

## UNCERTAINTY IN THE FLUCTUATIONS OF THE PRICE OF STOCKS

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We report on a study of the Tehran Price Index (TEPIX) from 2001 to 2006 as an emerging market that has been affected by several political crises during the recent years, and analyze the non-Gaussian probability density function (PDF) of the log returns of the stock prices. We show that while the average of the index did not fall very much over the time period of the study, its day-to-day fluctuations strongly increased due to the crises. Using an approach based on multiplicative processes with a detrending procedure, we study the scale-dependence of the non-Gaussian PDFs, and show that the temporal dependence of their tails indicates a gradual and systematic increase in the probability

of the appearance of large increments in the returns on approaching distinct critical time scales over which the TEPIX has exhibited maximum uncertainty.

*Keywords:* Stock market; Castaing equation; non-Gaussian distributions; fluctuations.

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## 1. Introduction

In recent years, financial markets have been a focus of physicists' attempts for applying the existing knowledge from statistical mechanics to economic problems.<sup>1-3</sup> The markets, though largely varying in the details of their trading rules and the traded goods, may be characterized by some generic features of the time series that describe the fluctuations in the prices of various stocks and commodities. An important and challenging problem is to understand and evaluate risk in the markets, which must be done through the analysis of such time series. The aim of the analysis is to characterize the statistical properties of the time series, with the hope that a better understanding of its underlying stochastic dynamics would provide useful information that can be used for creating new models that are able to reproduce experimental facts (i.e., the actual recorded prices and their fluctuations).

A considerable amount of data and numerous studies indicate the possibility that the financial time series may exhibit self-similarity (and/or self-affinity) at short time scales which, however, apparently break down at much longer times. Such features are usually modeled in terms of various statistical distributions with truncated tails. Recent studies indicated, however, that an approach based on Brownian motion,<sup>5,6</sup> or other more elaborated descriptions, such as those based on the Lévy and truncated Lévy distributions,<sup>1</sup> may not be suitable for properly describing the statistical features of the fluctuations in stock prices. Such models have been constructed based on the premise that the financial time series may be viewed as *additive* processes that are built up over time. There is now increasing evidence that an approach based on *multiplicative* processes might be a more fruitful way of pursuing an accurate analysis of the financial time series. This approach lends itself in a natural way to multifractality<sup>7</sup> (see below). Such an idea was, in fact, suggested some years ago when the intermittency phenomenon in the returns fluctuations was observed at different scales, which motivated some efforts for establishing a link between analysis of the financial time series and other areas of physics, such as turbulence.<sup>8-11</sup> We remind the reader that, if  $p_i$  represents the value of a stochastic variable at (time)  $i$ , the returns  $r_i$  are defined by  $r_i = \ln(p_{i+1}/p_i)$ . Nowadays, however, we know that there are important differences between the two phenomena, such as, for example, the differences between their spectra of frequencies.

Based on the recent efforts for characterization of the various stages of the development of markets,<sup>12-14</sup> it is clear that the Tehran stock exchange represents an emerging market. It has witnessed considerable activities over the past several years, but it is still far from an efficient and developed market. Over a two-year period, it lost more than 30% of its value (from 13 750 units in September 2004

to 9150 units in August 2006) and, on average, the price of the stocks' units has decreased from \$0.92 to \$0.49 (which, percentage-wise, represents an even steeper decline than that of the units by which the market has lost value), even without considering the rate of inflation. In addition, over the past six months alone (up to the time of writing this paper), the volume and values of the traded stocks have decreased by more than 60%. Compared with the S&P 500, the Tehran stock exchange is still not a completely developed market,<sup>13</sup> with its index exhibiting stronger non-stationary features.

In this paper, we provide comprehensive evidence of the existence of distinct critical time scales over which the Tehran Price Index (TEPIX) has exhibited maximum uncertainty. Moreover, at several critical times over the past few years, Tehran stock exchange has been affected rather strongly by several political crises. These features provide a good opportunity to test the method of analysis suggested by Kiyono *et al.*<sup>15</sup> for an emerging market. More specifically, by analyzing the temporal evolution of the index dynamics, we demonstrate the strongly non-Gaussian behavior of the logarithmic returns of the TEPIX and scale-dependent behavior (data collapse) of their probability density function (PDF). The critical time scales are found to be in the vicinity of large index movements, consistent with the high probability of multiscale events at critical points. From the observed non-Gaussian behavior of the index, we numerically estimate the unexpectedly high probability of a large price change near the critical times. Such estimates are of importance to risk analysis, as they represent a central issue for the understanding of the statistics of price changes.

The rest of this paper is organized as follows. In the next section we present the data that we consider and describe how we analyze them. The conclusions are summarized in Sec. 3.

## 2. Analysis of the Data

Figure 1 shows the TEPIX over a period of over 4 1/2 years, from December 20, 2001 to August 10, 2006. The data had been recorded on each trading day. We show in the lower panel of Fig. 1 the one-day log returns, i.e.,  $r_s(t) = \ln[p(t+s)/p(t)]$ , where  $s = 1$  day. We then analyze the PDF of the detrended log returns over different time scales. To remove the trends present in  $\{x(t)\}$ , where  $x(t) = \ln p(t)$ , we fit  $x(t)$  in each subinterval  $[1 + s(k - 1), s(k + 1)]$  of length  $2s$  (where  $k$  is the index of the subinterval to a linear function of  $t$  that represents the exponential trend of the original index in the corresponding time window. After the detrending procedure, we define detrended log returns on a scale  $s$  as  $\Delta_s p(t) = x^*(t + s) - x^*(t)$ , where  $1 + s(k - 1) \leq t \leq sk$ , and  $x^*(t)$  is the deviation from the fitting function.<sup>7</sup>

The scale-invariance properties of a fractal function  $\Delta_s p(t)$  are generally characterized by exponents  $\xi_q$  that govern the power-law scaling of the absolute moments of its fluctuations, i.e.,  $m(q, l) = K_q l^{\xi_q}$ , where, for example, one may choose  $m(q, l) = \sum_t |\Delta_s p(t+l) - \Delta_s p(t)|^q$ . As is well-known, if the exponents  $\xi_q$  are linear

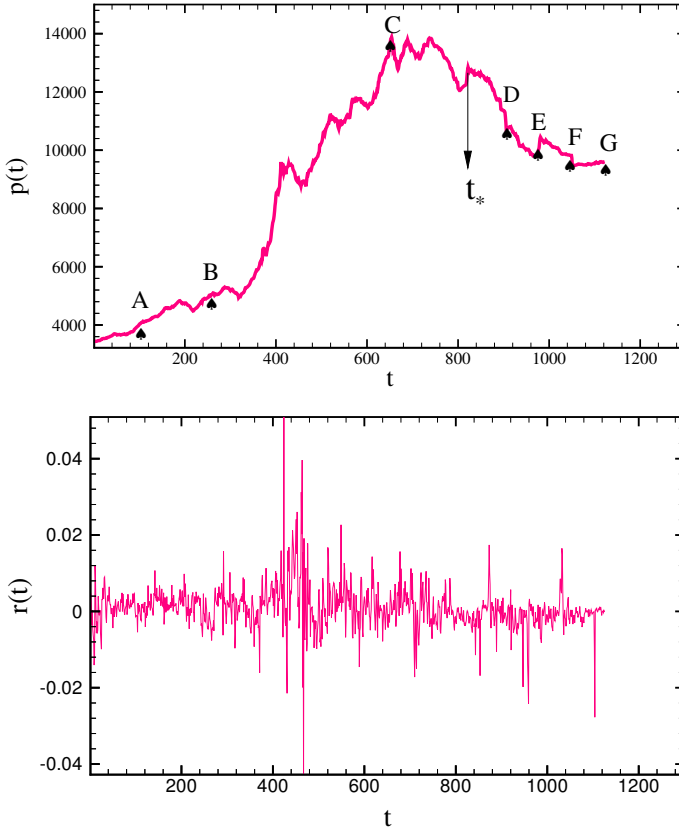


Fig. 1. Top: history (2001–2006) of daily deflated closure of the TEPIX. Bottom: one day log returns of the TEPIX.

in  $q$ , a single scaling exponent  $H$  suffices for characterizing the fractal properties with  $\xi_q = qH$ , in which case  $\Delta_s p(t)$  is said to be monofractal. If, on the other hand, the function  $\xi_q$  is not linear in  $q$ , the process  $\Delta_s p(t)$  is said to be multifractal. Some well-known monofractal stochastic processes are self-similar processes with the following property:

$$\Delta_{\lambda s} p(t) = \lambda^H \Delta_s p(t), \quad \forall s, \lambda > 0. \tag{1}$$

Widely-used examples of such processes are the fractional Brownian motion and the Lévy walk. One reason for their success is, as it is generally the case in experimental time series, that they do not involve any particular scale ratio [i.e., there is no constraint on  $s$  or  $\lambda_s$  in Eq. (1)].

In the same spirit, one can try to build multifractal processes that do not involve any particular scale ratio. A common approach, originally proposed in the field of fully-developed turbulence,<sup>8,15–18</sup> has been to describe such processes in terms of stochastic equations, in the scale domain, describing the cascading process that determines how the fluctuations evolve when one passes from the coarse to fine

scales. One can state that the fluctuations at scales  $s$  and  $\lambda_s$  are related (for fixed  $t$ ) through the cascading rule,

$$\Delta_{\lambda_s} p(t) = W_\lambda \Delta_s p(t), \quad \forall s, \lambda > 0, \tag{2}$$

where  $\ln(W_\lambda)$  is a random variable. Let us note that Eq. (2) can be viewed as a generalization of Eq. (1) with  $H$  being stochastic. Since Eq. (2) can be iterated, it implicitly forces the random variable  $W_\lambda$  to have a log infinitely-divisible law.<sup>19</sup> It has been demonstrated by Castaing *et al.*<sup>19</sup> that a non-Gaussian PDF with “fat” tails can be modeled by random multiplicative processes.

Thus, let us assume that the increments in the time series are represented by the following multiplicative form:<sup>7</sup>

$$\Delta_s p(t) = \zeta_s(t) \exp[\omega_s(t)], \tag{3}$$

where  $\zeta_s$  and  $\omega_s$ , assumed to be independent variables, are both Gaussian random variables with zero mean and variances  $\sigma_s^2$  and  $\lambda_s^2$ , respectively. The PDF of  $\Delta_s p(t)$  has fat tails, depending on the variance of  $\omega_s$ , and is expressed by:<sup>19</sup>

$$P_s(\Delta_s p) = \int F_s \left( \frac{\Delta_s p}{\sigma_s} \right) \frac{1}{\sigma_s} G_s(\ln \sigma_s) d \ln \sigma_s, \tag{4}$$

where we have assumed that  $F_s$  and  $G_s$  are both Gaussian with zero mean and variance  $\sigma_s$  and  $\lambda_s$ , i.e.,

$$G_s(\ln \sigma_s) = \frac{1}{\sqrt{2\pi\lambda_s}} \exp \left( -\frac{\ln^2 \sigma_s}{2\lambda_s^2} \right). \tag{5}$$

Thus, we may investigate the time scale-dependence of  $\lambda_s^2$ . In this case, the equation for  $P_s(\Delta_s p)$  is referred to as Castaing’s equation, whose solution converges to a Gaussian when  $\lambda \rightarrow 0$ .

The fit of the PDF of TEPIX increments to Castaing’s equation is indeed almost perfect, especially within  $\pm 3$  standard deviations, even for a single record. This is demonstrated in Fig. 2. Although Eq. (4) is equivalent to that for a log-normal cascade model — originally introduced to study fully-developed turbulence<sup>19</sup> — it approximately describes the non-Gaussian PDFs observed not only for turbulence, but also in a wide variety of other phenomena, ranging from rate of exchange of foreign currencies,<sup>8</sup> to heartbeat interval fluctuations.<sup>15,20</sup> Also shown in Fig. 2 is the fit of the data for  $s = 20$  days to a Gaussian distribution, which clearly fails to represent the data.

For a quantitative comparison, we fit the data (over the 4 1/2 years interval) to the above function [Eq. (4)], as illustrated in Figs. 2 and 3, and estimate the variance  $\lambda_s^2$  of  $G_s$ . As shown in Fig. 3, the standardized (variance = 1) PDF of the detrended log returns indicates the existence of a scaling law in the behavior of  $\lambda_s^2$  as a function of  $s$ , rather than logarithmic decay which is characteristic of classical cascade processes.<sup>17–19,21</sup> Figure 3 indicates that after  $s = 4$  days, there is a crossover in the behavior of  $\lambda_s$  as a function of  $s$ . For comparison, we have also calculated the variance  $\langle r(t + \tau)r(t) \rangle$  (which represents the width of the joint

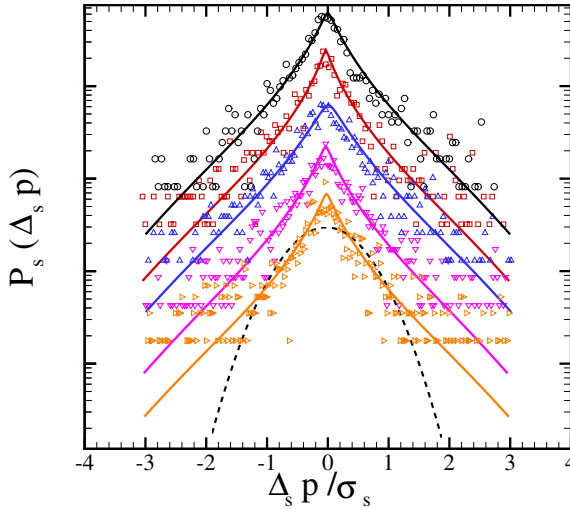


Fig. 2. Continuous formation of the increment probability distribution function's across scales for, from top to bottom,  $s = 4, 8, 12, 16$  and  $20$  days. Solid lines show the approximated PDF based on Casting's equation, the right-hand side of Eq. (4).

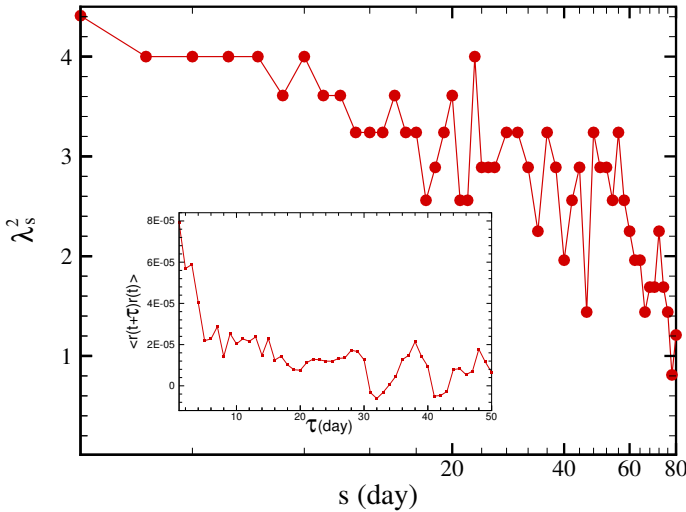


Fig. 3. The scale-dependence of the fitting parameter of Castaing's equation  $\lambda^2$  vs.  $\log s$ . The inset shows the results for  $\sigma = \langle r(t + \tau)r(t) \rangle$ . The results indicate that after  $s = 4$  days, there is crossover in the behavior of  $\lambda$  vs.  $s$ , and that of  $\sigma$  vs.  $\tau$ .

probability distribution). The results are shown in the inset of Fig. 3. Similar to  $\lambda_s$ , there is a crossover in the behavior of width. (No interesting results were found for the simple standard deviation.)

In the following, we identify a temporal region of complete departure from the cascade scenario to an instance of the critical-like behavior. We evaluate (in sliding

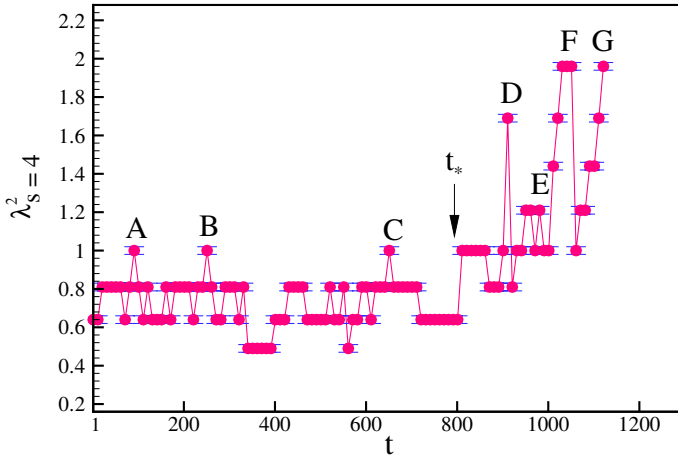


Fig. 4. The local temporal variation of  $\lambda_{s=4}^2$  days over a one-year period shows a gradual, systematic increase on approaching the critical time scales A–G.

time intervals  $[t - \Delta t/2, t + \Delta t/2]$  the temporal dependence of  $\lambda_s^2$ . The local temporal variation of  $\lambda_{s=4}^2$  days over a one-year period shows a gradual, systematic increase on approaching the critical time scales A–G identified in Fig. 4. It is beneficial to risk analysis to quantify the non-Gaussian nature of (detrended) price fluctuations on a relatively short time scale ( $\sim 4$  days), and not just the volatility at larger time scales,<sup>1</sup> which is what is normally analyzed. The important point is that large values of  $\lambda_s^2$  indicate a high probability of a large price change; this probability follows a sharp increase with growing  $\lambda_s^2$ .

The critical points are denoted by A to G in Fig. 4. To plot Fig. 4 we chose a moving window with length  $\Delta t = 150$  days. It may be interesting to note that these time scales are related to the political developments in Iran. There was an increasing trend in the price index over the time scale A, caused by privatization of Iran’s industries. B represents the time period from February 21, 2003, when the inspectors of the International Atomic Energy Agency (IAEA) and its Director-General, Dr. Mohammed ElBaradei, traveled to Iran, to June 16, 2003, when Dr. ElBaradei reported to the IAEA’s Board of Governors on what the IAEA had found in Iran. C represents the restart by Iran of production of centrifuges’ parts, used in uranium enrichment (UE), on July 31, 2004. D is the time period that included the European Union’s warning to Iran that it would cut off negotiations on May 11, 2005; Iran’s subsequent declaration on May 19, 2005 that its UE program was irreversible, and the election of Iran’s new president on June 26, 2005. E is the time period over which a new director for Tehran stock market was appointed, and the economic policies of Iran’s new president were declared. Finally, F is the time at which the IAEA reported to the United Nations Security Council Iran’s nuclear dossier on February 27, 2006. The time  $t_*$  represents the time at which Iran’s rejection of the IAEA demand for stopping work on the construction of a

heavy-water nuclear reactor in Arak was announced on February 13, 2005. It can be seen clearly in Fig. 4 that, after that time, the TEPIX entered a critical period that has continued up to now. Moreover, as Fig. 4 indicates, similar to most major stock markets around the world, the Tehran stock market has responded almost immediately to the political events on the dates indicated. As shown in Fig. 4, the trends in the TEPIX are essentially stable up to time scale C, but beyond C the average uncertainty increases.

To check the changing of the nature of fractal distribution of the returns, we plot (in a semi-logarithmic graph) the PDFs of one-day returns before and after the critical time  $t_*$ . The results are presented in Fig. 5. Relative to a Gaussian distribution, they exhibit sharp peaks, but not long tails. In Table 1, we compare the means, standard deviations, skewnesses and kurtosises of the returns time series before and after the time  $t_*$ , as given in Fig. 1. As Table 1 indicates, the mean value of the returns is negative after  $t_*$ , but positive before  $t_*$ . Moreover, the variance after  $t_*$  is smaller than its value before  $t_*$ , implying that, on average, the investors have lost their investments after  $t_*$  but, with smaller risk, had gained before  $t_*$ .

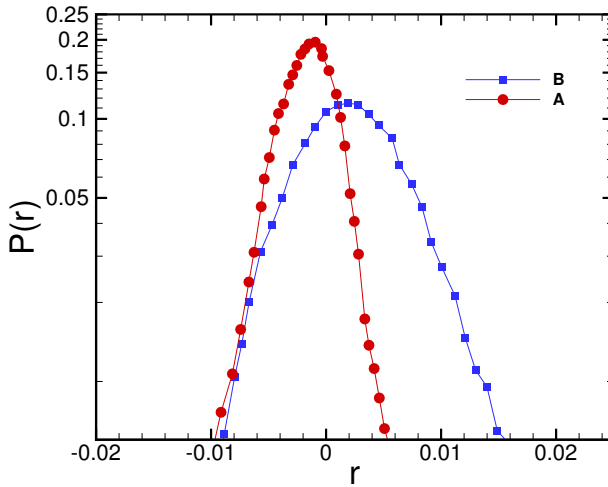


Fig. 5. The probability distribution functions of the TEPIX returns before (B) and after (A) critical time  $t_*$ .

Table 1. Comparing the general parameters of the one-day return series.

	Mean	Standard Deviations	Skewness	Kurtosis
$t < t_*$	0.00172	0.006	0.84	16.65
$t > t_*$	-0.00104	0.004	-0.66	12.46



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