Solution method

1- Converting mathematical program with equilibrium constraints (MPEC) to MILP

In order to convert the proposed bi-level problem into a MILP which guaranties obtaining the global optimum solution, three steps should be followed:

i) Substituting the lower level problem with its KKT optimality conditions

In the proposed bi-level model, the lower level problem is linear since TVPP bidding price is specified for ISO. Therefore, the lower level problem is convex and substituting it with its KKT optimality conditions causes the upper and lower level problems to solve simultaneously. In order to substitute the lower level problem by its KKT conditions, at first Lagrange equation is built as (a1).

$$\begin{aligned} L = & \Big(\sum_{i\in I, f\in F, t\in T} \pi^{ifts_R s_D s_C} q_{DA}^{ifts_R s_D s_C} - \sum_{d\in D, f\in F, t\in T} \pi^{dfts_R s_D s_C} l_{DA}^{dfts_R s_D s_C} \Big) \\ & + \sum_{b_T\in B_T} \lambda_{DA}^{b_T ts_R s_D s_C} \Big(\sum_{d\in DB_T, f\in F} l_{DA}^{dfts_R s_D s_C} - \sum_{i\in IB_T, f\in F} q_{DA}^{ifts_R s_D s_C} + \sum_{b'_T\in B_T} B_X(b_T, b'_T) \delta_T^{b'_T ts_R s_D s_C} \Big) \\ & + \sum_{i\in I, f\in F} \mu_{min}^{ifts_R s_D s_C} (-q_{DA}^{ifts_R s_D s_C}) + \sum_{i\in I, f\in F} \mu_{max}^{ifts_R s_D s_C} (q_{DA}^{ifts_R s_D s_C} - Q^{ifts_R s_D s_C}) \\ & + \sum_{d\in D, f\in F} \mu_{min}^{dfts_R s_D s_C} (-l_{DA}^{dfts_R s_D s_C}) + \sum_{d\in D, f\in F} \mu_{max}^{dfts_R s_D s_C} (l_{DA}^{dfts_R s_D s_C} - L^{dfts_R s_D s_C}) \\ & + \sum_{l_T\in L_T} \gamma_{min}^{l_T ts_R s_D s_C} (-S_{max\_T}^{l_T} - FP_T^{l_T ts_R s_D s_C}) + \sum_{l_T\in L_T} \gamma_{max}^{l_T ts_R s_D s_C} (FP_T^{l_T ts_R s_D s_C} - S_{max\_T}^{l_T}) \\ & + \sum_{b_T\in B_T} \zeta_{min}^{b_T ts_R s_D s_C} (-\pi - \delta_T^{b_T ts_R s_D s_C}) + \sum_{b_T\in B_T} \zeta_{max}^{b_T ts_R s_D s_C} (\delta_T^{b_T ts_R s_D s_C} - \pi) \\ & + \chi^{ts_R s_D s_C} (-\delta_T^{1ts_R s_D s_C}) \end{aligned} \tag{a1}$$

Based on Lagrange equation, KKT optimality conditions are attained for $\forall i \in IB_T, d \in DB_T, f \in F, t \in T, s_R \in S_R, s_D \in S_D, s_C \in S_C$ as follows.

$$\pi^{ifts_R s_D s_C} - \lambda_{DA}^{b_T ts_R s_D s_C} - \mu_{min}^{ifts_R s_D s_C} + \mu_{max}^{ifts_R s_D s_C} = 0 \tag{a2}$$

$$-\pi^{dfts_R s_D s_C} + \lambda_{DA}^{b_T ts_R s_D s_C} - \mu_{min}^{dfts_R s_D s_C} + \mu_{max}^{dfts_R s_D s_C} = 0 \tag{a3}$$

$$\sum_{i\in IB_T, f\in F} q_{DA}^{ifts_R s_D s_C} - \sum_{d\in DB_T, f\in F} l_{DA}^{dfts_R s_D s_C} = \sum_{b'_T\in B_T} B_X(b_T, b'_T) \delta_T^{b'_T ts_R s_D s_C} \tag{a4}$$

$$\sum_{b'_T\in B_T} \lambda_{DA}^{b'_T ts_R s_D s_C} B_X(b'_T, b_T) - \sum_{l_T\in L_T} \gamma_{min}^{l_T ts_R s_D s_C} F(l_T, b_T) + \sum_{l_T\in L_T} \gamma_{max}^{l_T ts_R s_D s_C} F(l_T, b_T) - \zeta_{min}^{b_T ts_R s_D s_C} + \zeta_{max}^{b_T ts_R s_D s_C} = 0, \quad b_T \neq 1 \tag{a5}$$

$$q_{DA}^{ifts_R s_D s_C} \geq 0 \quad \perp \quad \mu_{min}^{ifts_R s_D s_C} \geq 0 \tag{a6}$$

$$Q^{ifts_R s_D s_C} - q^{ifts_R s_D s_C} \geq 0 \quad \perp \quad \mu_{max}^{ifts_R s_D s_C} \geq 0 \tag{a7}$$

$$l_{DA}^{dfts_R s_D s_C} \geq 0 \quad \perp \quad \mu_{min}^{dfts_R s_D s_C} \geq 0 \tag{a8}$$

$$L^{dfts_R s_D s_C} - l_{DA}^{dfts_R s_D s_C} \geq 0 \quad \perp \quad \mu_{max}^{dfts_R s_D s_C} \geq 0 \tag{a9}$$

$$S_{max\_T}^{l_T} + FP_T^{l_T ts_R s_D s_C} \geq 0 \quad \perp \quad \gamma_{min}^{l_T ts_R s_D s_C} \geq 0 \tag{a10}$$

$$S_{max\_T}^{l_T} - FP_T^{l_T ts_R s_D s_C} \geq 0 \quad \perp \quad \gamma_{max}^{l_T ts_R s_D s_C} \geq 0 \tag{a11}$$

$$\pi + \delta_T^{b_T ts_R s_D s_C} \geq 0 \quad \perp \quad \zeta_{min}^{b_T ts_R s_D s_C} \geq 0 \tag{a12}$$

$$\pi - \delta_T^{b_T ts_R s_D s_C} \geq 0 \quad \perp \quad \zeta_{max}^{b_T ts_R s_D s_C} \geq 0 \tag{a13}$$

$$\delta_T^{1ts_Rs_Ds_C} = 0 \tag{a14}$$

It should be noted that matrix $F(l_T, b_T)$ in equation (a5) represents the relation between the power flow of lines and voltage angel of buses through the equations of $FP_T^{l_T ts_Rs_Ds_C} = F(l_T, b_T) \times \delta_T^{b_T ts_Rs_Ds_C}$.

However, the outcome formulation is an MPEC which suffers nonlinearity of upper level objective function -(1) in associated paper- and nonlinearity of KKT complementary conditions (a6)-(a14). Therefore, parts *ii* and *iii* utilize linearization of these nonlinearities to convert MPEC to MILP [A].

ii) Linearizing the upper level objective function

The upper level problem objective function nonlinearity is due to the terms $\lambda_{DA}^{(b_T=b_T^{TVPP})ts_Rs_D}.q_{DA}^{ifts_Rs_D}$ and $\lambda_{DA}^{(b_T=b_T^{TVPP})ts_Rs_D}.q_{DA}^{ifts_Rs_D}$, each one consists of multiplication of two variables. Using KKT optimality conditions and strong duality theory, linearized form of nonlinear parts of objective function is represented in (a15).

$$\begin{aligned}
&\sum_{i\in I_s, f\in F} \lambda_{DA}^{(b_T=b_T^{TVPP})ts_Rs_D}\,.\,q_{DA}^{ifts_Rs_D} - \sum_{d\in D_s, f\in F} \lambda_{DA}^{(b_T=b_T^{TVPP})ts_Rs_D}\,.\,l_{DA}^{dfts_Rs_D} \\
&= -\sum_{i\in I_R, f\in F} \pi^{ifts_R}\,.\,q_{DA}^{ifts_Rs_D} - \sum_{i\in I_R, f\in F} \mu_{\max}^{ifts_Rs_D}\,.\,Q^{ifts_R} \\
&\quad + \sum_{d\in D_R, f\in F} \pi^{dfts_R}\,.\,l_{DA}^{dfts_Rs_D} - \sum_{d\in D_R, f\in F} \mu_{\max}^{dfts_Rs_D}\,.\,L^{ifts_R} \\
&\quad - \sum_{l_T\in L_T} \gamma_{\min}^{l_T ts_Rs_D} S_{\max_T}^{l_T} - \sum_{l_T\in L_T} \gamma_{\max}^{l_T ts_Rs_D} S_{\max_T}^{l_T} - \sum_{b_T\in B_T} \zeta_{\min}^{b_T ts_Rs_D}\pi \\
&\quad - \sum_{b_T\in B_T} \zeta_{\max}^{b_T ts_Rs_D}\pi, \qquad \forall t\in T,\, s_R\in S_R, s_D\in S_D
\end{aligned} \tag{a15}$$

iii) Linearizing the KKT complementary conditions

Relations (a6)-(a14) represent KKT complementary conditions which are nonlinear. In order to linearizing these relations, binary variable series *w* and sufficient large amount series *M* are defined. For example, linearization of KKT complementary condition (a7) is done through following four relations where $w_{max}^{ifts_Rs_Ds_C}$, M^p and M^{pu} are binary variable for upper limit of $q_{DA}^{ifts_Rs_D}$ and sufficient large numbers, respectively [A]. Other KKT complementary conditions can be linearized in the same way.

$$Q^{ifts_Rs_Ds_C} - q_{DA}^{ifts_Rs_Ds_C} \geq 0 \tag{a16}$$

$$\mu_{max}^{ifts_Rs_Ds_C} \geq 0 \tag{a17}$$

$$Q^{ifts_Rs_Ds_C} - q_{DA}^{ifts_Rs_Ds_C} \leq (1 - w_{max}^{ifts_Rs_Ds_C})M^p \tag{a18}$$

$$\mu_{max}^{ifts_Rs_Ds_C} \leq w_{max}^{ifts_Rs_Ds_C} M^{pu} \tag{a19}$$

[A] C. Ruiz and A. J. Conejo, “Pool strategy of a producer with endogenous formation of locational marginal prices,” *IEEE Trans. Power Syst.*, vol. 24, no. 4, pp. 1855–1866, Nov. 2009.