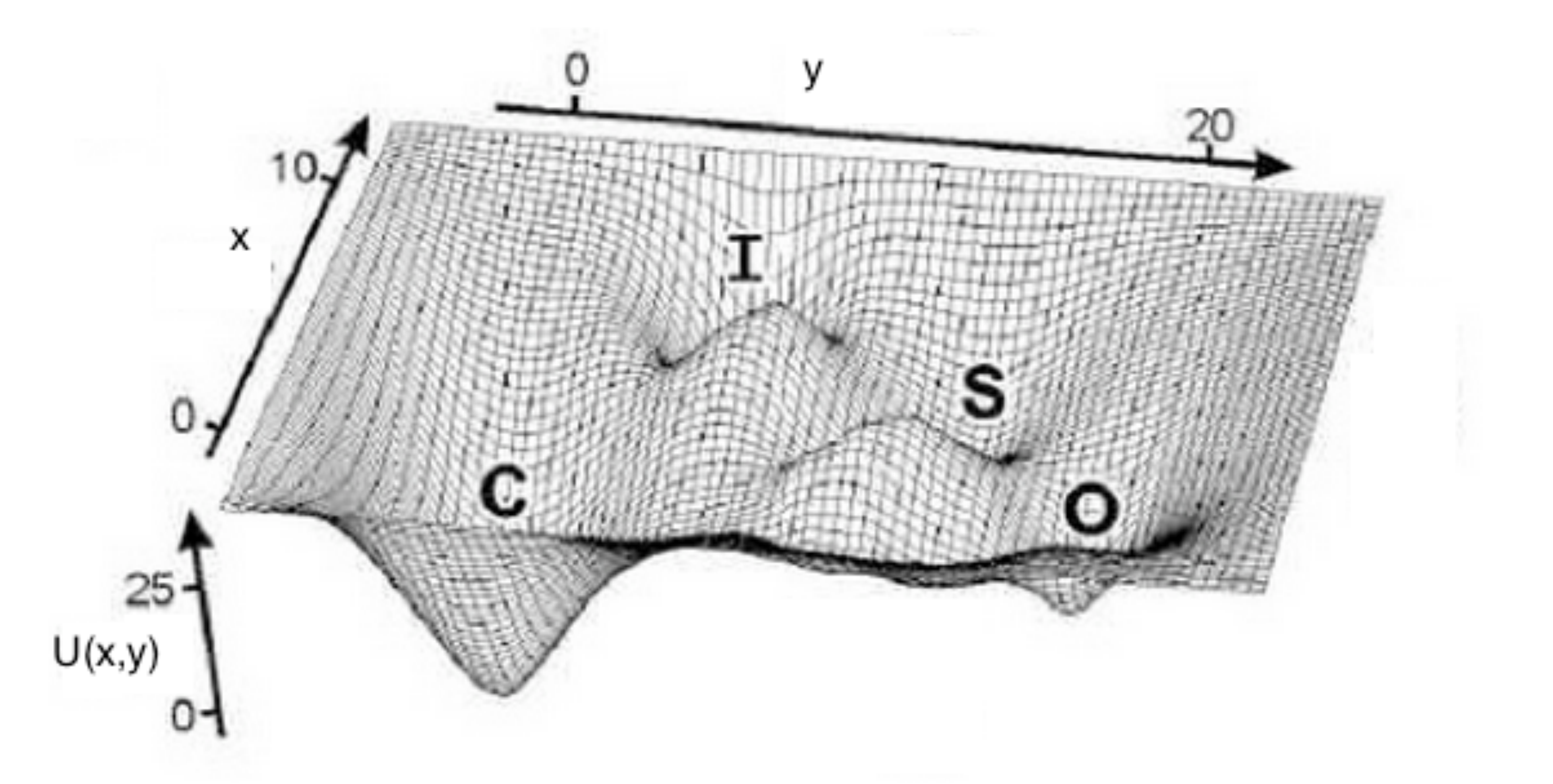
Two Dimensional Potentials

$$U(r) = \frac{1}{2}k\ r^2 = \frac{1}{2}k\ (x^2 + y^2)$$

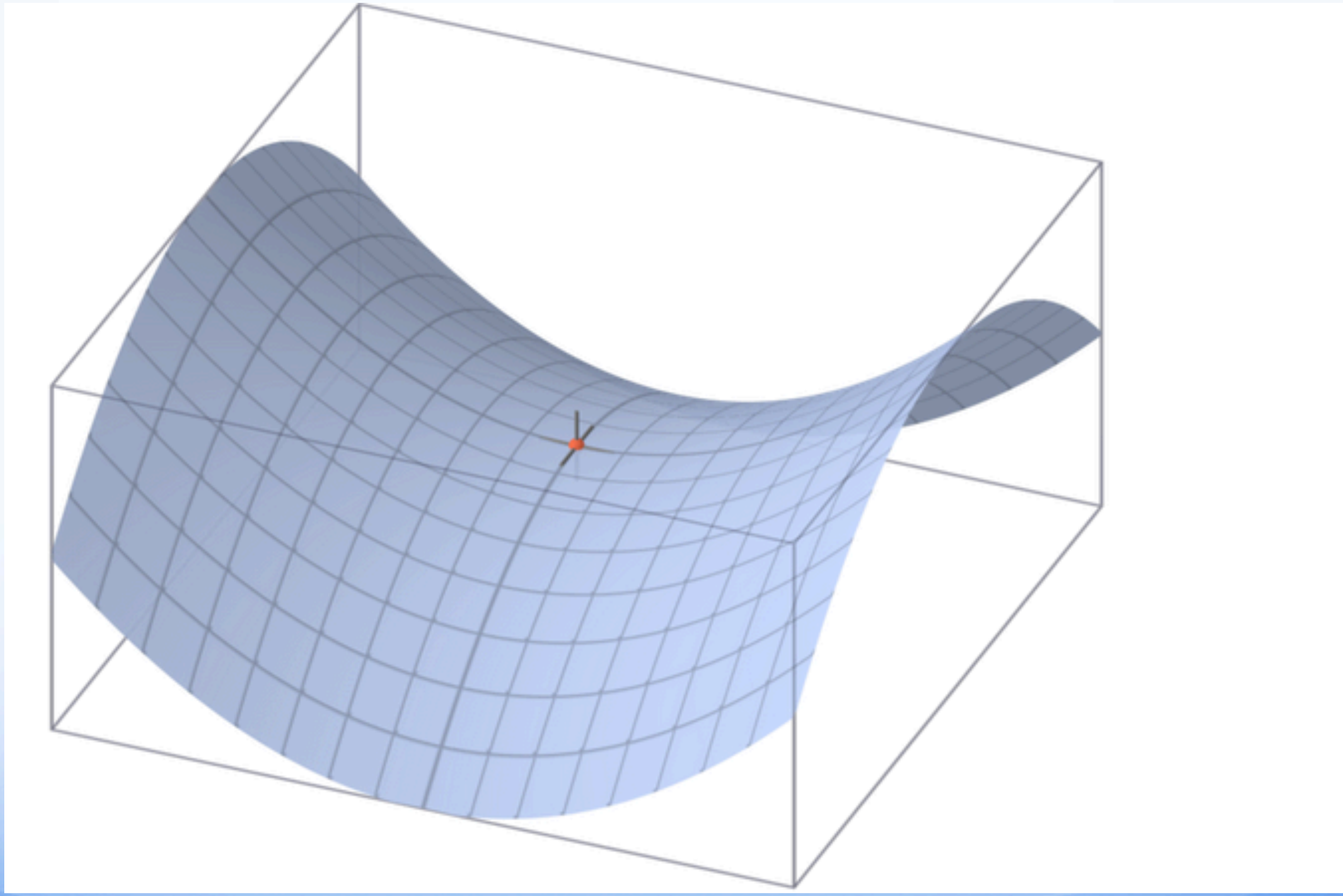




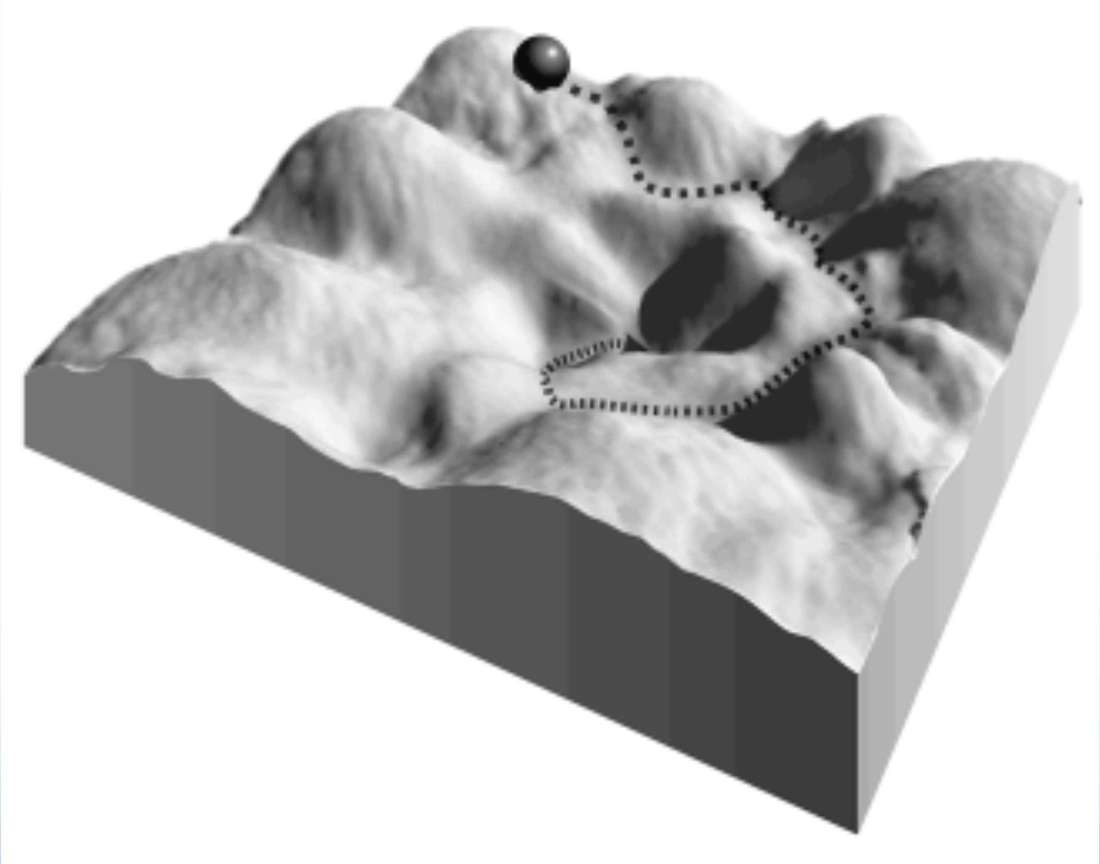
Example

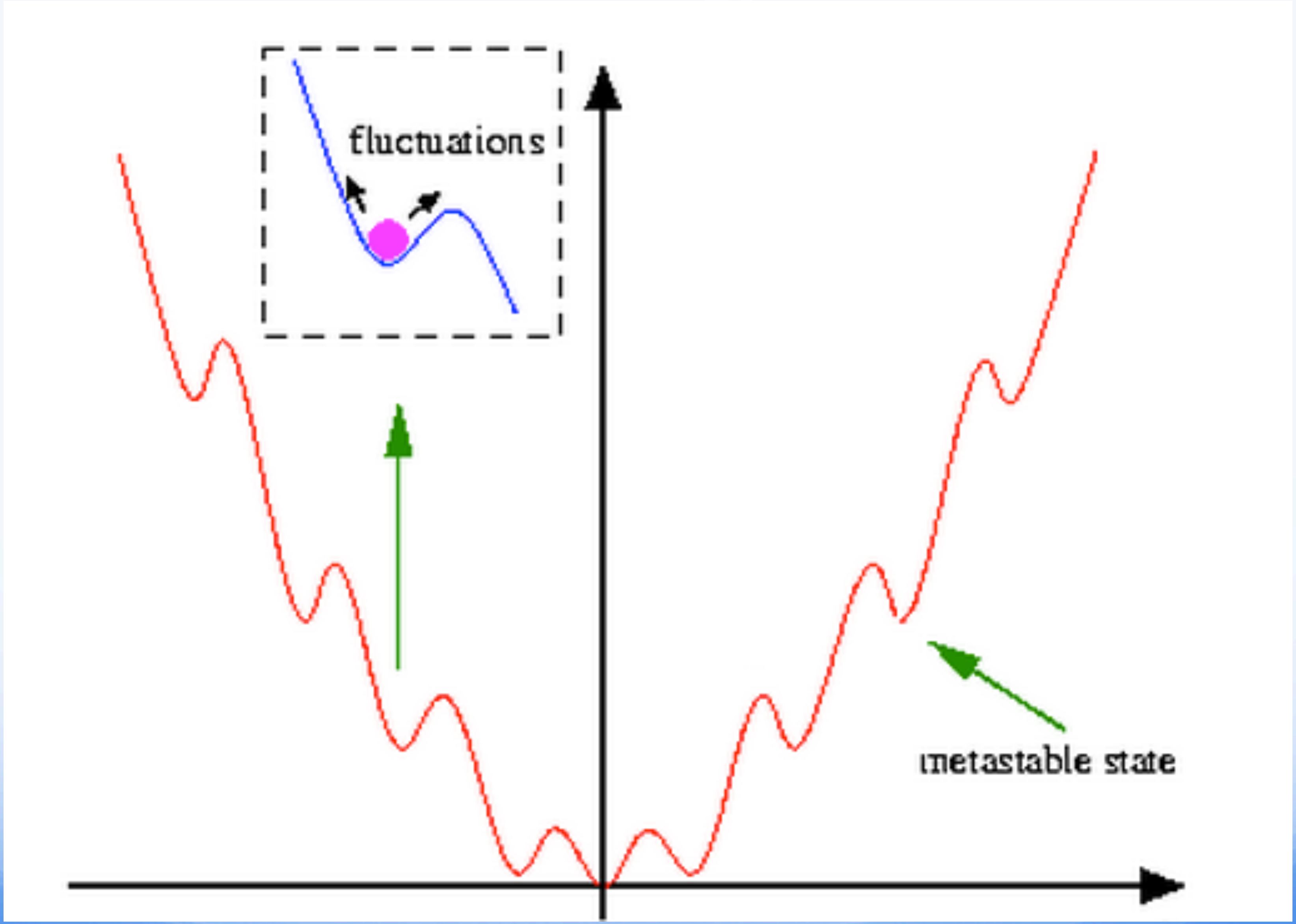


Saddle Point



Energy Landscape




fluctuations
metastable state

Example

Example: A particle of mass m, moving in the x-direction, is acting on by a potential

$$U(x) = -U_1\left(\left(\frac{x}{x_1}\right)^3 - \left(\frac{x}{x_1}\right)^2\right), \qquad (10)$$

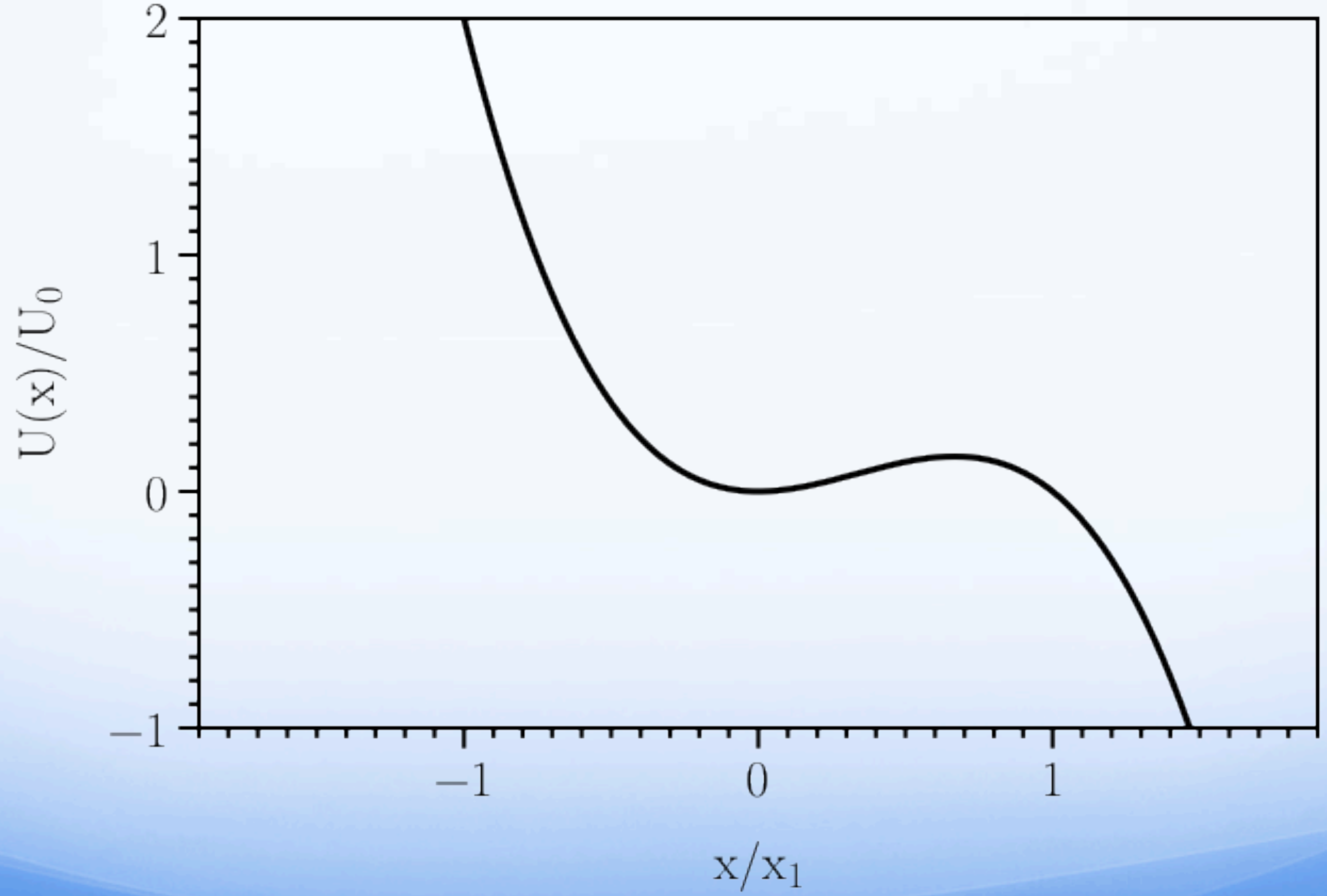
where U_1 and x_1 are positive constants and $U(0) = 0$.

a) Sketch $U(x)/U_1$ as a function of x/x_1.

b) Find the points where the force on the particle is zero. Classify them as stable or unstable. Calculate the value of $U(x)/U_1$ at these equilibrium points.

c) For energies E that lies in $0 < E < (4/27)U_1$ find an equation whose solution yields the turning points along the x-axis about which the particle will undergo periodic motion.

d) Suppose $E = (4/27)U_1$ and that the particle starts at $x = 0$ with speed v_0. Find v_0.



2
1
0
−1
U(x)/U0
−1
0
1
x/x1

b) The force on the particle is zero at the minimum of the potential which occurs at

$$F_x(x) = -\frac{dU}{dx}(x) = U_1\left(\left(\frac{3}{x_1{}^3}\right)x^2 - \left(\frac{2}{x_1{}^2}\right)x\right) = 0 \tag{11}$$

which becomes

$$x^2 = (2x_1 / 3)x\,. \tag{12}$$

We can solve Eq. (12) for the extrema. This has two solutions

$$x = (2x_1 / 3) \quad \text{and} \quad x = 0\,. \tag{13}$$

The second derivative is given by

$$\frac{d^2U}{dx^2}(x) = -U_1\left(\left(\frac{6}{x_1{}^3}\right)x - \left(\frac{2}{x_1{}^2}\right)\right). \tag{14}$$

Evaluating the second derivative at $x = (2x_1 / 3)$ yields a negative quantity

$$\frac{d^2U}{dx^2}(x = (2x_1 / 3)) = -U_1\left(\left(\frac{6}{x_1{}^3}\right)\frac{2x_1}{3} - \left(\frac{2}{x_1{}^2}\right)\right) = -\frac{2U_1}{x_1{}^2} < 0 \tag{15}$$

indicating the solution $x = (2x_1 / 3)$ represents a local maximum and hence is an unstable point. At $x = (2x_1 / 3)$, the potential energy is given by the value $U((2x_1 / 3)) = (4 / 27)U_1$.

Evaluating the second derivative at $x = 0$ yields a positive quantity

$$\frac{d^2U}{dx^2}(x = 0) = -U_1\left(\left(\frac{6}{x_1^3}\right)0 - \left(\frac{2}{x_1^2}\right)\right) = \frac{2U_1}{x_1^2} > 0 \qquad (16)$$

indicating the solution $x = 0$ represents a local minimum and is a stable point. At the local minimum, $x = 0$, the potential energy $U(0) = 0$.

c) Because the kinetic energy $K(x) = E - U(x) > 0$ must be always be positive, for energies in the range of

$$U(0) = 0 < E < U(2x_1/3) = \frac{4U_1}{27} . \qquad (17)$$

the particle will undergo periodic motion, between the values $x_a < x < x_b < 2x_1/3$, where x_a and x_b are the turning points and are solutions to the equation

$$E = U(x) = -U_1\left(\left(\frac{x}{x_1}\right)^3 - \left(\frac{x}{x_1}\right)^2\right) . \qquad (18)$$

For $E > U(2x_1/3) = \frac{4U_1}{27}$, Eq. (18) has only one solution x_a and for all values of $x > x_a$ the kinetic energy $K(x) = E - U(x) > 0$ which means that the particle can "escape" to infinity but can never enter the region $x < x_a$.

For $E < U(0) = 0$, the kinetic energy is negative for all values of x i.e. $K(x) = E - U(x) < 0; \; -\infty < x < +\infty$. All regions of space are forbidden.

d) If the particle has speed v_0 at $x = 0$ where the potential energy is zero $U(0) = 0$, the energy of the particle is constant and equal to kinetic energy

$$E = K(0) = \frac{1}{2} m v_0^2 . \tag{19}$$

Therefore

$$(4/27)U_1 = \frac{1}{2} m v_0^2 \tag{20}$$

which we can solve for the speed v_0,

$$v_0 = \sqrt{8U_1 / 27m} . \tag{21}$$

Work Done on a System by an External Force

Work is energy transferred to or from a system by means of an external force acting on that system.

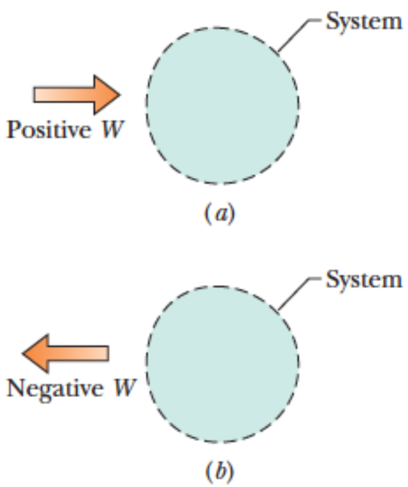


Figure 8-11 (*a*) Positive work *W* done on an arbitrary system means a transfer of energy to the system. (*b*) Negative work *W* means a transfer of energy from the system.

No Friction Involved

$$W = \Delta K + \Delta U,$$

$$W = \Delta E_{mec} \quad \text{(work done on system, no friction involved),}$$

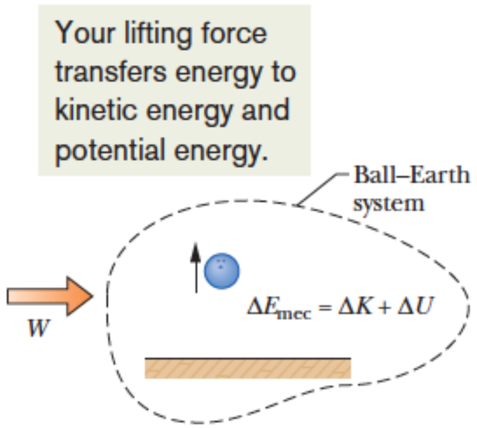
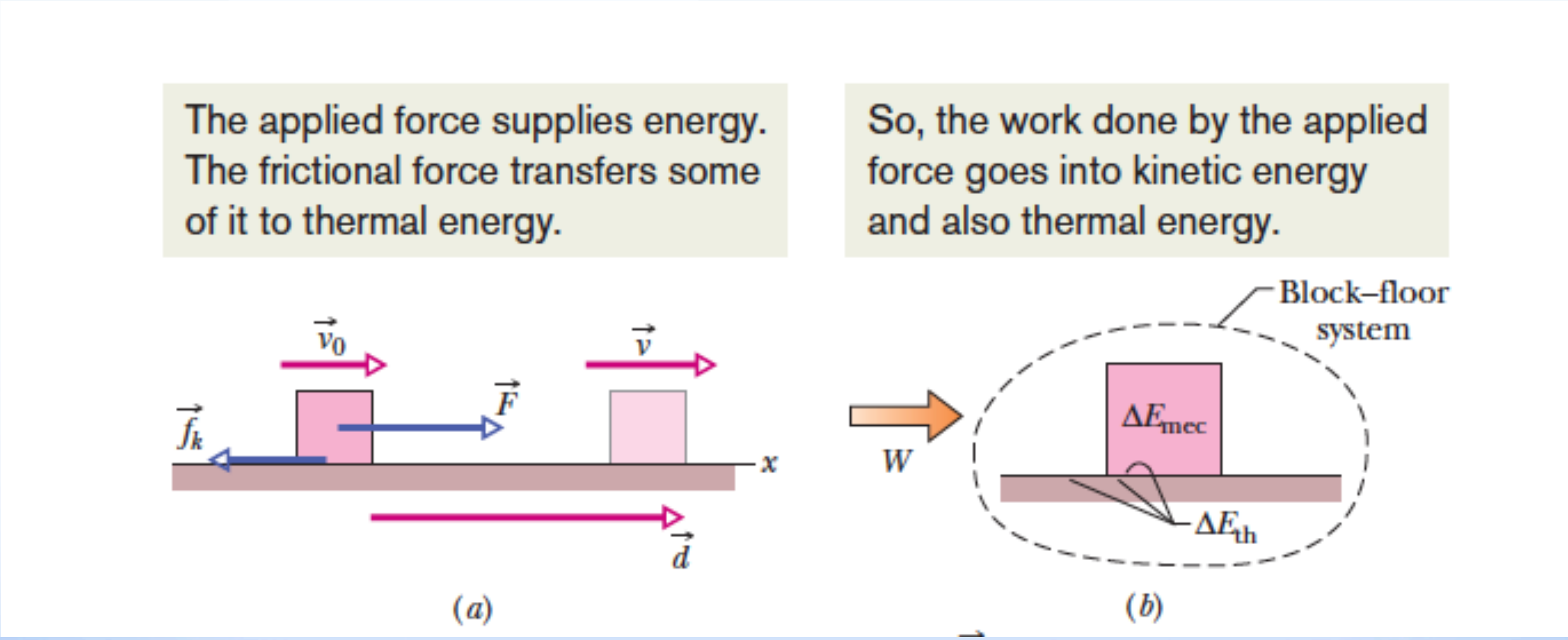


Figure 8-12 Positive work W is done on a system of a bowling ball and Earth, causing a change ΔE_{mec} in the mechanical energy of the system, a change ΔK in the ball's kinetic energy, and a change ΔU in the system's gravitational potential energy.

Friction Involved



$$F - f_k = ma.$$

Because the forces are constant, the acceleration $\vec{a}$ is also constant.

$$v^2 = v_0^2 + 2ad.$$

$$Fd = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 + f_k d$$

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = \Delta K$$

$$Fd = \Delta K + f_k d.$$

$$Fd = \Delta E_{\text{mec}} + f_k d.$$

$$\Delta E_{\text{th}} = f_k d \quad \text{(increase in thermal energy by sliding).}$$

$$Fd = \Delta E_{\text{mec}} + \Delta E_{\text{th}}.$$

Conservation of Energy

$$W = \Delta E_{mec} + \Delta E_{th} \quad \text{(work done on system, friction involved).}$$

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int},$$

Isolated System

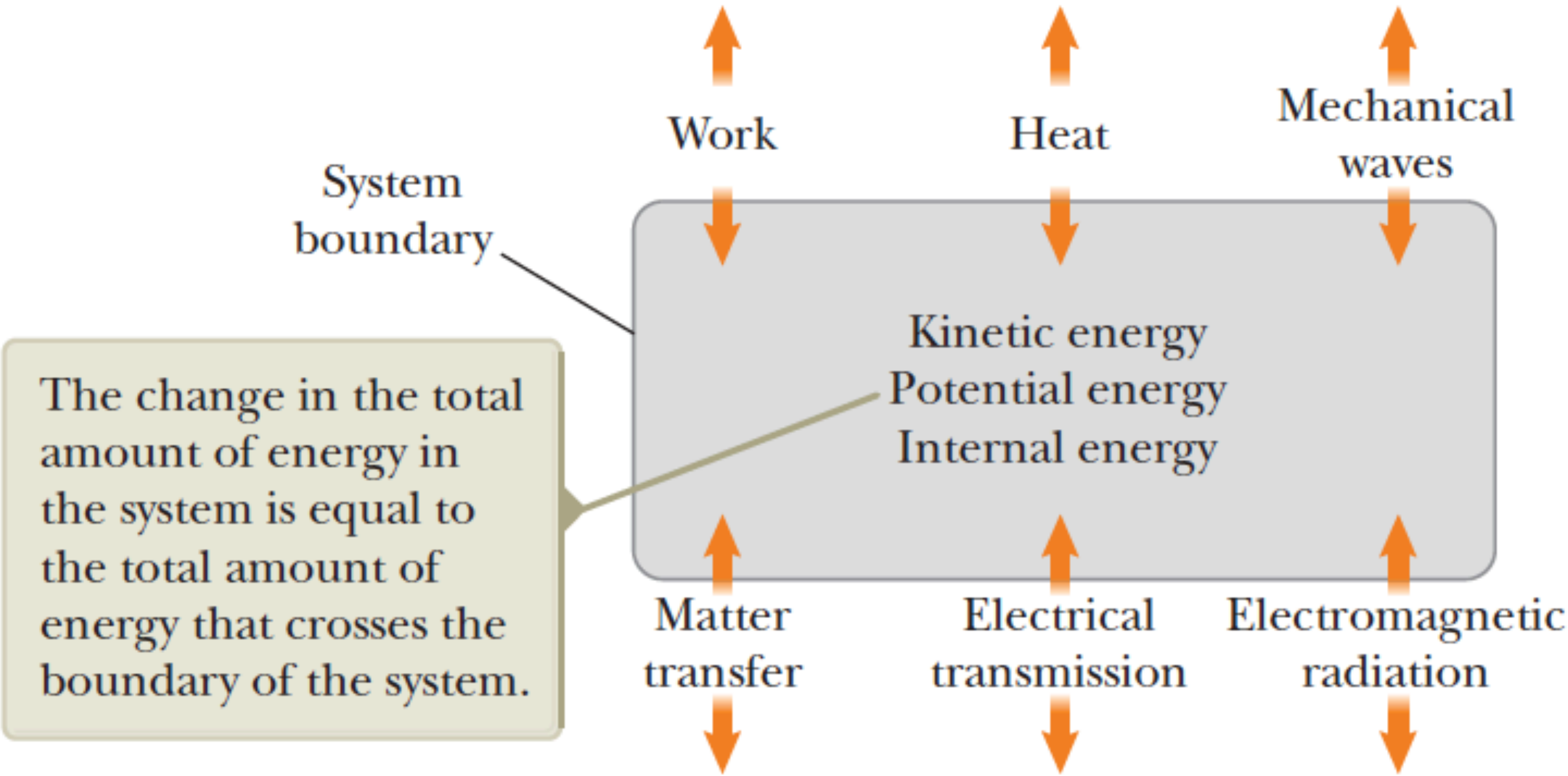
The total energy E of an isolated system cannot change.

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0 \quad \text{(isolated system).}$$

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

$$\Delta E_{mec} = E_{mec,2} - E_{mec,1},$$

$$E_{mec,2} = E_{mec,1} - \Delta E_{th} - \Delta E_{int}.$$

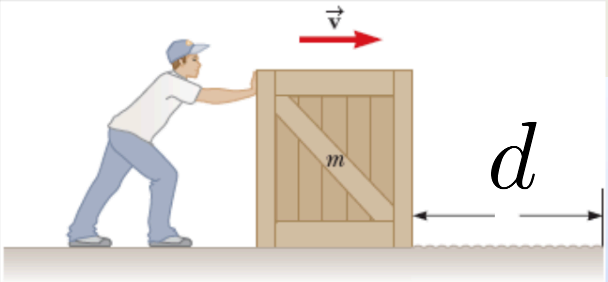


Work
Heat
Mechanical waves
System boundary
Kinetic energy
Potential energy
Internal energy
The change in the total amount of energy in the system is equal to the total amount of energy that crosses the boundary of the system.
Matter transfer
Electrical transmission
Electromagnetic radiation

Example

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14$ kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50$ m, the speed of the crate decreases from $v_0 = 0.60$ m/s to $v = 0.20$ m/s.

(a) How much work is done by force $\vec{F}$, and on what system does it do the work?



(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

Calculation: Substituting given data, including the fact that force $\vec{F}$ and displacement $\vec{d}$ are in the same direction, we find

$$W = Fd\cos\phi = (40\ \text{N})(0.50\ \text{m})\cos 0°$$
$$= 20\ \text{J}. \quad \text{(Answer)}$$

Therefore, the system on which the work is done is the crate–floor system, because both energy changes occur in that system.

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8\text{-}34)$$

Calculations: We know the value of W from (a). The change ΔE_{mec} in the crate's mechanical energy is just the change in its kinetic energy because no potential energy changes occur, so we have

$$\Delta E_{\text{mec}} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2.$$

Substituting this into Eq. 8-34 and solving for ΔE_{th}, we find

$$\Delta E_{\text{th}} = W - (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2) = W - \tfrac{1}{2}m(v^2 - v_0^2)$$
$$= 20\ \text{J} - \tfrac{1}{2}(14\ \text{kg})[(0.20\ \text{m/s})^2 - (0.60\ \text{m/s})^2]$$
$$= 22.2\ \text{J} \approx 22\ \text{J}. \quad \text{(Answer)}$$

Example

Figure 8-17 shows a water-slide ride in which a glider is shot by a spring along a water-drenched (frictionless) track that takes the glider from a horizontal section down to ground level. As the glider then moves along ground-level track, it is gradually brought to rest by friction. The total mass of the glider and its rider is $m = 200$ kg, the initial compression of the spring is $d = 5.00$ m, the spring constant is $k = 3.20 \times 10^3$ N/m, the initial height is $h = 35.0$ m, and the coefficient of kinetic friction along the ground-level track is $\mu_k = 0.800$. Through what distance L does the glider slide along the ground-level track until it stops?

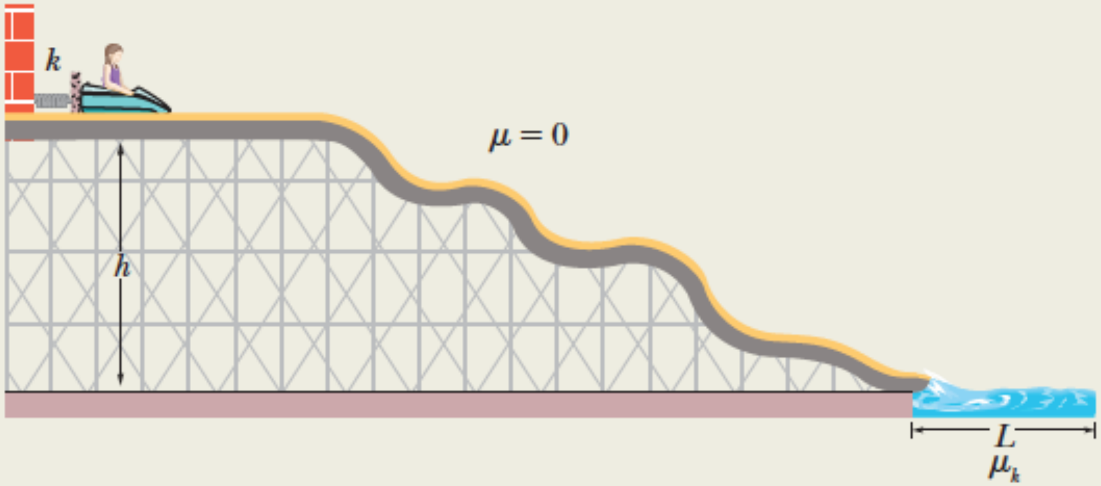


Figure 8-17 A spring-loaded amusement park water slide.

$$E_{\text{mec},1} = K_1 + U_{e1} + U_{g1}$$
$$= 0 + \tfrac{1}{2}kd^2 + mgh.$$

In the final state, with the spring now in its relaxed state and the glider again stationary but no longer elevated, the final mechanical energy of the system is

$$E_{\text{mec},2} = K_2 + U_{e2} + U_{g2}$$
$$= 0 + 0 + 0. \qquad (8\text{-}44)$$

$$\Delta E_{\text{th}} = \mu_k mgL.$$

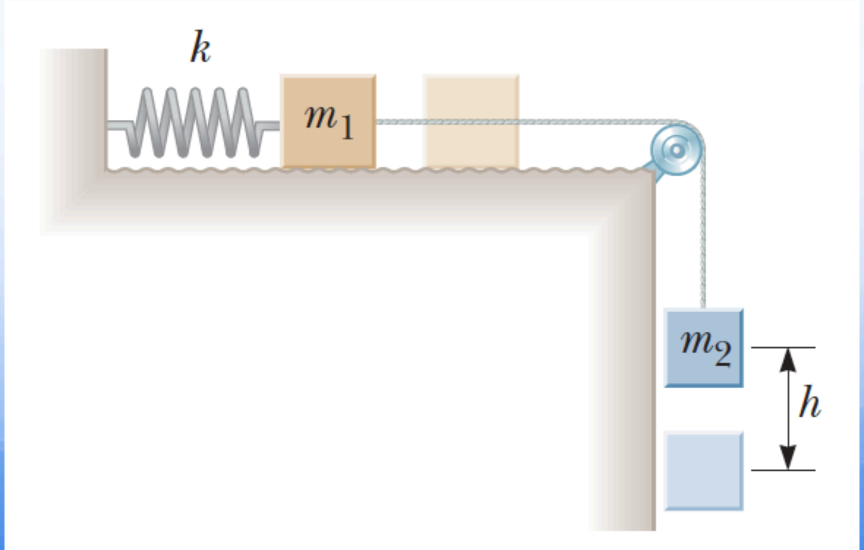
$$0 = \tfrac{1}{2}kd^2 + mgh - \mu_k mgL, \qquad (8\text{-}46)$$

and

$$L = \frac{kd^2}{2\mu_k mg} + \frac{h}{\mu_k}$$
$$= \frac{(3.20 \times 10^3\ \text{N/m})(5.00\ \text{m})^2}{2(0.800)(200\ \text{kg})(9.8\ \text{m/s}^2)} + \frac{35\ \text{m}}{0.800}$$
$$= 69.3\ \text{m}. \qquad \text{(Answer)}$$

Example

Two blocks are connected by a light string that passes over a frictionless pulley as shown in Figure 8.12. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



$$\Delta K + \Delta U + \Delta E_{th} = 0$$

$$\Delta U_g + \Delta U_s + \Delta E_{th} = 0$$

$$\Delta U_s = U_{sf} - U_{si} = \frac{1}{2}kh^2 - 0$$

$$\Delta U_g = U_{gf} - U_{gi} = 0 - m_2 gh$$

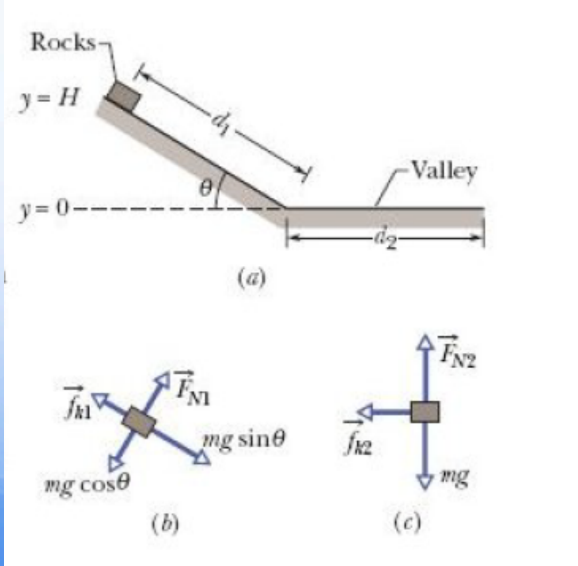
$$\Delta E_{th} = f_k h = (\mu_k n)h = \mu_k m_1 gh$$

$$-m_2 gh + \frac{1}{2}kh^2 + \mu_k m_1 gh = 0$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

Example

Figure 8-19*a* shows the mountain slope and the valley along which a rock avalanche moves. The rocks have a total mass m, fall from a height $y = H$, move a distance d_1 along a slope of angle $\theta = 45°$, and then move a distance d_2 along a flat valley. What is the ratio d_2/H of the runout to the fall height if the coefficient of kinetic friction has the reasonable value of 0.60?



Calculations: The final mechanical energy $E_{mec,2}$ is equal to the initial mechanical energy $E_{mec,1}$ minus the amount ΔE_{th} lost to thermal energy:

$$E_{mec,2} = E_{mec,1} - \Delta E_{th}. \tag{8-43}$$

Initially the rocks have potential energy $U = mgH$ and kinetic energy $K = 0$, and so the initial mechanical energy is $E_{mec,1} = mgH$. Finally (when the rocks stop) the rocks have potential energy $U = 0$ and kinetic energy $K = 0$, and so $E_{mec,2} = 0$. The amount of energy transferred to thermal energy is $\Delta E_{th,1} = f_{k1}d_1$ during the slide down the slope and $\Delta E_{\text{th},2} = f_{k2}d_2$ during the runout across the valley. Substituting these expressions into Eq. 8-43, we have

$$0 = mgH - f_{k1}d_1 - F_{k2}d_2. \tag{8-44}$$

From Fig. 8-19*a*, we see that $d_1 = H/(\sin\theta)$. To obtain expressions for the kinetic frictional forces, we use Eq. 6-2 ($f_k = \mu_k F_N$). Recall from Chapter 6 that on an inclined plane the normal force offsets the component $mg\cos\theta$ of the gravitational force (Fig. 8-19*b*). Similarly, recall from Chapter 5 that on a horizontal surface the normal force offsets the full magnitude mg of the gravitational force (Fig. 8-19*c*). Substituting these expressions into Eq. 8-44 and solving for the ratio d_2/H, we find

$$0 = mgH - \mu_k(mg\cos\theta)\frac{H}{\sin\theta} - \mu_k mgd_2$$

and

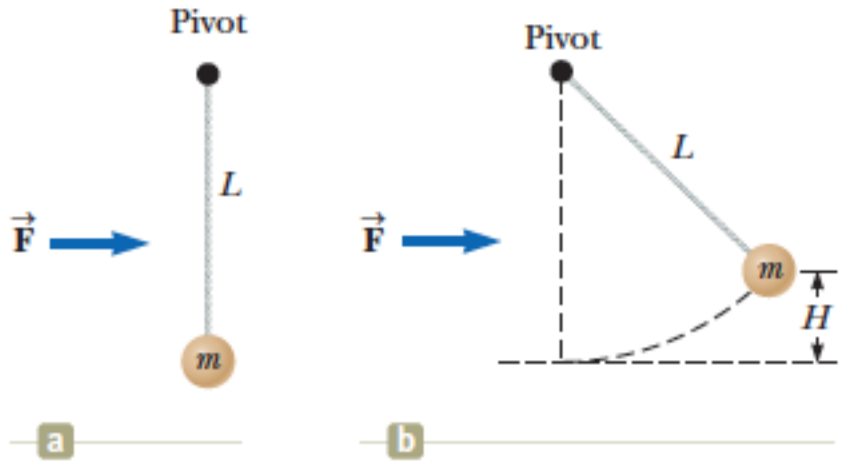
$$\frac{d_2}{H} = \left(\frac{1}{\mu_k} - \frac{1}{\tan\theta}\right). \tag{8-45}$$

Substituting $\mu_k = 0.60$ and $\theta = 45°$, we find

$$\frac{d_2}{H} = 0.67. \qquad \text{(Answer)}$$

Example

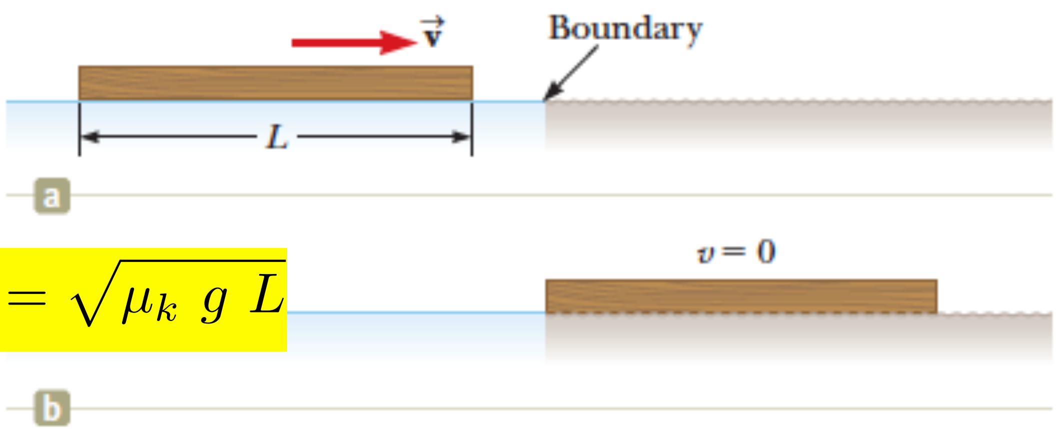
78. A ball of mass $m = 300$ g is connected by a strong string of length $L = 80.0$ cm to a pivot and held in place with the string vertical. A wind exerts constant force F to the right on the ball as shown in Figure P8.78. The ball is released



from rest. The wind makes it swing up to attain maximum height H above its starting point before it swings down again. (a) Find H as a function of F. Evaluate H for (b) $F = 1.00$ N and (c) $F = 10.0$ N. How does H behave (d) as F approaches zero and (e) as F approaches infinity? (f) Now consider the equilibrium height of the ball with the wind blowing. Determine it as a function of F. Evaluate the equilibrium height for (g) $F = 10$ N and (h) F going to infinity.

Example

75. **S** **Review.** A uniform board of length L is sliding along a smooth, frictionless, horizontal plane as shown in Figure P8.75a. The board then slides across the boundary

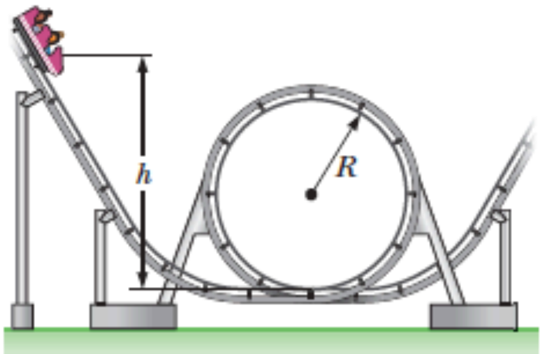


$$a = -\frac{\mu_k\ g\ x}{L}, \quad v = \sqrt{\mu_k\ g\ L}$$

with a rough horizontal surface. The coefficient of kinetic friction between the board and the second surface is μ_k. (a) Find the acceleration of the board at the moment its front end has traveled a distance x beyond the boundary. (b) The board stops at the moment its back end reaches the boundary as shown in Figure P8.75b. Find the initial speed v of the board.

Example

72. **S** A roller-coaster car shown in Figure P8.72 is released from rest from a height h and then moves freely with negligible friction. The roller-coaster track includes a circular loop of radius R in a vertical plane. (a) First suppose the car barely makes it around the loop; at the top of the loop, the riders are upside down and feel weightless. Find the required height h of the release point above the bottom of the loop in terms of R. (b) Now assume the release point is at or above the minimum required height. Show that the normal force on the car at the bottom of the loop exceeds the normal force at the top of the loop by six times the car's weight. The normal force on each rider follows the same rule. Such a large normal force is dangerous and very uncomfortable for the riders. Roller coasters are therefore not built with circular loops in vertical planes. Figure P6.19 (page 159) shows an actual design.



Example

67. **S** A pendulum, comprising a light string of length L and a small sphere, swings in the vertical plane. The string hits a peg located a distance d below the point of suspension (Fig. P8.67). (a) Show that if the sphere is released from a height below that of the peg, it will return to this height after the string strikes the peg. (b) Show that if the pendulum is released from rest at the horizontal position ($\theta = 90°$) and is to swing in a complete circle centered on the peg, the minimum value of d must be $3L/5$.

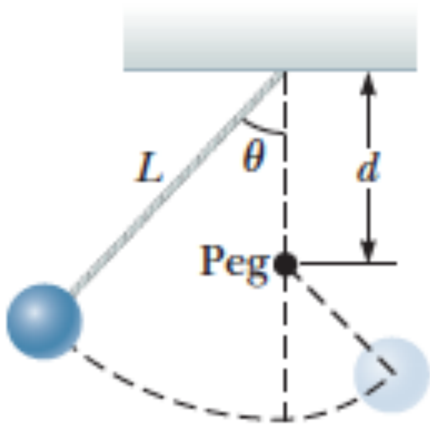
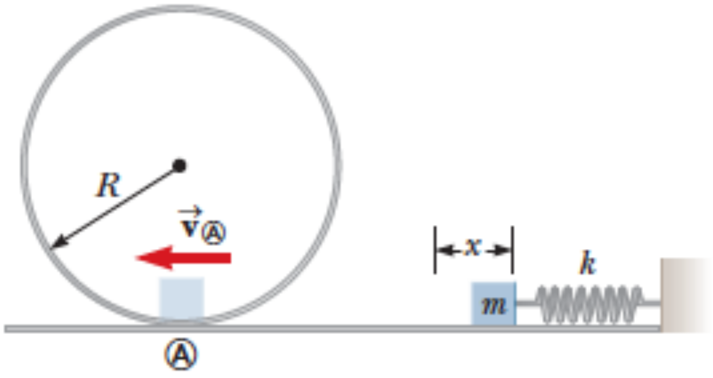


Figure P8.67

Example

65. Q|C A block of mass 0.500 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance x (Fig. P8.65). The force constant of the spring is 450 N/m. When it is released, the block travels along a frictionless, horizontal surface to point Ⓐ, the bottom of a vertical circular track of radius $R = 1.00$ m, and continues to move up the track. The block's speed at the bottom of the track is $v_{Ⓐ} = 12.0$ m/s, and the block experiences an average friction force of 7.00 N while sliding up the track.



(a) What is x? (b) If the block were to reach the top of the track, what would be its speed at that point? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top?

Example

64. **M** A block of mass $m_1 = 20.0$ kg is connected to a block of mass $m_2 = 30.0$ kg by a massless string that passes over a light, frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of $k = 250$ N/m as shown in Figure P8.64. The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled a distance $h = 20.0$ cm down the incline of angle $\theta = 40.0°$ and released from rest. Find the speed of each block when the spring is again unstretched.

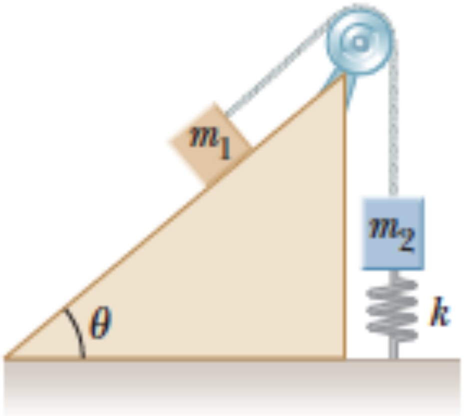


Figure P8.64

Example

63. A 10.0-kg block is released from rest at point Ⓐ in Figure P8.63. The track is frictionless except for the portion between points Ⓑ and Ⓒ, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2 250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between points Ⓑ and Ⓒ.

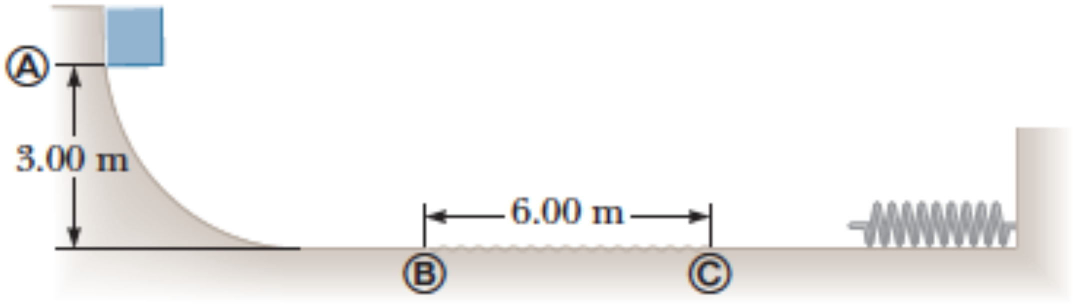


Figure P8.63

Example

41. A small block of mass $m = 200$ g is released from rest at point Ⓐ along the horizontal diameter on the inside of a frictionless, hemispherical bowl of radius $R = 30.0$ cm (Fig. P8.41). Calculate (a) the gravitational potential energy of the block–Earth system when the block is at point Ⓐ relative to point Ⓑ, (b) the kinetic energy of the block at point Ⓑ, (c) its speed at point Ⓑ, and (d) its kinetic energy and the potential energy when the block is at point Ⓒ.

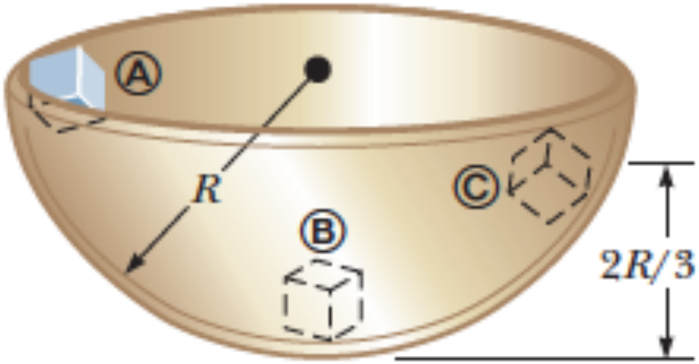


Figure P8.41 Problems 41 and 42.

Example

91 GO Two blocks, of masses $M = 2.0$ kg and $2M$, are connected to a spring of spring constant $k = 200$ N/m that has one end fixed, as shown in Fig. 8-69. The horizontal surface and the pulley are frictionless, and the pulley has negligible mass. The blocks are released from rest with the spring relaxed. (a) What is the combined kinetic energy of the two blocks when the hanging block has fallen 0.090 m? (b) What is the kinetic energy of the hanging block when it has fallen that 0.090 m? (c) What maximum distance does the hanging block fall before momentarily stopping?

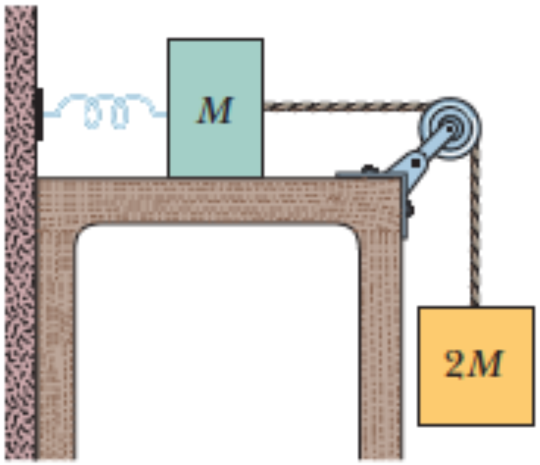


Figure 8-69 Problem 91.

Example

87 SSM A massless rigid rod of length L has a ball of mass m attached to one end (Fig. 8-68). The other end is pivoted in such a way that the ball will move in a vertical circle. First, assume that there is no friction at the pivot. The system is launched downward from the horizontal position A with initial speed v_0. The ball just barely reaches point D and then stops. (a) Derive an expression for v_0 in terms of L, m, and g. (b) What is the tension in the rod when the ball passes through B? (c) A little grit is placed on the pivot to increase the friction there. Then the ball just barely reaches C when launched from A with the same speed as before. What is the decrease in the mechanical energy during this motion? (d) What is the decrease in the mechanical energy by the time the ball finally comes to rest at B after several oscillations?

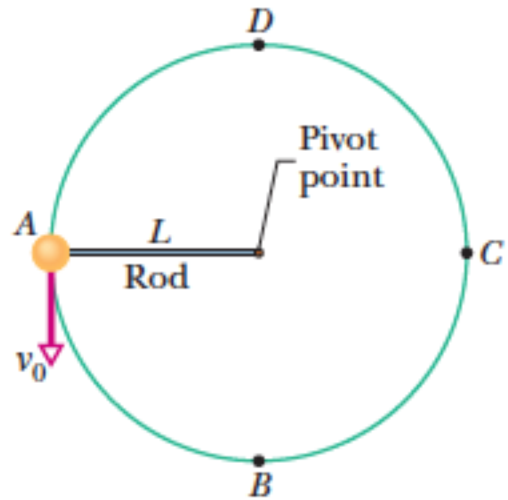


Figure 8-68 Problem 87.

Example

93 A playground slide is in the form of an arc of a circle that has a radius of 12 m. The maximum height of the slide is $h = 4.0$ m, and the ground is tangent to the circle (Fig. 8-70). A 25 kg child starts from rest at the top of the slide and has a speed of 6.2 m/s at the bottom. (a) What is the length of the slide? (b) What average frictional force acts on the child over this distance? If, instead of the ground, a vertical line through the *top of the slide* is tangent to the circle, what are (c) the length of the slide and (d) the average frictional force on the child?

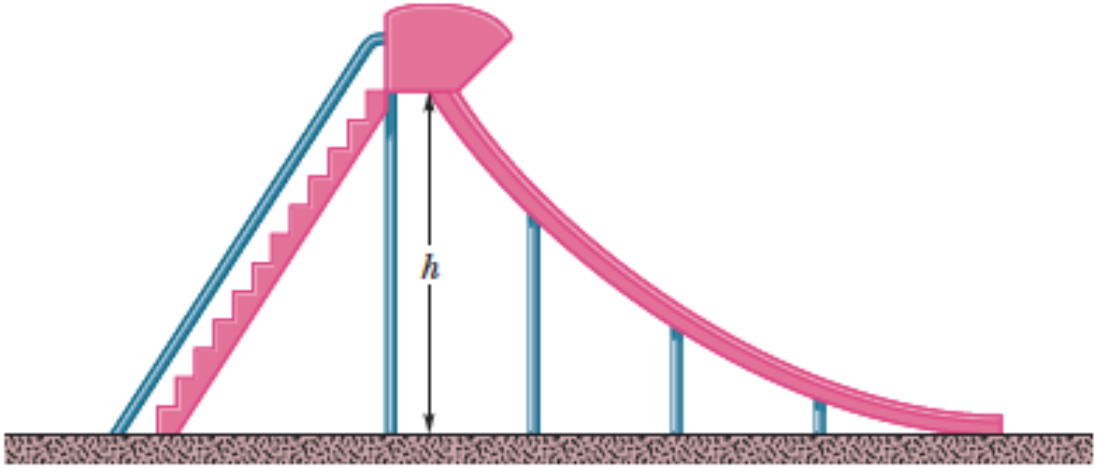
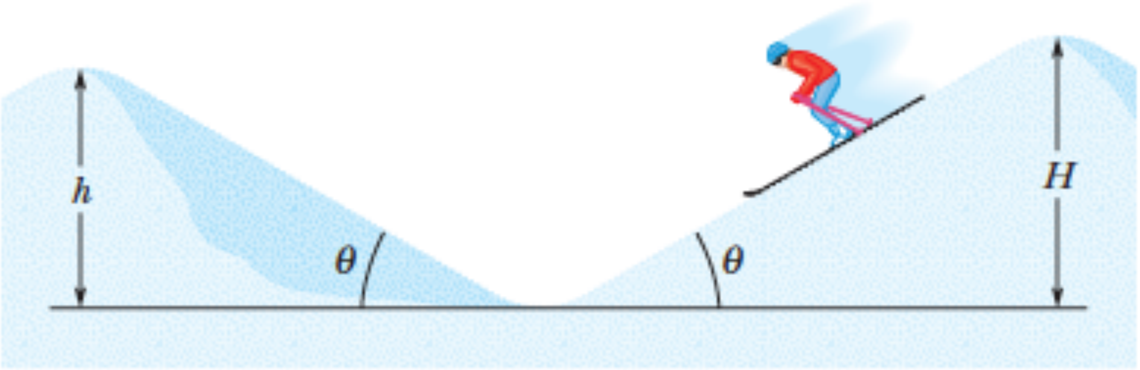


Figure 8-70 Problem 93.

Example

72 Two snowy peaks are at heights $H = 850$ m and $h = 750$ m above the valley between them. A ski run extends between the peaks, with a total length of 3.2 km and an average slope of $\theta = 30°$ (Fig. 8-61). (a) A skier starts from rest at the top of the higher peak. At what speed will he arrive at the top of the lower peak if he coasts without using ski poles? Ignore friction. (b) Approximately what coefficient of kinetic friction



between snow and skis would make him stop just at the top of the lower peak?

Example

•••34 GO A boy is initially seated on the top of a hemispherical ice mound of radius $R = 13.8$ m. He begins to slide down the ice, with a negligible initial speed (Fig. 8-47). Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?

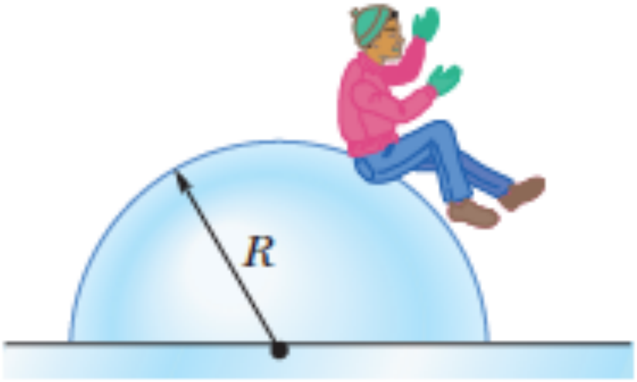


Figure 8-47 Problem 34.

Example

••32 In Fig. 8-45, a chain is held on a frictionless table with one-fourth of its length hanging over the edge. If the chain has length $L = 28$ cm and mass $m = 0.012$ kg, how much work is required to pull the hanging part back onto the table?

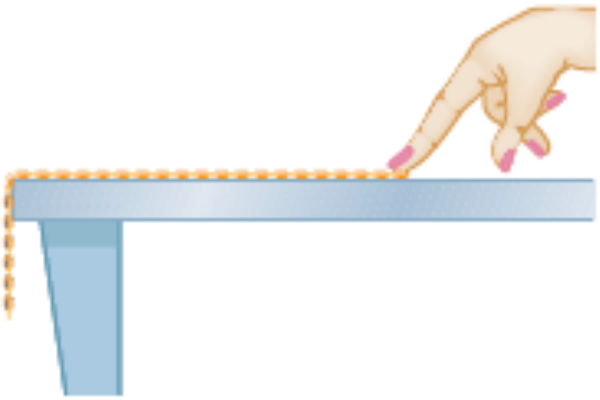


Figure 8-45 Problem 32.

Example

••19 GO Figure 8-36 shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone. (a) What is the spring constant? (b) The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release? (c) What is the change in the gravitational potential energy of the stone–Earth system when the stone moves from the release point to its maximum height? (d) What is that maximum height, measured from the release point?

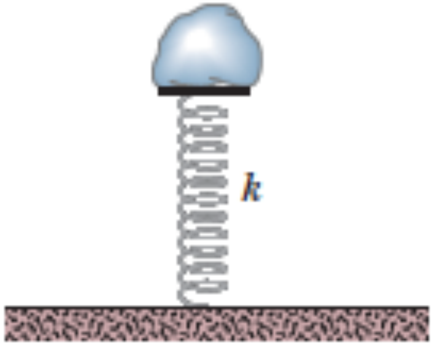


Figure 8-36 Problem 19.