Condensed Matter II

Take home exam (due 5 Tir 84)

- Consider the following extended Hubbard model in one dimension:

$$H = \sum_{k\sigma} \varepsilon_{k\sigma} c^{+}_{k\sigma} c_{k\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + \frac{1}{2} J \sum_{<ij>} S^{z}_{i} S^{z}_{j}$$

where the energy dispersion is that of a simple chain of hopping t: $\varepsilon_k = 2t\cos(ka)$

Write down the many body ground state wavefunction Ψ and total energy $E[\Psi]$ (within the mean field approximation, and in the second quantized form) in the following cases: **paramagnetic, ferromagnetic, antiferromagnetic and charge density wave phase of period 2a**. For the last 3 cases, you need to define an order parameter (OP) and write Ψ and $E[\Psi]$ down as a function of the OP.
Find the stability of these phases in the limiting cases (U and J much larger or smaller than t and U<<J or J<<U etc…) and plot the phase diagram in the corresponding boundaries of the (U/t,J/t) plane. **Discuss the results.**

- Consider the two dimensional graphite lattice, with one π orbital per atom and one electron per orbital (half-filled) within the nearest neighbor tight-binding approximation. Plot using Mathematica the energy surfaces $E_{\pm}(k_x, k_y)$ and the real and imaginary parts of the Lindhard function $\chi^0(\vec{k}, \omega)$ for $\vec{k}$ along the directions (1,0) and $(\sqrt{3},1)$.
Using the random phase approximation, calculate and plot the real and imaginary parts of the dielectric function (take $v(q) = 2\pi / q$ as the Fourier transform of the Coulomb interaction in 2D).
Do the same for a "doped" graphite where the Fermi level is $\mu = 0.05\ t$, and **discuss the results and the difference between them**.