Risk Management in Financial Markets

Dr. Hassan Nojumi

SOURCES OF RISK

Uncertainty

SOURCES OF RISK

Uncertainty

due to lack of complete information

SOURCES OF RISK

Uncertainty

due to lack of complete information

leads to stochastic behavior,

SOURCES OF RISK

Uncertainty

due to lack of complete information

leads to stochastic behavior,

which is the source of risk.

RISK MANAGEMENT STEPS

RISK MANAGEMENT STEPS

1. **Risk Identification**

RISK MANAGEMENT STEPS

1. Risk Identification
2. Risk Assessment and Measurement

RISK MANAGEMENT STEPS

1. Risk Identification
2. Risk Assessment and Measurement
3. Design of Risk Management System

RISK MANAGEMENT STEPS

1. Risk Identification
2. Risk Assessment and Measurement
3. Design of Risk Management System
4. Implementation

RISK MANAGEMENT STEPS

1. Risk Identification
2. Risk Assessment and Measurement
3. Design of Risk Management System
4. Implementation
5. Maintenance and Review

RISK QUANTIFICATION

RISK QUANTIFICATION

1. Frequency of Events

RISK QUANTIFICATION

1. Frequency of Events
2. Magnitude or Severity of Events

RISK QUANTIFICATION

1. Frequency of Events
2. Magnitude or Severity of Events
3. Available Information

RISK QUANTIFICATION

1. Frequency of Events
2. Magnitude or Severity of Events
3. Available Information
4. Confidence in Available Information

RISK QUANTIFICATION

1. Frequency of Events
2. Magnitude or Severity of Events
3. Available Information
4. Confidence in Available Information
5. Available Knowledge

RISK QUANTIFICATION

1. Frequency of Events
2. Magnitude or Severity of Events
3. Available Information
4. Confidence in Available Information
5. Available Knowledge
6. Available Experience

RISK TYPES

Avoidable Risk

Unavoidable Risk

RISK TYPES

Avoidable Risk

NOT REWARDED

Unavoidable Risk

RISK TYPES

Avoidable Risk
NOT REWARDED

Unavoidable Risk
REWARDED

RISK TYPES

Avoidable Risk — Nonsystematic Risk

NOT REWARDED

Unavoidable Risk

REWARDED

RISK TYPES

Avoidable Risk
NOT REWARDED

Nonsystematic Risk

Unavoidable Risk
REWARDED

RISK TYPES

Avoidable Risk **NOT REWARDED**	Nonsystematic Risk
Unavoidable Risk **REWARDED**	Systematic Risk

RISK TYPES

Avoidable Risk **NOT REWARDED**	Nonsystematic Risk
Unavoidable Risk **REWARDED**	Systematic Risk Systemic Risk

RISK TYPES

- Pure Risk
- Speculative Risk

RISK TYPES

- Objective Risk
- Subjective Risk

RISK TYPES

- Static Risk
- Dynamic Risk

TYPES OF FINANCIAL RISKS

TYPES OF FINANCIAL RISKS

- Credit Risk

TYPES OF FINANCIAL RISKS

- Credit Risk
- Commodity Risk

TYPES OF FINANCIAL RISKS

- Credit Risk
- Commodity Risk
- Interest Rate Risk

TYPES OF FINANCIAL RISKS

- Credit Risk
- Commodity Risk
- Interest Rate Risk
- Foreign Exchange Rate Risk

VALUE AT RISK

At significance level α**, value at risk** VaR **is the value such that**

$$P(\ Maximum\ Loss > VaR\ )\ =\ \alpha$$

VALUE AT RISK

At significance level α, value at risk VaR is the smallest value of x satisfying

$$P(\ Maximum\ Loss > x\ )\ \leq\ \alpha$$

VALUE AT RISK

At significance level α, value at risk VaR is the largest value of x satisfying

$$P(\ Maximum\ Loss < x\ )\ \geq\ 1 - \alpha$$

CAPM

CAPM

CAPITAL

ASSET

PRICING

MODEL

CAPM

Consider a portfolio:

1. θ **relative units of asset** A

2. $1 - \theta$ **relative units of "the market".**

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

$r_A =$ **return on asset** A

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

$r_A =$ **return on asset** A

$r_M =$ **return on the whole market**

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

$r_A =$ **return on asset** A

$r_M =$ **return on the whole market**

$r_P =$ **return on portfolio** $= \theta r_A + (1-\theta) r_M$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A

2. $1-\theta$ **relative units of "the market".**

$\sigma_A =$ **risk of asset** $A := \sqrt{\text{Var } r_A}$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

$\sigma_A =$ **risk of asset** $A := \sqrt{\text{Var } r_A}$

$\sigma_M =$ **risk of the market** $:= \sqrt{\text{Var } r_M}$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

$\sigma_A =$ **risk of asset** $A := \sqrt{\text{Var } r_A}$

$\sigma_M =$ **risk of the market** $:= \sqrt{\text{Var } r_M}$

$\sigma_P =$ **risk of the portfolio** $:= \sqrt{\text{Var } r_P}$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

$$\rho_{A,M} := \text{Corr}\left(r_A, r_M\right)$$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A

2. $1-\theta$ **relative units of "the market".**

$$\rho_{A,M} := \text{Corr}\,(r_A, r_M) = \frac{\text{Cov}\,(r_A, r_M)}{\sigma_A\ \sigma_M}$$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

$$\sigma_P = \sqrt{\text{Var}[\ \theta r_A + (1-\theta) r_M\ ]}$$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A

2. $1-\theta$ **relative units of "the market".**

$$\sigma_P = \sqrt{\text{Var}\left[\ \theta r_A + (1-\theta) r_M\ \right]}$$

$$\sigma_P = \sqrt{\theta^2 \sigma_A^2 + (1-\theta)^2 \sigma_M^2 + 2\theta(1-\theta)\rho_{A,M}\sigma_A\sigma_M}$$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

Sharpe ratio of the portfolio:

$$\frac{\text{Portfolio's Excess Return}}{\text{Portfolio's Risk}}$$

CAPM

Consider a portfolio:

1. θ relative units of asset A
2. $1-\theta$ relative units of "the market".

Sharpe ratio of the portfolio:

$$\frac{r_P - r_f}{\sigma_P}, \qquad r_f = \text{risk-free interest rate}$$

CAPM

Consider a portfolio:

1. θ relative units of asset A
2. $1-\theta$ relative units of "the market".

Obtain θ maximizing the Sharpe ratio:

CAPM

Consider a portfolio:

1. **θ relative units of asset A**
2. **$1-\theta$ relative units of "the market".**

Obtain θ maximizing the Sharpe ratio:

$$\frac{\partial}{\partial\theta}\left(\frac{r_P - r_f}{\sigma_P}\right) = 0$$

CAPM

Consider a portfolio:

1. θ relative units of asset A
2. $1-\theta$ relative units of "the market".

Obtain θ maximizing the Sharpe ratio:

$$\frac{\partial}{\partial\theta}\left(\frac{\theta(r_A - r_f) + (1-\theta)(r_M - r_f)}{\sigma_P} \right) = 0$$

CAPM

Consider a portfolio:

1. θ **relative units of asset** A

2. $1-\theta$ **relative units of "the market".**

CAPM Assumes that in equilibrium condition of the market, the optimal portfolio consists of only the "market".

CAPM

Consider a portfolio:

1. θ **relative units of asset** A
2. $1-\theta$ **relative units of "the market".**

In other words:

$$\frac{\partial}{\partial\theta}\left(\frac{\theta(r_A-r_f)+(1-\theta)(r_M-r_f)}{\sigma_P}\right)\Bigg|_{\theta=0}=0$$

CAPM

Security Market Line (SML) :

$$r_A - r_f \;=\; \beta_{A/M}\left(r_M - r_f\right)$$

$$\beta_{A/M} := \frac{\text{Cov}\,(r_A, r_M)}{\text{Var}\; r_M} = \frac{\text{Cov}\,(r_A, r_M)}{\sigma_M^2}$$

CAPM

Oscillatory behavior through the cases:

$r_A - r_f > \beta_{A/M}\left(r_M - r_f\right)$ **(A attractive)**

$r_A - r_f = \beta_{A/M}\left(r_M - r_f\right)$ **(A neutral)**

$r_A - r_f < \beta_{A/M}\left(r_M - r_f\right)$ **(A non-attractive)**

CAPM

higher beta $\longleftrightarrow$ riskier asset

CAPM

higher beta $\longleftrightarrow$ riskier asset

Examples:

$$\beta_{\mathrm{IBM/S\&P500}} \approx 1.4 \qquad (1995 - 2002)$$

CAPM

higher beta $\longleftrightarrow$ riskier asset

Examples:

$$\beta_{\text{IBM/S\&P500}} \approx 1.4 \qquad (1995 - 2002)$$

$$\beta_{\text{SONY/TOPIX}} \approx 1.45 \quad (1995 - 2002)$$

PORTFOLIO OPTIMIZATION

With market having n products, each portfolio can be characterized be a vector

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \cdot \\ \theta_n \end{pmatrix} \qquad \theta_k = \begin{array}{c} \text{Relative share of} \\ \text{the } k\text{th product} \\ \text{in the portfolio} \end{array}$$

PORTFOLIO OPTIMIZATION

With market having n products, each portfolio can be characterized be a vector

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \cdot \\ \theta_n \end{pmatrix}$$

$$0 \leq \theta_k \leq 1$$

$$\sum_{k=1}^{n} \theta_k = 1$$

PORTFOLIO OPTIMIZATION

Return of the portfolio $= r_P = \sum_{k=1}^{n} \theta_k r_k$

Risk of the portfolio

$$\sigma_P = \sqrt{\text{Var } r_P}$$

$$= \sqrt{\theta' C \theta}$$

$C :=$ Covaiance Matrix $= \left[\ Cov\left(r_i, r_j\right)\ \right]_{n \times n}$

PORTFOLIO OPTIMIZATION

Unconditional minimization of portfolio risk:

$$\begin{cases} \text{Minimize } \theta' \; C \; \theta \\ \\ \text{Subject to } \theta' \; u = 1 \end{cases} \qquad r = \begin{pmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ \cdot \\ r_n \end{pmatrix} \; u = \begin{pmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{pmatrix}$$

PORTFOLIO OPTIMIZATION

Solution with method of Lagrange multiplier

$$L := \theta' C \theta + \lambda(\theta' u - 1)$$

$$\left\{ \begin{array}{l} \dfrac{\partial L}{\partial \theta} = 0 \\ \\ \dfrac{\partial L}{\partial \lambda} = 0 \end{array} \right. \Rightarrow \; \theta_{\text{opt}} = \frac{1}{u' C^{-1} u} \; C^{-1} u$$

PORTFOLIO OPTIMIZATION

Return of the optimal portfolio

$$r_P = \theta'_{\text{opt}} \; r$$

Risk of the optimal portfolio

$$\sigma_P = \sqrt{\theta'_{\text{opt}} \; C \; \theta_{\text{opt}}}$$

PORTFOLIO OPTIMIZATION

With A Target Return α

$$\begin{cases} \text{Minimize } \theta' \; C \; \theta \\ \\ \text{Subject to} \begin{cases} \theta' \; r = \alpha \\ \\ \theta' \; u = 1 \end{cases} \end{cases}$$

PORTFOLIO OPTIMIZATION

Solution with method of Lagrange multiplier

$$L := \theta' C \theta + \lambda_1(\theta' r - \alpha) + \lambda_2(\theta' u - 1)$$

$$\frac{\partial L}{\partial \theta} = 0 \qquad \frac{\partial L}{\partial \lambda_1} = 0 \qquad \frac{\partial L}{\partial \lambda_2} = 0$$

PORTFOLIO OPTIMIZATION

$$\theta_{\text{opt}} = x \ C^{-1} r + y \ C^{-1} u$$

with x **and** y **solutions of**

$$\begin{cases} (r' \ C^{-1} \ r) \ x + (r' \ C^{-1} \ u) \ y = \alpha \\ (u' \ C^{-1} \ r) \ x + (u' \ C^{-1} \ u) \ y = 1 \end{cases}$$

PORTFOLIO OPTIMIZATION

Return of the optimal portfolio

$$r_P = \theta'_{\text{opt}}\ r$$

Risk of the optimal portfolio

$$\sigma_P = \sqrt{\theta'_{\text{opt}}\ C\ \theta_{\text{opt}}}$$

PORTFOLIO OPTIMIZATION

With A Target Risk ξ

$$\begin{cases} \text{Maximize } \theta' \; r \\ \\ \text{Subject to} \begin{cases} \theta' \; C \; \theta = \xi^2 \\ \\ \theta' \; u = 1 \end{cases} \end{cases}$$

PORTFOLIO OPTIMIZATION

Solution with method of Lagrange multiplier

$$L := \theta' r + \lambda_1(\theta' C\theta - \xi^2) + \lambda_2(\theta' u - 1)$$

$$\frac{\partial L}{\partial \theta} = 0 \qquad \frac{\partial L}{\partial \lambda_1} = 0 \qquad \frac{\partial L}{\partial \lambda_2} = 0$$

PORTFOLIO OPTIMIZATION

$$a = (u' \, C^{-1} \, u) - \xi^2 (u' \, C^{-1} \, u)^2$$

$$b = 2 \, (u' \, C^{-1} \, r) \left(\xi^2 (u' \, C^{-1} \, u) - 1 \right)$$

$$c = (r' \, C^{-1} \, r) - \xi^2 (u' \, C^{-1} \, r)^2$$

PORTFOLIO OPTIMIZATION

$$\theta_{\text{opt}} = \frac{1}{x}\, C^{-1} (y\ u + r)$$

with x **and** y **solutions of**

$$a y^2 - b y + c = 0$$

$$x = (u'\ C^{-1}\ u)\ y + (u'\ C^{-1}\ r)$$

PORTFOLIO OPTIMIZATION

Return of the optimal portfolio

$$r_P = \theta'_{\text{opt}} \ r$$

Risk of the optimal portfolio

$$\sigma_P = \sqrt{\theta'_{\text{opt}} \ C \ \theta_{\text{opt}}}$$

PORTFOLIO OPTIMIZATION

Dual Problems:

- Optimization With A Target Return
- Optimization With A Target Risk

PORTFOLIO OPTIMIZATION

Dual Problems:

- Optimization With A Target Return
- Optimization With A Target Risk

DUALITY OF THE SOLUTIONS

LEVY PROCESSES

LEVY PROCESSES

- **Starting from zero**

LEVY PROCESSES

- **Starting from zero**

- **Independent Increments Property**

LEVY PROCESSES

- **Starting from zero**
- **Independent Increments Property**
- **Stationary Increments Property**

LEVY PROCESSES

- **Starting from zero**
- **Independent Increments Property**
- **Stationary Increments Property**
- **Stochastic Continuity**

LEVY PROCESSES

Characteristic Function with Levy triplet (γ, σ, ν)

$$\Psi_{X_t}(u) = t \ \psi_{X_t}(u)$$

$$\psi_{X_t}(u) \ = \ iu\gamma - \frac{1}{2}\sigma^2 u^2$$

$$+ \int_{\mathbb{R}} \left(e^{iux} - 1 - iux 1_{|x| \leq 1} \right) \nu(dx)$$

LEVY PROCESSES

$$c_1(X_t) = \Psi_{X_t}^{(1)}(0) = \mathsf{E}X_t$$

LEVY PROCESSES

Mean Function

$$\mathsf{E}X_t \;=\; t\;\left[\;\gamma + \int_{|x|>1} x\nu(dx)\;\right]$$

LEVY PROCESSES

Mean Function

$$\mathsf{E}X_t = t \left[\gamma + \int_{|x|>1} x\nu(dx) \right]$$

Mean increases with time linearly.

LEVY PROCESSES

$$c_2(X_t) = \Psi_{X_t}^{(2)}(0) = \mathrm{Var} X_t$$

LEVY PROCESSES

Variance Function

$$\mathrm{Var} X_t \; = \; t \; \left[\; \sigma^2 + \int_{\mathbb{R}} x^2 \nu(dx) \; \right]$$

LEVY PROCESSES

Variance Function

$$\text{Var} X_t \ = \ t \ \left[\ \sigma^2 + \int_{\mathbb{R}} x^2 \nu(dx) \ \right]$$

Variance increases with time linearly.

LEVY PROCESSES

$$
\begin{aligned}
c_3(X_t) &= \Psi_{X_t}^{(3)}(0) \\
&= (\mathrm{Var} X_t)^{3/2} \ \mathrm{Skw} X_t
\end{aligned}
$$

LEVY PROCESSES

Skewness Function

$$\text{Skw}X_t = \frac{1}{\sqrt{t}} \frac{\int_{\mathbb{R}} x^3 \nu(dx)}{\left[\sigma^2 + \int_{\mathbb{R}} x^2 \nu(dx) \right]^{3/2}}$$

LEVY PROCESSES

Skewness Function

$$\text{Skw}X_t \;=\; \frac{1}{\sqrt{t}}\,\frac{\int_{\mathbb{R}} x^3 \nu(dx)}{\left[\ \sigma^2 + \int_{\mathbb{R}} x^2 \nu(dx)\ \right]^{3/2}}$$

Skewness decreases to zero with time, asymptotically like $1/\sqrt{t}$.

LEVY PROCESSES

$$c_4(X_t) = \Psi_{X_t}^{(4)}(0)$$

$$= (\mathrm{Var}\,X_t)^2\ (\ \mathrm{Kur}\,X_t - 3\ )$$

LEVY PROCESSES

Kurtosis Function

$$\text{Kur} X_t = 3 + \frac{1}{t} \frac{\int_{\mathbb{R}} x^4 \nu(dx)}{\left[\sigma^2 + \int_{\mathbb{R}} x^2 \nu(dx) \right]^2}$$

LEVY PROCESSES

Kurtosis Function

$$\text{Kur}\, X_t \;=\; 3 + \frac{1}{t}\, \frac{\int_{\mathbb{R}} x^4 \nu(dx)}{\left[\; \sigma^2 + \int_{\mathbb{R}} x^2 \nu(dx) \;\right]^2}$$

Kurtosis decreases to its *normal* **value 3 with time, asymptotically like** $1/t$**.**

LEVY PROCESSES

Kurtosis Function

$$\text{Kur} X_t \;=\; 3 + \frac{1}{t} \, \frac{\int_{\mathbb{R}} x^4 \nu(dx)}{\left[\, \sigma^2 + \int_{\mathbb{R}} x^2 \nu(dx) \, \right]^2}$$

All non-Brownian Levy Processes ($\nu \neq 0$) are leptokurtic.

BENEFITS OF RISK PRESENCE

BENEFITS OF RISK PRESENCE

- **Makes life more rewarding**

BENEFITS OF RISK PRESENCE

- **Makes life more rewarding**
- **Makes life more exciting**

BENEFITS OF RISK PRESENCE

- Makes life more rewarding
- Makes life more exciting

So Risk avoidance is not an effective and efficient risk management strategy, and in most cases not possible.

BENEFITS OF RISK PRESENCE

Risk must be understood and then handled skillfully in the direction of making profits and making progress.