

به نام خدا

تمرین سری ۶ ریاضی مهندسی

۱) هر یک از مساله های ناهمگن زیر را حل کنید :

1-1)

$$\begin{aligned} u_{tt} - 9u_{xx} &= x, \quad 0 < x < \pi, t > 0 \\ u(x, 0) &= 2 \quad u_t(x, 0) = x + 1 \\ u_x(0, t) &= t \quad u_x(\pi, t) = 2t \end{aligned}$$

1-2)

$$\begin{aligned} u_{xx} + u_{yy} &= x + 2y, \quad 0 < x < \pi, 0 < y < \pi \\ u(x, 0) &= x \quad u(x, \pi) = 2 \quad 0 \leq x \leq \pi \\ u(0, y) &= y \quad u(\pi, y) = \cos(y) \quad 0 \leq y \leq \pi \end{aligned}$$

1-3)

$$\begin{aligned} u_{xx} &= u_t - x, \\ u(0, t) &= 0 \quad u(1, t) = \frac{5}{6} \\ u(x, 0) &= -\frac{1}{6}x^3 \end{aligned}$$

1-4)

$$\begin{aligned} 9u_{xx} &= u_t + xt, \quad 0 < x < 1 \quad t > 0 \\ u(0, t) &= t \quad u(1, t) = 1 \\ u(x, 0) &= 2x \end{aligned}$$

1-5)

$$\begin{aligned} u_{xx} &= u_{tt} - e^{-t} \sin(3x), \quad 0 < x < \pi \quad t > 0 \\ u(0, t) &= 0 \quad u(\pi, t) = 1 \\ u(x, 0) &= \frac{x}{\pi} \quad u_t(x, 0) = 0 \end{aligned}$$

۲) مسایل پواسن زیر را حل کنید :

2-1)

$$\begin{aligned} u_{xx} + u_{yy} &= x + 1, \quad 0 < x < a, 0 < y < b \\ u(0, y) &= u_y(x, b) = u_y(x, 0) = 0 \\ u(a, y) &= \alpha(y) \end{aligned}$$

2-2)

$$\begin{aligned} u_{xx} + u_{yy} &= xy, \quad 0 < x < \pi, 0 < y < 1 \\ u(0, y) &= u(\pi, y) = 0 \\ u(x, 0) &= 0 \quad u(x, 1) = 0 \end{aligned}$$

2-3)

$$\begin{aligned} \nabla^2 u &= 1 \quad 0 < x < \pi, 0 < y < \pi \\ u(0, y) &= 0 \quad u_x(\pi, y) = 0 \quad 0 \leq y \leq \pi \\ u_y(x, \pi) + hu(x, \pi) &= x^2 \quad u_y(x, 0) = 0 \end{aligned}$$