Assignment Number 4

1) In axisymmetric stress analysis, See Figure 1, the strain energy is given by the following expression:

$$U=\int_0^{2\pi}\int_0^{L}\int_{R1}^{R2}\frac{E(1-\upsilon)}{(1+\upsilon)(1-2\upsilon)}[(\frac{\partial w}{\partial z})^2+(\frac{\partial u}{\partial r})^2+(\frac{u}{r})^2+\frac{2\upsilon}{1-\upsilon}(\frac{\partial w}{\partial z}\frac{\partial u}{\partial r}+\frac{u}{r}\frac{\partial w}{\partial z}+\frac{u}{r}\frac{\partial u}{\partial r})+\frac{(1-2\upsilon)}{2(1+\upsilon)}(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r})^2]rdrd\theta dz$$

Also note that for an axisymmetric problem:

$$\varepsilon_{zz}=\frac{\partial w}{\partial z}\qquad \varepsilon_{rr}=\frac{\partial u}{\partial r}\qquad \varepsilon_{\theta}=\frac{u}{r}\qquad \gamma_{rz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}$$

And the elasticity matrix is given by: (E=modulus of elasticity, ν=poisson's ratio)

$$[D]=\frac{E(1-\upsilon)}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix}1 & \frac{\upsilon}{(1-\upsilon)} & \frac{\upsilon}{(1-\upsilon)} & 0\\ \frac{\upsilon}{(1-\upsilon)} & 1 & \frac{\upsilon}{(1-\upsilon)} & 0\\ \frac{\upsilon}{(1-\upsilon)} & \frac{\upsilon}{(1-\upsilon)} & 1 & 0\\ 0 & 0 & 0 & \frac{(1-2\upsilon)}{2(1-\upsilon)}\end{bmatrix}$$

Answer the following questions for a finite element formulation.

a) How many rigid body modes are expected?
b) What continuity requirement will be sufficient for convergence in the energy sense?
c) Choose an appropriate isoparametric finite element of your choice to model the thick-walled cylinder with curved walls as shown in Figure 1.
d) Briefly describe the steps involved in the derivation of the stiffness matrix for isoparametric finite element of your choice.

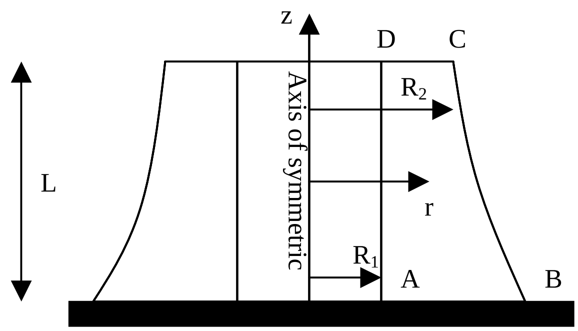


Figure 1. Solid of revolution, Area ABCD revolved about z

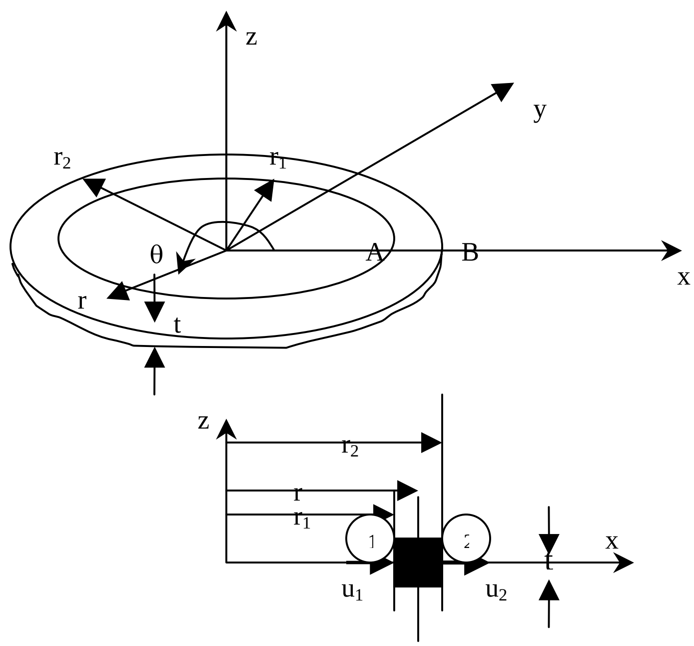
2) The total potential energy expression for a axisymmetric angular ring given by:

$$\pi = \frac{Et}{2(1-\nu^2)}\int_0^{2\pi}\int_{r_1}^{r_2}[(\frac{du}{dr})^2 + 2\nu(\frac{du}{dr})(\frac{u}{r}) + (\frac{u}{r})^2]rdrd\theta - t\int_0^{2\pi}[(r_1 p_i u(r_1) - r_2 p_0 u(r_2)]d\theta$$

Where E= modulus of elasticity, v=poisson's ratio, t=thickness, r_1=inner radius, r_2 =outer radius, u=displacement along r, p_i=pressure on the ring inside and p_0=pressure on the ring out side. See Figure 2.

Answer the following questions for a finite element formulation.

a) How many rigid body modes are expected?
b) What continuity of displacement u is required for convergence in the energy sense?
c) Consider the ring finite element shown in figure 2 with two degrees of freedom for u(r), derive the stiffness matrix for the ring element.
d) For pressure p_i constant on the inside and p_0=0, derive the load vector.
e) Do you expect any problems with stiffness matrix in part d) when r_1=0



3) Consider a plane stress problem with normal and tangential springs on part of the boundary as shown in Figure 4. You have decided to use 8-node quadrilateral isoparametric element. If the spring stiffness K_N and K_T vary linearly, i.e.:

$$K_N(t) = \frac{1}{2}[K_{Ni}(1-t) + K_{Nj}(1+t)]$$

$$K_T(t) = \frac{1}{2}[K_{Ti}(1-t) + K_{Tj}(1+t)]$$

a) Derive the contributions to the stiffness matrix of a boundary element due to these springs. Show your steps clearly in terms of matrices and line integrals.

b) Is it possible to use two point numerical integration? If so, would this integral be exact?

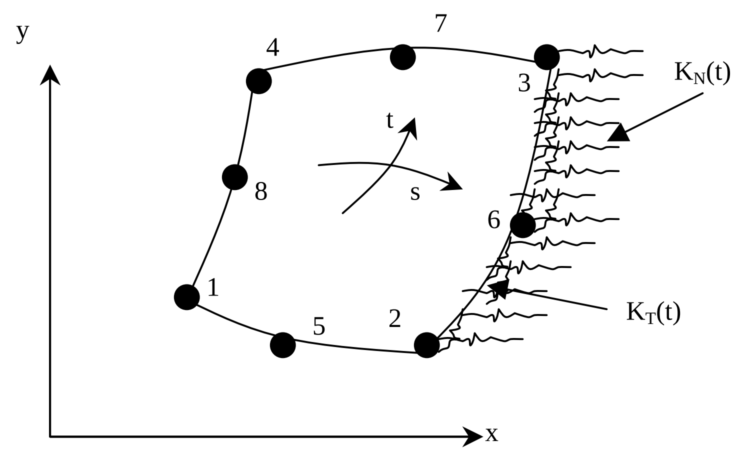


Figure 3

4) You have decided to use 8-node quadrilateral isoparametric element for a plane stress problem (Figure3). A uniform distributed load with b_x intensity is applied at edge2-3.

a) Determine the nodal forces of nodes 2, 3 and 6.

b) Describe the procedure of obtaining the stiffness matrix for this element.

c) Is it possible to use two point numerical integration? If so, would this integral be exact?

d) Calculate K55 (a member of the stiffness matrix) using Gause Numerical integration (n=2).