Stochastic Processes	Student ID:
Department of Computer Engineering	
Sharif International University of Technology	Student Name and Family:
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In The Name of God, the Compassionate, the Merciful

Midterm Exam

Time: 120 min

I. Random Process Basic Concepts (30 points = 6+9+9+6)

a) Find mean, autocorrelation and variance of the random process **X=** t.**U**; where **U** is a uniform random variable on [0,1].

b) Given a random process $\mathbf{X}(t) = A.\cos\left(wt + \frac{1}{n}j\right)$ with $j \sim U(0,2mp)$. Where A, w are constant and ***m***, ***n*** are arbitrary but fixed integers. Find mean and autocorrelation of **X**(t). Is it WSS? Why?

c) Let **X** and **Y** be independent exponential random variables with $l = 1$. (i.e. $f_X(t) = f_Y(t) = e^{-t}$ for $t \geq 0$). If $\mathbf{U} = \mathbf{X}^2$ and $\mathbf{V} = \mathbf{X}^2 + \mathbf{Y}$, find the PDF of **U** and **V**.

d) Prove that for a stationary process **X**(t) we have $|R_{XX}(t)| \leq R_{XX}(0)$.

II. Special Random Processes (25 points = 15+10)

a) For a normal random process **X**(t) with $m_X = 1$ and $s_X{}^2 = 9$, find $a = P\left(\mathbf{X}(2t) \geq \frac{\mathbf{X}(t) + \mathbf{X}(3t)}{2}\right)$ without any integration.

b) Assume the random processes **X**(t) and **Y**(t) which are uniformly distributed over $[0, 2 + \sin(2p\,t)]$ and $[\sin(2p\,t), 2]$ respectively. Let **Z**(t)=**X**(t)+**Y**(t) be another random process. Find and plot PDF of **Z**(t).

III. System Analysis (10 Points)

Suppose we have an LTI system with impulse response $h(t) = \begin{cases} 1 & 0 \leq t \leq 4 \\ 0 & O.W. \end{cases}$. Assume that the input of the system is stationary and has autocorrelation function $R(t) = 1 + \sin(2t)$. Find autocorrelation function of the output.

IV. Estimation Theory (35 points = 10+10+15)

a) If we observed some i.i.d random variables $\mathbf{X_1, X_2, ..., X_n}$ with log-normal PDF. (i.e. $f_X(x) = \frac{1}{xs\sqrt{2p}} \exp\left(-\frac{(Ln(x)-m)^2}{2s^2}\right)$). Find an ML estimator for $q = \begin{pmatrix} m \\ s^2 \end{pmatrix}$.

b) b) b is an exponential random variable with PDF $f_b(b) = l\, e^{-lb}$ where l is a constant. We observe output samples of a system $\mathbf{X_1, X_2, ..., X_n}$ that are i.i.d and have Rayleigh PDF with parameter b . (i.e. $f_X(x) = \frac{x}{b^2} \exp\left(-\frac{x^2}{2b^2}\right)$). Find an MAP estimator for b .

c) We observe i.i.d random variables $\mathbf{X_1, X_2, ..., X_n}$ with the PDF $f_X(x) = \frac{g}{p\left((x-m)^2 + g^2\right)}$ and want to estimate g when m is fixed.

1) Find an unbiased estimator for g .

2) Find Cramer-Rao bound for this estimation.

V. Extra Points (15 points)

Let $\mathbf{X} \sim \text{Binomial}(2, q)$ where $0 \le q \le 1$ and $g(q) = q^2$.

a) Show that ***h(x)*** is an unbiased estimator of $g(q)$, where $h(x) = \begin{cases} 0 & x = 0,1 \\ 1 & x = 2 \end{cases}$.

b) Is ***h(x)*** the UMVUE for $g(q)$? Why or why not?

c) Show that $Var(h(x)) = q^2(1-q^2)$.

Good Luck

Hamid R. Rabiee