The Inefficiency of Equilibria in a Network Creation Game with Packet Forwarding

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Abstract. We study a novel variation of network creation games in which the players (vertices) form a graph by building undirected edges to each other with the goal of reducing their costs of using the network. The model we introduce assumes that a minimal set of nodes with high reachability from others are handed the responsibility of routing the traffic alongside the network. For this purpose, we suggest that a minimum dominating set (MDS) of the graph would be a reasonable choice as the intermediate nodes, thus the players in one such set would incur an extra cost for forwarding.

We study the Nash equilibrium in this model assuming an extra cost of β is evenly shared among all the nodes in a MDS. We prove upper bounds on the *price of anarchy*, the worst-case ratio of the social cost of Nash equilibria of the network to that of socially optimum solution, for different values of β. Specifically, we show this inefficiency is modest for $\beta = n$ since the price of anarchy is $O(n^{1/3})$. We also prove a tight upper bound of $\Theta(n)$ for $\beta = n^2$, and also give some upper bounds when β takes a value between n and n^2.

Key words: Network Creation, Nash Equilibrium, Price of Anarchy, Packet Forwarding

1 Introduction

In recent years, a majority of literature has been dedicated to *network design* for its prominent position in computer science and operations research. The goal in this line of research is to take the role of a central authority and find a minimum-cost structure for the network that satisfies some specific criteria. As in the case of real computer networks such az the Internet, it is usually the interaction of self-interested independent agents that creates the network, rather than a central authority. Therefore, a novel game theoretic approach has also been proposed [6, 1, 4] to the traditional network design in which it is assumed that each agent has a certain objective of her own, and attempts to optimize the cost she incurs in the network, regardless of what extra cost her actions may impose to others.

The new approached first proposed by Fabrikant, Luthram Maneva, Papadimitriou, and Shenker takes into account both the creation and the usage cost of the network, in that every player's goal is to minimize the sum of her shortest-path distances to other nodes plus the price she pays for building links (edges) to other players. We propose a variation based on this model which emphasizes on the usage cost. The intuition behind this new model is that although the players would incur a cost for link creation, once the edges are laid down, each player is constantly charged for using the network. Therefore, in a long run, it is the usage portion of the cost that matters most. Furthermore, there are so many real-world applications in which the cost of network creation is negligible. For instance, in the case of social networks such as the web graph, the cost of link creation for a player is merely including a hyperlink in her web page to any other node she desires. In such applications, it is reasonable to assume the player only wish to minimize their usage costs, the cost of routing their traffic to intended destinations.

Many different routing and broadcasting protocols have been devised in different types of networks. A critical challenge for designing any such protocol is a concept known as the *broadcast storm*. Roughly speaking, a broadcast storm is a state in which the network is overwhelmed with many consecutive broadcasts or multicasts. To overcome this dilemma, many routing and broadcasting protocols deploy a common strategy: they first determine a *forwarding set* for the whole network, which consists of a set of highly reachable nodes of the graph. These nodes are then given the responsibility of broadcasting or multicasting the messages across the network. This way, the number of messages required for a piece of information to be broadcasted reduces dramatically. In a vast number of networks, such as the wireless ad hoc networks, *dominating sets* are used as a virtual backbone for broadcasting and/or multicasting the messages [9]. That is, a dominating set of the graph is chosen as the forwarding set, and once a node wishes to send a message to everyone else, instead of broadcasting it, she simply routes it to the forwarding set, an then the forwarding set (which is a dominating set) sends a copy of the original message to every other node.

In the model we introduce in this paper, we consider the selfishness of the nodes in the context of packet forwarding. In other words, we assume that in presence of a protocol which uses a dominating set as the forwarding set for the network, every player wishes to reduce her cost on the network, and therefore wishes to escape the responsibility of forwarding, while trying to minimize her shortest-path distances to others. Thus, each player faces a trade-off between being reachable from all other node players, and the excessive load of work for forwarding the packets of other players. To the best of our knowledge, this is the first time in the literature that such an approach to network creation games is taken.

In this paper, we consider the pure Nash equilibria of the game; the states of the game in which none of the players can reduce their cost on the network by unilateral changes of their strategies. We study the *price of anarchy*, a measure of inefficiency of Nash equilibria, in our model of network creation. The price of anarchy is defined to be the worst-case ration of the social cost of a Nash equilibrium to that of a socially optimum solution, taken over all Nash equilibria of the game. More precisely, we assume that the players in a *minimum dominating set (MDS)* of the network are charged an extra cost of β. Although this assumption is somewhat unrealistic in that finding a MDS of the graph is known to be NP-hard, it simplifies the model so that some nice results on the price of anarchy can be proved. We hope the simple model we introduce here can bring an insight into more complicated process of topology formation in real networks such as *Mobile Ad hoc Networks (MANETs)*, and might lead to better theoretic results under more realistic assumptions.

Our Results. The model of network creation based on the concept of dominating set is considered in this paper for the first time. We study the price of anarchy in our model for different values of β, the extra cost players incur for being placed in a MDS. We first prove that for $\beta = n$, the price of anarchy in this model is always $O(d)$, where d is the diameter of the network. The range of $\beta \approx n$ perhaps is interesting because it corresponds to the cases where the extra cost put on the nodes in a MDS is roughly in balance with her *covering degree*, i.e., the (average) number of nodes outside a MDS whose traffic is assigned to a single member of a minimum dominating set for forwarding. We show that the inefficiency of Nash equilibria is modest in this range of values for β by showing that the diameter of the stable graph[3] is $O(n^{1/3})$ in this case, and therefore the price of anarchy is in fact $O(n^{1/3})$. We also prove the upper bound of $O(n)$ for $\beta = n^2$, and show its tightness by bringing examples of the stable graph which actually embrace the price of anarchy of n. For the values of β between n and n^2, we prove the upper bound of $O(max\left\{n^{\epsilon}, n^{(1+\epsilon)/3}\right\})$ for $1 \leq \epsilon \leq 2$.

Related Work. The first model of network creation was proposed by Fabrikant et al. in [6]. They studied the pure Nash equilibria of their model alongside the inefficiency of the equilibria, and proved the price of anarchy in their model was $O(\sqrt{n})$. As their experiments had suggested, they also postulated a *tree conjecture*, stating that for adequately large edge prices, every Nash equilibrium of the game was a tree. The game theoretic approach to network design has also been taken in economic literature for analyzing social networks, where instead of using Nash equilibrium as the notion of stability, the concept of *pairwise stability* is used. Pairwise stability applies to the settings in which for building an edge, the consent of both its incident vertices is required. This concept has been studied by Corbo and Parkes in [3]. The price of anarchy of network creation games has been studied more in depth in [1] where Albers et al. improved the upper bounds of [6] to $O(n^{1/3})$, while disproving the *tree conjecture* first stated in [6]. They also studied other variation of the original network creation games, and proved bounds on the price of anarchy for them. Demaine et al., improved both the results of [1] and [3]. The also proved the first $O(n^{\epsilon})$ for general range of values for the edge price, and study the Nash equilibria in some variations of net-

[3] A graph which is a Nash equilibrium of the game

work formation in their own rights. Finally, the idea of using a dominating set of the graph as a forwarding set has been considered in many routing/broadcasting protocols. For instances of this, see [2, 5, 7–9].

2 The Model

In the model of network creation we propose here, the nodes of a graph are the selfish players, who establish undirected links (edges) to others while seeking to minimize the cost they incur on the network. Their objective is a contradicting one: on one hand, they wish to minimize their shortest-path distances to all other nodes, which once done, leads to making them highly reachable nodes of the network, and on the other hand, they have a tendency to escape the responsibility of forwarding the packets of other nodes by trying not to be placed in a minimum dominating set of the graph.

Formally, given is a set of vertices, $V = \{1, 2, \dots, n\}$, which correspond to the players of the network. Each player v, chooses a subset of $V \backslash v$ to whom she established undirected edges. This subset, is actually the strategy of player v in the network, and is denoted by S_v. We also show the joint strategy vector for all players of the network by $\overrightarrow{S} = (S_1, S_2, \dots, S_n)$, in which S_i denotes the strategy of player i for $i = 1, 2, \dots, n$. We denote the graph generated by such a joint strategy by G_S. In fact, we make the use of the definition of [1] for the graph, and let $G_S = (V, E)$, in which $E = \bigcup_{i=1}^{n} \bigcup_{j \in S_i} (i, j)$. We assume that the players do not incur any cost for building edges, instead they are charged and extra amount of β if they happen to be placed in (one of) the minimum dominating set(s) of the graph. This extra cost is evenly shared among all the players in that MDS. The cost for a player in a MDS of the graph is the sum of her shortest-path distances to all other nodes, plus β over the size of the minimum dominating set. More precisely, if player $v \in V$ is contained in a minimum dominating set of the graph, then the cost she incurs would be

$$cost_S(v) = \sum_{i=1}^{n} dist_S(v, i) + \frac{\beta}{s},$$

where $cost_S(v)$ is the cost player v incurs under the strategy S, $dist_S(v, i)$ is the length of the shortest path between the nodes v and i in G_S, and s is the size of the minimum dominating set of the graph. For a node u that is not contained inside any MDS of the graph, the cost would be merely the sum of her distances to all other players. The social cost of the network is defined to be the sum of the costs of all the players:

$$SC(G_S) = \sum_{i=1}^{n} cost_S(i),$$

where $SC(G_S)$ denote the social cost of the graph G_S.

A joint strategy $\overrightarrow{S}$ is said to be *stable* if, for any other joint strategy $\overrightarrow{T}$ which differs from $\overrightarrow{S}$ only in the ith element, we have that $cost_S(i) < cost_T(i)$.

Likewise, a graph G_s is said to be stable if, the strategy S that generates it is stable.

Finally, the price of anarchy in this model is defined to be the worst-case ratio of the social cost of a Nash equilibrium of the game to that of the socially optimum solution. Let $\overrightarrow{NE}$ be a stable joint strategy, and $\overrightarrow{OPT}$ be a joint strategy that minimizes the social cost. The price of anarchy is then the maximum of $SC(G_{NE})/SC(G_{OPT})$, taken over all stable strategies $\overrightarrow{NE}$. Before we can talk about the price of anarchy in this model, we need to show that, in the settings we wish to discuss here, Nash equilibria actually exist.

Proposition 1. *The aforementioned model of network creation always admits a stable joint strategy $\overrightarrow{S}$ for input values of $n \geq 2$ and $n \leq \beta \leq n^2$.*

Proof. We prove this proposition by constructing a stable graph for $n \leq \beta \leq n^2$. First suppose n, the number of vertices, to be even. Consider the complete graph K_n. We know that this graph has a perfect matching, say M. Let $\overrightarrow{S}$ be a joint strategy that generates $K_n \backslash M$. We claim that G_S is stable. No one node is placed in a MDS by herself, and every pair of nodes of the graph form a minimum dominating set. If the node $v \in V$ lays down a new edge, then she would make a new MDS of size 1, and would incur an extra cost of $\beta - \frac{\beta}{2} \geq \frac{n}{2} \geq 1$, while her distance cost is reduced by exactly 1. If v decides to remove some of her other edges, she would increase her distance to others, while she still remains in a MDS of size 2. The only way for her to end up in a dominating set of size larger than 2 is by removing all the edges she had laid down, hence incurring a cost of ∞. So no player of the network would be willing to change her strategy. If n is odd, consider a $K_{n-1} \backslash M'$ where M' is a perfect matching of K_{n-1}. We add an extra node v^* to this structure and have all other nodes lay down edges to her. We also let $\overrightarrow{S'}$ be the joint strategy that generates such a graph. An instance of this construction is shown in Fig. 2 for $n = 7$. It is clear that v^* cannot change her strategy. Now, consider the nodes other than v^*. As the MDS of the graph only contains v^*, they do not wish to remove any of their edges. Also, they do not wish to create any new edge either, because this change of strategy would make any of them a minimum dominating set by herself, resulting in an extra cost of $\beta \geq n > 1$. So, $G_{S'}$ is stable.

3 The Price of Anarchy for $\beta = n$

In this section, we prove some upper bounds on the price of anarchy of our model for the cases where $\beta = n$. First, we need a definition.

Definition 1. *For a node v in a MDS of the graph G_S, the* ***covering degree*** *is defined to be the number of edges incident to v, whose other incident node is outside that MDS. For the cases where v is placed in more than one such MDS, the average number of such edges is considered instead.*

As mentioned before, $\beta \approx n$ is an interesting range of values for β since the MDS-penalty imposed on each network is roughly in the balance with the average covering degree. More specifically, if s, the number of nodes in a MDS is significantly small, s times the average covering degree of the members of MDS would be approximately n, the number of nodes. If we assume that each node in a MDS only incurs an extra cost proportional to her covering degree, then this extra cost equals $\frac{n}{s} = \frac{\beta}{s}$. Note that $\beta \approx n$ is the least amount of penalty put on any node inside a MDS for in reality, the nodes in a forwarding set might have to face an extra amount of traffic proportional to total number of the nodes. As we are interested in studying the price of anarchy in the new model, we take the somewhat normal approach of relating the social cost to the network diameter as in [1, 4, 6]. More precisely, we first show that the price of anarchy is upper-bounded by the diameter of the network.

Theorem 1. *Suppose that $\overrightarrow{S}$ is a joint strategy (which might be stable), and that G_S, is the graph generated by $\overrightarrow{S}$. If $\beta = n$, it is always the case that $\frac{SC(G_S)}{SC(G_{OPT})} \leq d+1$, where $\overrightarrow{OPT}$ is the socially optimum joint strategy, and d is the diameter of G_S.*

Proof. The social cost of the graph G_S is defined as

$$SC(G_S) = \sum_{i=1}^{n} cost_S = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} dist_S(i,j) + t_i \cdot \frac{\beta}{s} \right),$$

where t_i is an integer that indicates whether i belongs to a MDS by taking the value of 1, or otherwise by taking the values of 0. In the worst case, distance portion of the social cost would be $n^2 d$ as there are n^2 pairs of nodes, at most at the distance of d from each other. The worst case for the MDS-penalty portion of the social cost occurs when each node is placed in a MDS with the cardinality of 1. Therefore we have that

$$SC(G_S) \leq n^2 d + n\beta = n^2(d+1).$$

As for the socially optimum joint strategy (hereafter called the *optimum strategy*), $n(n-1)$ is a lower bound on the distance portion of the social cost sine there are $n(n-1)$ pairs of nodes which are at least at distance 1 from each other. The MDS-penalty portion of the social cost can be lower bounded as well. This portion of the costs embraces its minimum value if all the nodes are placed in a minimum dominating set. Therefore

$$SC(G_{OPT}) \geq n(n-1) + n \cdot \frac{\beta}{n} = n^2.$$

Thus, as for the price of anarchy

$$PoA \leq \frac{SC(G_S)}{SC(G_{OPT})} \leq \frac{n^2(d+1)}{n^2} = d+1.$$

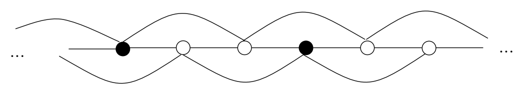
To prove the upper bound of $O(n^{1/3})$ on the price of anarchy for $\beta = n$, we first need these two lemmas.

Lemma 1. *[10] Given a graph H, let I be an independent set in H^2. Then, $|I| \leq |MDS(H)|$, where $|MDS(H)|$ is the size of minimum dominating set of H.*[4]

Lemma 2. *Given a graph G with diameter d and the size of minimum dominating set equal to s, we have that $s \geq \frac{d}{3}$.*

Proof. consider the path with length d in G. It would take the form of Fig. 1 in G^2. It is easy to see that the marked nodes in this figure form an independent set for G^2. Based on Lemma 1, $|I| \leq s$, where I denotes an independent set in G^2. Meanwhile, as the marked nodes are selected one out of every three, we have that $|I| \geq \frac{d}{3}$. Therefore, $s \geq \frac{d}{3}$.

Fig. 1. The path with maximum length in G^2.



Theorem 2. *Given a stable joint strategy $\overrightarrow{S}$ and $\beta = n$, $d \leq (12n)^{1/3}$, where d is the diameter of G_S.*

Proof. Consider two nodes u and v whose distance to each other is exactly d. We consider the following two cases:

1. **At least one of the two nodes is inside a MDS.** Without loss of generality, assume that v belongs to a MDS. If v creates an edge to u, she would decrease her distance cost by $(d-1)+(d-3)+\ldots+3+1 = (\frac{d}{2})^2$ if d is even, or $(d-1)+(d-3)+\ldots+4+2 \leq (\frac{d+1}{2})^2$ if d is odd. Anyway, node v saves at least $\frac{d^2}{4}$ in distance by building this edge. Since v has not built this edge, the MDS-penalty must have increased in case this edge has been bought. As v is places inside a MDS, by adding this edge, a node of this MDS covering u exclusively (or maybe u itself) would have been sent out of the MDS, resulting in reducing the size of the minimum dominating set of the graph by 1. So, the node v would incur an extra cost of $\frac{\beta}{s-1} - \frac{\beta}{s} = \frac{ns-ns+n}{s(s-1)} = \frac{n}{s(s-1)}$. This extra cost must have been larger than the benefit of laying down the (v, u) edge for the node v, so it is the case that

$$\frac{d^2}{4} < \frac{n}{s(s-1)} < \frac{n}{(s-1)^2},$$

[4] By the square of a graph, H^2, we mean a graph H' with the same vertex set, whose edge set would be the union of the original edge set and new edges between vetices in H which are at distance 2 from each other.

By the Lemma 2,

$$\frac{d^2}{4} < \frac{9n}{(d-3)^2},$$

Therefore,

$$(d-3)^4 < 36n \Rightarrow d < (36n)^{1/4} + 3 < (12n)^{1/3}.$$

2. **Neither of v and u is inside a MDS.** In this case, the benefit of adding the (v,u)-edge would be similarly at least $\frac{d^2}{4}$ to her. Since v has not created such an edge, it must have been the case that by adding this edge to S_v, she must have been placed inside a MDS. We claim the size of this MDS is s. For the sake of contradiction, suppose that the size if this MDS is less than or equal to $s-1$. As the newly added edge could at most cover one node, v must have been in a dominating set of at most size s, which contradicts the assumption of this case. So if v builds this new edge, she would be places in a MDS of size s, resulting in an extra cost of $\frac{\beta}{s} = \frac{n}{s}$. As she has not done so, we conclude that

$$\frac{d^2}{4} < \frac{n}{s}.$$

By Lemma 2,

$$\frac{d^2}{4} < \frac{3n}{d} \Rightarrow d < (12n)^{1/3}.$$

In any case, the claim of the theorem is true.

Corollary 1. *Given $\beta = n$, the price of anarchy in this model of network creation is $O(n^{1/3})$.*

Proof. Using Theorems 1 and 2, the claim is obvious.

4 The Price of Anarchy for $\beta = n^2$

In this section, we prove tight upper bounds on the price of anarchy of our model for $\beta = n^2$. The penalty of n^2 for every node inside a MDS was the worst case of broadcast storm that we could come up with, and happens when every node of the graph has a piece of information to disseminate to all. For this case, we prove the following theorem by construction.

Theorem 3. *For $\beta = n^2$, the price of anarchy in the proposed model of network creation is $\Theta(n)$.*

Proof. We first prove an upper bound on the price of anarchy. Quite similar to the proof of Theorem 1, the worst case for the distance portion of the social cost would be $n^2 d$, and the worst case for the MDS-penalty portion is $n\beta$. As in the optimal strategy these portions have a lower bound of n^2 and β respectively, by putting these two portion together we conclude

$$
\begin{aligned}
PoA &\leq \frac{SC(G_S)}{SC(G_{OPT})} \\
&\leq \frac{n^2 d + n\beta}{n^2 + \beta} \\
&= \frac{n^2 d + n^3}{2n^2} \\
&\leq \frac{2n^3}{2n^2} = n.
\end{aligned}
$$

Now, we show that there are networks in which, the price of anarchy would actually reach as high as this upper bound. The graphs we construct here are different for odd and even values of n. Therefore, we consider the two following cases.

1. **If n is even.** Let $\overrightarrow{S}$ be a joint strategy that generates $K_n \backslash M$, where M is a perfect matching K_n. According to Proposition 1, G_S is stable. The social cost of G_S is

$$SC(G_S) = n \cdot \frac{n^2}{2} + n(n+1)$$

as every node would incur a cost of $\frac{\beta}{s} = \frac{n^2}{2}$ as MDS-penalty cost, and $n+1$ as the shortest-path distances to others. Now, we give an upper bound on the optimum social cost using a specific network structure. Suppose that we add $\frac{n}{2}$ new vertices and edges to the complete graph $K_{\frac{n}{2}}$ such that, every vertex of $K_{\frac{n}{2}}$ is connected to exactly one new vertex via one new edge. Assume the $\overrightarrow{T}$ is a joint strategy that generates such structure. The minimum dominating set for G_T would include $\frac{n}{2}$ nodes since each of the $\frac{n}{2}$ nodes with degree 1 should be covered with at least one distinct vertex. So the MDS-penalty portion of it social cost would be $n \cdot \frac{2n^2}{n} = 2n^2$. It is easy to show that the distance portion of its social cost is $\frac{2n^2}{2} - 3n$. As the social cost of G_T is greater than or equal to that of the optimal strategy, we can conclude $PoA \geq \frac{SC(G_S)}{SC(G_{OPT})} \geq \frac{SC(G_S)}{SC(G_T)}$. Thus,

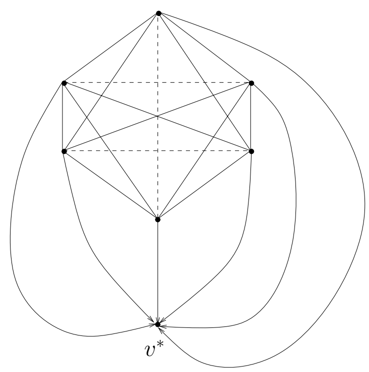
$$
\begin{aligned}
PoA &\geq \frac{SC(G_S)}{SC(G_T)} \\
&= \frac{n \cdot \frac{n^2}{2} + n(n+1)}{2n^2 + \frac{2n^2}{2} - 3n} \\
&\geq \frac{\frac{n^3}{2}}{\frac{6n^2}{2}},
\end{aligned}
$$

Therefore,

$$PoA \geq \frac{n}{6} \Rightarrow PoA = \Theta(n).$$

2. **If n is odd.** In this case, let $\overrightarrow{S'}$ be the same as in Proposition 1. Again, due to this proposition, $G_{S'}$ is stable. The MDS-penalty portion if the social cost of $G_{S'}$ is n^2. The distance of V^* to others is 1. As for other nodes, they have a distance of 1 to $n-2$ other nodes, and a distance of 2 to one node (their adjacent node in M'). So the distance portion of the social cost would be $n-1+(n-1)\cdot(n-2+2)=(n-1)(n+1)$. For this case, we let T' be a joint strategy that generates $K_{\frac{n+1}{2}}$ with $\frac{n-1}{2}$ new edges and vertices, and compare $SC(G_{S'})$ against $SC(G_{T'})$. We omit the calculations as they are quite the same as the case of even values of n, and jump straightly to the main result that in this case, $PoA = \Theta(n)$ as well.

Fig. 2. Tightness of the PoA for $\beta = n^2$ and odd values of n.



Thus, the proof is complete.

5 The Price of Anarchy for $n < \beta < n^2$

In the previous two sections, we studies the price of anarchy for both extreme values of β; the best case when the members of a forwarding set would incur the minimum cost of n, and the worst case, where they incur a maximum cost of n^2. In this section, we prove upper bounds on the price of anarchy for other members of this spectrum.

Theorem 4. *Given $n < \beta < n^2$, the price of anarchy in the proposed model is $O(max\left\{n^{\epsilon}, n^{(1+\epsilon)/3}\right\})$, where $0 < \epsilon < 1$, and $\beta = n^{1+\epsilon}$.*

Proof. The proof is similar to that of Theorem 2. Define $\beta = n^{1+\epsilon}$, where $0 < \epsilon < 1$. We first prove the following lemma.

Lemma 3. *Given a stable joint strategy $\overrightarrow{S}$ and $n < \beta < n^2$, we have that $d \leq 12^{1/3} \cdot n^{\frac{1+\epsilon}{3}}$, where d is the diameter of G_S, and $\beta = n^{1+\epsilon}$.*

Proof. Here again, we assume that v and u are the nodes with distance d from each other. We only prove the case where neither of them coincides with a MDS. The other case can be proven easily. With the same analysis as in Theorem 2, we can conclude v has not established a link to u because the fierce penalty of forwarding. Therefore

$$\frac{d^2}{4} \leq \frac{n^{1+\epsilon}}{s} \leq \frac{3n^{1+\epsilon}}{d},$$

thus

$$d \leq (12)^{1/3} n^{\frac{1+\epsilon}{3}}.$$

This, completes the proof.

Now that the lemma is proved, we can proceed with the price of anarchy.

$$\begin{aligned} PoA \ &< \frac{SC(G_S)}{SC(G_{OPT})} \leq \frac{n\beta + n^2 d}{\beta + n^2} = \frac{n^{2+\epsilon} + n^{2+\frac{1+\epsilon}{3}}}{n^{1+\epsilon} + n^2} \\ &= \frac{n^{1+\epsilon} + n^{\frac{4+\epsilon}{3}}}{n^{\epsilon} + n} \leq \frac{n^{1+\epsilon} + n^{\frac{4+\epsilon}{3}}}{n} \leq n^{\epsilon} + n^{\frac{1+\epsilon}{3}}. \end{aligned}$$

Therefore, the proof is complete.

Corollary 2. *Given $\beta = n^{1+\epsilon}$, the price of anarchy is of $O(n^{\epsilon})$ for $1 > \epsilon \geq 1/2$.*

Corollary 3. *Given $\beta = n^{1+\epsilon}$, the price of anarchy is of $O(n^{\frac{1+\epsilon}{3}})$ for $0 \leq \epsilon < 1/2$.*

6 Conclusion

In this paper, we proposed a new model of network creation game for the networks with packet forwarding. We incorporated the concept of minimum dominating sets in our model, and assumed the nodes selected as members of the forwarding set incur an extra cost of β. The range of values of β considered in this paper was $[n, n^2]$, for their resemblance to best and worst case of penalty charged against the members of the forwarding set in reality. We proved upper bound of $O(n^{1/3})$ for $\beta = n$, tight upper bound of $\Theta(n)$ for $\beta = n^2$, and upper bound of $O(max\left\{n^{\epsilon}, n^{(1+\epsilon)/3}\right\})$ with $0 < \epsilon < 1$ for the rest of the spectrum.

Perhaps the most important drawback to our model is that penalty is assigned to all nodes in any arbitrary MDS, and a node not contained in any MDS is free of any extra charges. In fact, both these assumption are inaccurate in real settings. The penalty model should be refined in future studies, via assigning the packet forwarding job to a connected MDS instead of any minimum dominating set for instance. Although we did not find the ranges of $\beta < n$ and $\beta > n^2$ applicable to networks with packet forwarding, they are still theoretically interesting, and also challenging. As en extension to this paper, we suggest finding upper bounds on the price of anarchy of these ranges. We also assumed, for the sake of simplicity, that the nodes are provided with global information about the topology of the network, and that they can determine whether they belong to a MDS or not in a reasonable amount of time, which is somewhat unrealistic. In future studies, the model can be extended to cases that the routing protocol actually chooses an approximate connected dominating set based on local information, and the nodes intend to mislead the protocol so as to minimize their costs on the network.

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