Valuation of American Call Option;

A Free Boundary Problem

Dr. Hassan Nojumi

Continuous Compounding

$A(t)$ = **investment after time** t **with interest added** *continuously*.

$$A(t) = e^{rt} A_0$$

Corollary.

The asset with value K at time T has value equal to $e^{-r(T-t)}K$ at time t.

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Quantity $e^{-r(T-t)}$ is the ***discount factor*****.**

In the real world, all quantities vary with time in random, unexpected ways.

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Hence an accurate modeling of financial markets requires methods of stochastic analysis.

Forward Contract

A contract between two parties by which one party has the *obligation* **to buy a specified asset, called** *underlying asset*, **at a specified amount** K, **called** *exercise value*, **at a specified time** T, **called** *maturity date* **from the other party, and the other party has the** *obligation* **to buy.**

Forward contract is a *symmetric* **contract; both sides are in similar situations. Hence the value of the forward contract is zero.** *Theoretically*, **one can enter a forward contract without having to pay anything.**

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Futures Contract

A formal forward contract traded under supervision of a financial institution called *Exchange*.

Forward and Futures Contracts

Risky trades for both sides:

S_t **:= Price of underlying asset at time** t**; a stochastic process for** $t \geq 0$**.**

Each side has a potential gain and a potential loss of $|S_T - K|$**, which can be quite large.**

Call Option

A contract between two parties by which one party, called *option holder* **or** *party in the long position*, **has the** *right*, **but not the obligation, to buy a specified asset, called** *the underlying asset*, **at a specified amount** K**, called** *exercise value*, **at a specified time** T, **called** *expiration date*, **from the other party.**

Call Option

If the holder decides to *exercise* **his right, then the other party, called** *option writer* **or** *party in short position* **has the** *obligation* **to sell the asset.**

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With this contract, the holder *insures* **himself against the large potential loss, but still has a large potential gain.**

Put Option

A contract between two parties by which one party, called *option holder* **or** *party in the long position*, **has the** *right*, **but not the obligation, to sell a specified asset, called** *the underlying asset*, **at a specified amount** K, **called** *exercise value*, **at a specified time** T, **called** *expiration date*, **from the other party.**

Put Option

If the holder decides to *exercise* **his right, then the other party, called** *option writer* **or** *party in short position* **has the** *obligation* **to buy the asset.**

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Various types of Options include :

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- European Option

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Why would anyone want to be the writer of an option?

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The *right* **possessed by the holder has some value. The holder should pay this value, called** *option premium*, **to enter a option contract.**

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Call option is not exercised if $S_t < K$.

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Call option is not exercised if $S_t < K$**.**

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Why would anyone want to be the writer of an option?

Call option is not exercised if $S_t < K$**.**

Put option is not exercised if $S_t > K$**.**

In case the option is not exercised, the writer will have gained the premium.

Dynamics of asset price

$$dS_t = a(S_t, t)\ dt$$

$a(S_t, t)\ dt$ **drift term**

Dynamics of asset price

$$dS_t = a(S_t, t)\ dt + b(S_t, t)\ dW_t$$

$a(S_t, t)\ dt$ **drift term**

$b(S_t, t)\ dW_t$ **diffusion term**

Standard Wiener Process

A stochastic process $\{W_t : t \geq 0\}$ **with the properties:**

- $W_0 = 0$
- $t \mapsto W_t$ **is a continuous function of** t
- **independent increments property**
- **stationary increments property**
- **For every** $t \geq 0$**:**

$$W_t \sim \text{Normal}(0, t), \quad \text{Var } W_t = t$$

Standard Wiener Process

Very important property: $dW_t^2 = dt$

an abbreviation for $\int_0^T dW_t^2 = T = \int_0^T dt$
which means

$$\lim_{n\to\infty} E\left(\left[\sum_0^{n-1} \left(W_{t_{i+1}} - W_{t_i}\right)^2 - T\right]^2\right) = 0$$

The problem of valuation of call option is determining the option value process C_t, and thereon the option premium C_0.

$$C_t = C(S_t, t)$$
$$= \text{Price of a Call Option at time } t.$$

$$
\begin{aligned}
dC_t \;=\; & \frac{\partial C_t}{\partial t}\; dt + \frac{\partial C_t}{\partial S_t}\; dS_t \\
& + \frac{1}{2}\frac{\partial^2 C_t}{\partial t^2}\; dt^2 + \frac{1}{2}\frac{\partial^2 C_t}{\partial S_t^2} dS_t^2 \\
& + \frac{\partial^2 C_t}{\partial t\;\partial S_t}\; dt\;\, dS_t + \cdots
\end{aligned}
$$

Lognormal Asset Price Model

$$dS_t = \mu S_t \ dt + \sigma S_t \ dW_t$$

Lognormal Asset Price Model

$$dS_t = \mu S_t \; dt + \sigma S_t \; dW_t$$

$$\begin{aligned} dS_t^2 &= (\mu \; S_t dt + \sigma S_t \; dW_t)^2 \\ &= \mu^2 S_t^2 \; dt^2 + \sigma^2 S_t^2 \; dW_t^2 + 2\mu\sigma S_t^2 \; dt \; dW_t \\ &= \sigma^2 S_t^2 dt + o(dt) \end{aligned}$$

Itô's formula for the Option Price

$$\begin{aligned} dC_t &= \frac{\partial C_t}{\partial t}\ dt + \frac{\partial C_t}{\partial S_t}\ dS_t + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} dt \\ &= \left(\frac{\partial C_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} + \mu S_t \frac{\partial C_t}{\partial S_t} \right) dt \\ &\quad + \left(\sigma S_t \frac{\partial C_t}{\partial S_t} \right) dW_t \end{aligned}$$

Now we construct a risk-free portfolio:

1. . . .

2. . . .

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1. One unit of this call option

2. . . .

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2. Δ **units of the underlying asset**

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We select $\triangle$ **such that this portfolio is risk-free.**

Now we construct a risk-free portfolio:

1. **One unit of this call option**
2. **Δ units of the underlying asset,**

We select Δ such that this portfolio is risk-free.

Value of this portfolio:

$$\Pi_t = C_t + \Delta \ S_t$$

$$
\begin{aligned}
d\Pi_t &= dC_t + \Delta \; dS_t \\
&= \left(\frac{\partial C_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} + \mu S_t \frac{\partial C_t}{\partial S_t} + \Delta \mu S_t \right) dt \\
&\quad + \left(\sigma S_t \frac{\partial C_t}{\partial S_t} + \Delta \sigma S_t \right) dW_t
\end{aligned}
$$

$$d\Pi_t = dC_t + \Delta\ dS_t$$

$$= \left(\frac{\partial C_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} + \mu S_t \frac{\partial C_t}{\partial S_t} + \Delta \mu S_t\right) dt$$

$$+ \left(\sigma S_t \frac{\partial C_t}{\partial S_t} + \Delta \sigma S_t \right) dW_t$$

So $\Delta = ?$

$$
\begin{aligned}
d\Pi_t &= dC_t + \Delta \, dS_t \\
&= \left(\frac{\partial C_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} + \mu S_t \frac{\partial C_t}{\partial S_t} + \Delta \mu S_t \right) dt \\
&\quad + \left(\sigma S_t \frac{\partial C_t}{\partial S_t} + \Delta \sigma S_t \right) dW_t
\end{aligned}
$$

So $\Delta = -\dfrac{\partial C_t}{\partial S_t}$

$$
\begin{aligned}
d\Pi_t &= dC_t + \Delta\ dS_t \\
&= \left(\frac{\partial C_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2}\right) dt \\
&= \ \dots
\end{aligned}
$$

$$
\begin{aligned}
d\Pi_t &= dC_t + \Delta \ dS_t \\
&= \left(\frac{\partial C_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} \right) dt \\
&= r\Pi_t \ dt
\end{aligned}
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&= \left(\frac{\partial C_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2}\right) dt \\
&= r\Pi_t \; dt \\
&= r\left(C_t - S_t \frac{\partial C_t}{\partial S_t}\right) \; dt
\end{aligned}
$$

Black-Scholes Formulation

$$\mathcal{L}_{BS} = \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} + r S_t \frac{\partial C}{\partial S_t} - rC = 0$$

Black-Scholes Formulation

$$\begin{cases} \mathcal{L}_{BS} = \dfrac{\partial C}{\partial t} + \dfrac{1}{2}\sigma^2 S_t^2 \dfrac{\partial^2 C}{\partial S_t^2} + r S_t \dfrac{\partial C}{\partial S_t} - rC = 0 \\ \ldots \; final \;\; condition \; \ldots \\ \ldots \; boundary \;\; condition \; \ldots \\ \ldots \; boundary \;\; condition \; \ldots \end{cases}$$

Black-Scholes Formulation

$$\begin{cases} \mathcal{L}_{BS} = \dfrac{\partial C}{\partial t} + \dfrac{1}{2}\sigma^2 S_t^2 \dfrac{\partial^2 C}{\partial S_t^2} + r S_t \dfrac{\partial C}{\partial S_t} - rC = 0 \\ C(S_T, T) = \max(S_T - K, 0) \\ \ldots \ boundary \ condition \ \ldots \\ \ldots \ boundary \ condition \ \ldots \end{cases}$$

Black-Scholes Formulation

$$\begin{cases} \mathcal{L}_{BS} = \dfrac{\partial C}{\partial t} + \dfrac{1}{2}\sigma^2 S_t^2 \dfrac{\partial^2 C}{\partial S_t^2} + r S_t \dfrac{\partial C}{\partial S_t} - rC = 0 \\ C(S_T, T) = \max(S_T - K, 0) \\ C(0, t) = 0 \\ \ldots \textit{ boundary condition } \ldots \end{cases}$$

Black-Scholes Formulation

$$\begin{cases} \mathcal{L}_{BS} = \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} + r S_t \frac{\partial C}{\partial S_t} - rC = 0 \\ C(S_T, T) = \max(S_T - K, 0) \\ C(0, t) = 0 \\ C(S, t) \sim S \text{ as } S \to \infty \end{cases}$$

BS Price of European Call

$$C(S_t, t) = \qquad S_t \, N(d_1) - K \, e^{-r(T-t)} \, N(d_2)$$

$$d_1 := \frac{1}{\sigma \, \sqrt{T-t}} \left[\log\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t) \right]$$

$$d_2 := \frac{1}{\sigma \, \sqrt{T-t}} \left[\log\left(\frac{S_t}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t) \right]$$

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-s^2/2} ds$$

BS Price of European Call

$$C(S_t, t) = e^{-q(T-t)} S_t \, N(d_1) - K \, e^{-r(T-t)} \, N(d_2)$$

$$d_1 := \frac{1}{\sigma \, \sqrt{T-t}} \left[\log\left(\frac{S_t}{K}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)(T-t) \right]$$

$$d_2 := \frac{1}{\sigma \, \sqrt{T-t}} \left[\log\left(\frac{S_t}{K}\right) + \left(r - q - \frac{1}{2}\sigma^2\right)(T-t) \right]$$

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-s^2/2} ds$$

Valuation of American Call

$C_t = C(S_t, t)$

$=$ **Price of American call at time** t

Valuation of American Call

$C_t = C(S_t, t)$

$=$ **Price of American call at time** t

$\geq$ **payoff of European call with expiry** t

Valuation of American Call

$C_t = C(S_t, t)$

$=$ **Price of American call at time** t

$\geq$ max $(S_t - K, 0)$

Valuation of American Call

And:

return from delta-hedged portfolio

$\leq$ **return from risk free bank account**

Valuation of American Call

For European Call: $d\Pi_t = r\Pi_t \, dt$

Valuation of American Call

For European Call: $d\Pi_t = r\Pi_t \, dt$

For American Call: $d\Pi_t \leq r\Pi_t \, dt$

Valuation of American Call

For European Call: $\mathcal{L}_{BS} C_t = 0$

Valuation of American Call

For European Call: $\mathcal{L}_{BS} C_t \; = \; 0$

For American Call: $\mathcal{L}_{BS} C_t \; \leq \; 0$

Valuation of American Call

$$C_t \geq \max\ (S_t - K, 0)$$

$$\mathcal{L}_{BS} C_t \leq 0$$

Valuation of American Call

For every $0 \leq t \leq T$ **there exists a** *free boundary* $S_f(t)$ **such that:**

in case $S_t < S_f(t)$ **we have:**

$$C_t \geq \max(S_t - K, 0) \qquad \mathcal{L}_{BS} C_t = 0$$

Valuation of American Call

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In this case, the optimal policy at time t **is to keep the option.**

Valuation of American Call

For every $0 \leq t \leq T$ **there exists a** *free boundary* $S_f(t)$ **such that:**

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$$C_t = \max(S_t - K, 0) \qquad \mathcal{L}_{BS} C_t \leq 0$$

Valuation of American Call

For every $0 \leq t \leq T$ there exists a *free boundary* $S_f(t)$ **such that:**

in case $S_t > S_f(t)$ we have:

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In this case, the optimal policy at time t is to exercise the option.

Valuation of American Call

Function $t \mapsto S_f(t)$ **is the** *free boundary* **function for this problem.**

Valuation of American Call

Function $t \mapsto S_f(t)$ **is the** *free boundary* **function for this problem.**

As a corollary the above arguments, it has the important property:

$$\frac{\partial C_t}{\partial S_t}\left(\ S_f(t)\ ,\ t\ \right) \ = \ 1$$

Valuation of American Call

Unknowns:

. . .

. . .

Valuation of American Call

Unknowns:

value of the option $C_t(S_t, t)$

. . .

Valuation of American Call

Unknowns:

value of the option $C_t(S_t, t)$

free boundary function $S_f(t)$

Valuation of American Call

So for every $0 \leq t \leq T$**:**

$$[\ C_t - \max(S_t - K, 0)\ ]\ [\ \mathcal{L}_{BS}\ C_t\ ] = 0$$

Valuation of American Call

So for every $0 \leq t \leq T$**:**

$$[\ C_t - \max(S_t - K, 0)\ ]\ \ [\ \mathcal{L}_{BS}\ C_t\ ] = 0$$

Linear Complementarity Formulation

Advantage: ?

Valuation of American Call

So for every $0 \le t \le T$:

$$[\ C_t - \max(S_t - K, 0)\ ]\ [\ \mathcal{L}_{BS}\ C_t\ ] = 0$$

Linear Complementarity Formulation

Advantage: No explicit free boundary

Valuation of American Call

Black-Scholes equation:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} + (r - q) S_t \frac{\partial C}{\partial S_t} - rC = 0$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

Black-Scholes equation:

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Valuation of American Call

Change of variables: $\quad t = T - \frac{1}{\frac{1}{\sigma^2}}\,\tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} + (k'-1)\frac{\partial c}{\partial x} - kc + f(x)$$

$$k := \frac{r}{\frac{1}{2}\sigma^2}, \;\; k' := \frac{r-q}{\frac{1}{2}\sigma^2}, \;\; f(x) := -(k-k')e^x + k$$

Valuation of American Call

Change of variables: $\quad t = T - \dfrac{1}{\frac{1}{\sigma^2}}\ \tau$

$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$

Free boundary in the new coordinates is $x_f(\tau)$ related to $S_f(t)$ via:

$$S_f\left(T - \frac{1}{\frac{1}{\sigma^2}}\ \tau\right) \;=\; K\ \exp\left(x_f(\tau)\right)$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$C_t(S_t, t) \;\geq\; \max\ (S_t - K, 0)$$

. . . .

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}} \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$C_t(S_t, t) \;\geq\; \text{max}\ (S_t - K, 0)$$

$$c(x,\tau) \;\geq\; \text{max}\ (e^x - 1, 0)$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$C\left(S_f(t), t\right) = S_f(t) - K$$

. . . .

Valuation of American Call

Change of variables: $\quad t = T - \dfrac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^{x}, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$C\left(S_f(t), t\right) = S_f(t) - K$$

$$c(\ x_f(\tau), \tau\ ) = 0 \quad \Longrightarrow \quad . \ . \ . \ . \ . \ .$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\,\tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$C\left(S_f(t), t\right) = S_f(t) - K$$

$$c(\ x_f(\tau), \tau\ ) = 0 \quad \Longrightarrow \quad \frac{\partial c}{\partial \tau}(\ x_f(\tau), \tau\ ) = 0$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$\frac{\partial C}{\partial S_t}\Big(S_f(t), t\Big) = 1$$

. . . .

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\,\tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$\frac{\partial C}{\partial S_t}\Big(S_f(t), t\Big) = 1$$

$$\frac{\partial c}{\partial x}\Big(x_f(\tau), \tau\Big) = 0$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} + (k' - 1)\frac{\partial c}{\partial x} - kc + f(x)$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

The only value of $x_f(0^+)$ satisfying $c(\ x_f(0^+), 0^+) = 0$ is the root of f.

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}} \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$f(x) := -(k - k')e^x + k$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$f(x) := -(k - k')e^x + k$$

$$x_f(0^+) \ = \ \log \ \frac{k}{k-k'}$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\ \tau$

$$S = Ke^{x}, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$x_f(0^+) \ = \ \log \ \frac{k}{k - k'}$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}} \tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$x_f(0^+) = \log \frac{r}{q}$$

$$S_f(T^-) = \log \frac{r}{q} K \quad . \; . \; .$$

Valuation of American Call

Change of variables: $t = T - \frac{1}{\frac{1}{\sigma^2}}\,\tau$

$$S = Ke^x, \quad C(S,t) = S - K + Kc(x,\tau)$$

$$x_f(0^+) \;=\; \log\,\frac{r}{q}$$

$$S_f(T^-) \;=\; \log\,\frac{r}{q}\,K \;\xrightarrow{q\to 0}\; +\infty$$

Valuation of American Call

Important Result:

An American option on an asset that pays no dividends is effectively a European option since it is never optimal to exercise. So it has the same value as the corresponding European option.

$$S_f(T^-) = \log \frac{r}{q} K \xrightarrow{q \to 0} +\infty$$

Computational Finance

- . . .
- . . .
- . . .

Computational Finance

- **Deterministic Analysis**

- . . .

- . . .

Computational Finance

- **Deterministic Analysis**
- **Stochastic Analysis**
- **. . .**

Computational Finance

- **Deterministic Analysis**
- **Stochastic Analysis**
- **Numerical Analysis**

Various types of Options include :

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Alternative way to show that American call on an asset that pays no dividends is never optimal to exercise:

Portfolio 1:

- . . .
- . . .

Portfolio 2.:

- . . .

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<u>Portfolio 1</u>:

- Long position in an American call
- . . .

<u>Portfolio 2.</u>:

- . . .

Alternative way to show that American call on an asset that pays no dividends is never optimal to exercise:

Portfolio 1:

- **Long position in an American call**
- **Amount of cash equal to** $e^{-r(T-t)}K$

Portfolio 2.:

- **. . .**

Alternative way to show that American call on an asset that pays no dividends is never optimal to exercise:

<u>Portfolio 1</u>:

- **Long position in an American call**
- **Amount of cash equal to** $e^{-r(T-t)}K$

<u>Portfolio 2.</u>:

- **One unit of the underlying asset**

What is the value of American option on an underlying asset that pays dividends?

. . .

What is the value of American option on an underlying asset that pays dividends?

What is the value of Bermudan option, discretized version of American option?