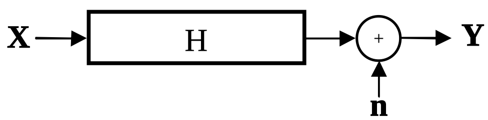
In The Name of God, the Compassionate, the Merciful

Midterm Project

Part (a)

The focus of this project is on a simple communication system in which a random signal is transmitted over an AWGN (Additive White Gaussian Noise) channel which deterministic impulse response. The objective of this project is to give you an applied sense of basic concepts of stochastic processes as well as linear systems and detection theory. You are supposed to carry out mathematical analysis as well as MATLAB simulations in this project. Consider the following model:



in which $\mathbf{X}(t)$ is a 2×1 vector process, H is a linear system modeling the channel, $\mathbf{n}(t)$ is a 2×1 noise vector and $\mathbf{Y}(t)$ is a 2×1 vector, which is observed by the receiver as the output of the channel. Let us assume $\mathbf{X}(t)$ and $\mathbf{n}(t)$ are independent and $\mathbf{n}(t)$ is a Gaussian random vector, i.e. $\mathbf{n} \sim N(0, S_n)$. Also, let us assume the following PDF for $\mathbf{X}(t)$, which is a discreet time process:

$$\mathbf{X}_i(kt) = \begin{cases} \frac{1}{\sqrt{2}} & p = 0.5 \\ -\frac{1}{\sqrt{2}} & p = 0.5 \end{cases} \quad \text{and 0 otherwise}$$

In other words, $\mathbf{X}(t)$ has k-1 zeros between each two samples or over-sampled with rate k. Also, assume $S_n = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ and $h(t) = e^{-\frac{t}{a}} u(t)$ as the impulse response of the channel. For MATLAB simulations, assume n_s=1000 samples. Find the following:

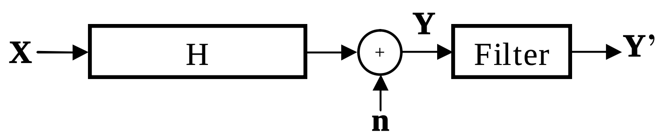
a. Derive $R_{XX}(t)$, the autocorrelation function of $\mathbf{X}(t)$, and $S_{XX}(w)$ mathematically and using MATLAB (include the plot in your report) for k=1 and 10 and a=1.

b. The density of **Y** assuming $a=1$ and $\boldsymbol{k}=1$ mathematically, then simulate the system for $\boldsymbol{n_s}$ samples using MATLAB and derive $\boldsymbol{f}_Y(\boldsymbol{y})$ empirically. Also derive $\boldsymbol{R_{YY}}(\boldsymbol{t})$ and $\boldsymbol{S_{YY}}(w)$ and plot the results of your MATLAB simulation for them.

c. Assuming the best sample of $\mathbf{Y}(\boldsymbol{t})$ is used to detect $\mathbf{X}(\boldsymbol{t})$ at the receiver, find the bit error rate (BER) as the probability of error in detecting the non-zero symbol sent be the transmitter. Assume perfect synchronization (i.e. receiver knows which symbol should be non-zero). Present the result using your MATLAB simulation and compare the results for various values of $\boldsymbol{k}$ and $a=1$.

d. What is the effect of $\boldsymbol{k}$ in BER? In order to have BER not greater than 10^{-8}, what should be the value of $\boldsymbol{k}$? What is the role of a?

e. Now, assume we filter $\mathbf{Y}(\boldsymbol{t})$ to improve BER or equivalently consider the below system:



Study about the ***Matched Filter*** concept and design a matched filter so that the BER is minimized. Derive the BER in this case. Implement the new model in MATLAB and compare the analytical results with simulation results for $\boldsymbol{k}=1$ and $a=1$. Also derive and plot $\boldsymbol{f}_{Y'}(\boldsymbol{y}')$, $\boldsymbol{R_{Y'Y'}}(\boldsymbol{t})$ and $\boldsymbol{S_{Y'Y'}}(w)$.

f. How much is the improvement in BER by applying the matched filter. Present a plot showing BER vs. $\boldsymbol{k}$ for both systems.

Part (b)

Project Objective: The objective of this project is to introduce motivated students to discrete event simulation, and to provode them with hands on experience in developing basic simulation models and analyzing output data. Simulation is a tool used to evaluate system performance, compare alternative system designs, quantify the effects of system parameters and workloads, and to validate analytically derived models. The crucial element common to all simulations is that they involve uncertainty. More specifically, certain aspect of system behavior are subject to events that are characterized according to random distributions. For example the input to a communication network is not known in advance, however, we may be able to model it according to the probability distribution of the time between successive 'arrivals' and the probability distribution of the length of each element, thus, determining the distribution of the time it takes to serve each element. The same characterization holds if we choose to model jobs arriving to a CPU or customers arriving for service at a bank. A simulation models the random behavior of the workload (inputs), however, it also models random aspects of the service. For example, in a contention based communication system the time it takes to transmit a packet of data depends on the packet length and data rate, which we assume are fixed, and a random delay due to contention for access to the shared transmission media. The simulation model is designed to approximate the response of the real system to each input event; however it should be clear that due to the stochastic nature of the inputs the observed responses are themselves random events.

As such, in the analysis of simulation output the observed responses must be treated as outcomes of random experiments. This is essential for students to understanding a stochastic system rare events occur. As such, without an understanding of the random nature of simulation output data it is easy to arrive at dubious conclusions. The significance of simulation output can only be assured through elimination of startup transients (response of a system that is initially empty) and careful statistical analysis based (typically) on multiple independent repetitions of the random experiment.

Queuing Theory: The study of “waiting in line" is a fundamental aspect of Engineering design and analysis across a range of application domains. Modern telecommunications networks and packet-switched networks (Internet), as well as computer architecture and distributed system design are all based on queueing systems. Hence, they benefit from the knowledge gained through the theory of queues. A generic queuing system consists of customers and servers-customers arrive to the system according to a stochastic process. Upon arrival a customer either enters service or enters the queue. Which of these happens depends on whether or not a server (many systems only have a single server) is idle, whether or not there are other customers already in line, and the nature of the service discipline. If the system is stable each customer will eventually receive service and depart the queue - the time it takes to serve a customer once it has entered service is also a stochastic process. In general the response of a queueing system is determined statistically based on the characteristics of the following: (1) the arrival process; (2) the number of servers; (3) the distribution of service times; (4) the service discipline; and, (5) the capacity (maximum number of customers) of the queue. Some of the questions that engineers seek to answer about a queueing
system include:

- What is the steady-state (long-term average) mean time that a customer is in the system, including both waiting and service time?
- What is the steady-state mean delay or waiting time experienced by a customer in the system?
- What is the steady-state mean number of customers in the system?
- What is the steady-state utilization (percentage of time spent servicing customers) of the system? (Note: this is unity minus the probability the system is empty)
- What are the state probabilities for a given number of customers in the system?
- What is the complete distribution of time that customers spend in the system (sojourn time)?

- What is the distribution of time before the system reaches a certain state, e.g. the time until the delay exceeds some threshold (first passage time)?

Project Overview: Although the analysis of queueing systems is rich with valuable techniques and results, many practical queueing systems prove intractable when it comes to steady-state and especially transient behavior. An alternative to queueing theory for the analysis of these systems is discrete-event simulation. In this project you should use a discrete-event simulation system (Open Source, commercial such as OPNET, etc.) to model a number of basic Markovian queues; for each such queue you will be asked to use your simulation model to collect sample data and perform statistical analysis to estimate a number of steady-state metrics (state-probabilities, mean delay, etc.). Simulation results will then be compared with the analytical results which you will be given. The objectives we hope to achieve in this exercise can be summarized as follows; we hope to develop understanding of:

- The process of modeling basic queueing systems with discrete-event simulation.
- The fundamental properties of Markovian queues-input and output processess, steady-state metrics, transient behavior, etc. (not queueing theory; general characteristics)
- The statistical variation in simulation output and how to use output data to correctly estimate system performance in our case to validate analytical models.

Project Requirements: In order to complete this part of the project, you should read a few references as required based on your background:

2. Develop model for the M-M-1 and M-G-1 queues. (Note: the M-G-1 queue can be used to model the M-M-1, M-D-1 and queues with other service time distributions).

3. For the M-M-1 queue service times are exponentially distributed and arrivals come to the system according to a Poisson process, which also means that the interarrival times are also exponentially distributed; you are asked to select the mean service time and then select 10 dfferent arrival rates. The arrival rates should be selected uniformly over a range that results in queue utilizations from 5% to 95%.

4. For the lowest and highest arrival rates you are asked to determine the approximate duration (in simulated time) to reach steady-state (this will be pretty fast for the M-M-1 queue). A methodology based on sliding window averages should be assumed. You will then use the larger of the two values as your simulation warm-up time.
5. For data collection you will be asked to use a common methodology known as replication and deletion, namely, you will repeat each experiment n times; wherein the first k seconds of data are ignored (deleted). The number of seconds, k, is the simulation warm-up time determined previously.
6. Using any tool you like, e.g. matlab, a calculator, etc., you are asked to estimate the steady-state mean number in the system, queuing delay, time in the system and system utilization for each arrival rate. Both point estimates (the sample mean) and interval estimates (95% confidence intervals) are required. For each metric you are asked to plot the simulation results along with the analytical results (the equations shall be provided to you).

Good Luck
Hamid R. Rabiee

References:

1. Queueing Theory Books and Notes Online:
http://www2.uwindsor.ca/~hlynka/qonline.html

2. Queueing Tutorial:
Prof. Dimitri P. Bertsekas, MIT; Traffic Behavior and Queuing in a QoS Environment:
http://www.mit.edu/people/dimitrib/OPNET_Full_Presentation.ppt