Governance of Social Welfare in Networked Markets

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Abstract—This paper aims to investigate how a central authority (e.g. a government) can increase social welfare in a network of markets and firms. In these networks, modeled using a bipartite graph, firms compete with each other *à la* Cournot. Each firm can supply homogeneous goods in markets which it has access to. The central authority may take different policies for its aim. In this paper, we assume that the government has a budget by which it can supply some goods and inject them into various markets. We discuss how the central authority can best allocate its budget for the distribution of goods to maximize social welfare. We show that the solution is highly dependent on the structure of the network. Then, using the network's structural features, we present a heuristic algorithm for our target problem. Finally, we compare the performance of our algorithm with other heuristics with experimentation on real datasets.

Index Terms—Networked Markets, Cournot Competition, Social Welfare, Governance, Optimization.

1 Introduction

Cournot Competition in the single-market setting has been vastly studied in the literature. For instance, refer to [1], [2], [3], [4]. In this oligopolistic model, each firm decides the quantity of the homogenous good they are willing to supply into the market. Then, according to the inverse demand function, the market-clearing price is determined based on the aggregate supply in the market. However, with the emergence of diverse and complicated economic scenarios, single-market models are inadequate for studying reality. In many settings, firms can compete in different markets, whether or not the good is identical in those markets. Typically, this situation is modeled using a bipartite graph in which one side of nodes represents firms and the other side depicts various markets. Each market is characterized by an inverse demand function. Multi-market competition is found abundantly in industries such as natural gas, water, electricity, airlines, cement, healthcare, etc; see [5], [6], [7], [8].

One question that arises naturally in the presence of strategic agents is the means by which it is possible to raise welfare measures like the ones used in [9], [10]. One such measure is social welfare, which captures the aggregate well-being in the environment, as discussed in [11], [12]. In this case, it is typically the government that seeks higher social welfare. While there have been many studies on interactions among firms and equilibria in networked markets (See e.g. [13], [14], [15]), to the best of our knowledge, there is little work on how to govern and control social welfare in networked markets. The prevalence of networked markets in real-life experiences motivates us to study social welfare in this model. Our paper takes one step forward towards this objective.

We consider a limited intervention budget for the government in the pursuit of higher social welfare. Therefore, we assume that the government is able to have a small amount of supply into every market. This small intervention setting enables us to use some techniques for the estimation of social welfare in terms of government's supplies. The simple structure of the approximation leads to a strategy for the government. However, it is good to note that the actions taken by the government, are specified by the structure of the network.

This problem arises from a real-world scenario in every country (especially in third-world countries) controlling their inflation by directly setting prices. Such intervention is absolutely incorrect and conflicts with the free-market economy. The framework offered in this paper gives a natural way for governing the markets and controlling the prices (maximizing social welfare) without violating the free-market economy, which concurrently makes firms happy.

In this paper, first, we mathematically model the target problem by using, which is typically an optimization problem. Then we propose a heuristic for solving this optimization problem and then compare the performance of our heuristic with other heuristics one may propose for solving this problem. The experiments are evaluated on a real-world dataset gathered by the research team and are one of the contributions of this paper.

1.1 Related Works

Our work is in essence related to several categories of papers. First, there have been many attempts in studying the strategic behavior of firms and equilibria in the competition; for example, refer to [16], [14], [17], [18]. One such study, which has been our first step-stone, is done by Bimpikis el al. [14], where they present a ”characterization of the production quantities at the unique equilibrium of the resulting game for any given network”, in terms of supply paths in the network. Furthermore, they introduce the *price-impact* matrix which enables them to explore the effect of changes in network structure on firms' profits and consumer welfare. These changes include entering of a firm in a new market and also merging of two firms. Their results challenge the standard beliefs in Cournot oligopoly that more competition necessarily leads to higher welfare. Relatedly, [15] turn their focus on finding algorithms that compute pure Nash equilibria in Cournot competitions in networks. Moreover, [13] study the impact of monopolies on social welfare in a certain model of Bertrand network competition.

Another group of studies relevant to ours are the ones that analyze interactions in the networks and their impact on aggregate measures, e.g. [19], [20], [21], [22]. Most relatedly, [20] have proposed a framework that paves the way to examine equilibria in such interdependent agents setup and discover the influence of small microeconomic shocks on the economy's aggregate performance. Acting as our main inspiring study, they use Taylor expansion to acquire insights on the impact of shocks. Their examinations yield different results about the economy's *ex ante* aggregate performance in the case of linear and non-linear worlds. To understand how the structure of the network shapes the economy's performance, they demonstrate that the *Bonacich centrality* measure can capture this effect when the nature of the interactions is linear. Such analysis is prevalent in economics. For example, the general notion of production networks demands consideration of dependencies and network effects [23].

Lastly, following the connections found by [14] and [24] between *Bonacich centrality* and network effects in network Cournot competition, studies about controlling centrality measures in networks can be considered related to ours. Generally, with an established relationship between centrality measures and social welfare in our setting, one might use these methods to change the structure of the competition such that social welfare increases. As such, [25], [26], [27] model the centrality control problem as an optimization problem and presents an algorithm to solve it.

2 Problem Formulation

Consider a network game $\mathcal{G}$ which consists of n firms $F = \{f_1, \cdots, f_n\}$ and m markets $M = \{m_1, \cdots, m_m\}$ in which the firms compete. Each firm has access to a set of markets, meaning that it can supply the good only in those certain markets. For firm f_i, let M_i be the set of those markets.

Similarly, let F_j denote the set of firms that have access to market m_j. The amount of good that firm f_i supplies in m_j is denoted by q_{ij}. Moreover, firm f_i would incur the production cost $C_i(q)$ (q is the vector of all q_{ij}s). Following the framework used by [14], we consider the inverse demand functions of the markets as affine. More specifically, the price of the good in market m_j, which we denote by $P_j(q)$, is governed by the relation

$$P_j(q) = \alpha_j - \beta_j \sum_{f_i \in F_j} q_{ij}. \tag{1}$$

Additionally, we assume

$$C_i(q) = c_i \cdot (\sum_{m_j \in M_i} q_{ij})^2. \tag{2}$$

For the sake of simplicity of our formulas, we suppose that for all $m_i \in M$, $\alpha_i = \alpha$ and $\beta_i = \beta$ and for all $f_j \in F$, $c_j = c$, where $\alpha, \beta, c > 0$. We model this economy with a bipartite graph $G = (V, E)$. An example of this graph can be seen in Figure 1.

It is essential to note that the structure of the cost functions C_is is what determines whether the analysis of different markets can be done separately. If the cost functions C_is are additive in terms of q_{ij}s, then there is no need for studying these interdependencies. However, with general cost functions, decision in different markets are coupled. Our considered quadratic form brings out the role of the underlying network structure. It is worth mentioning that while our approach is generally applicable on many other parametric assumptions, this form makes the calculations easier to follow.

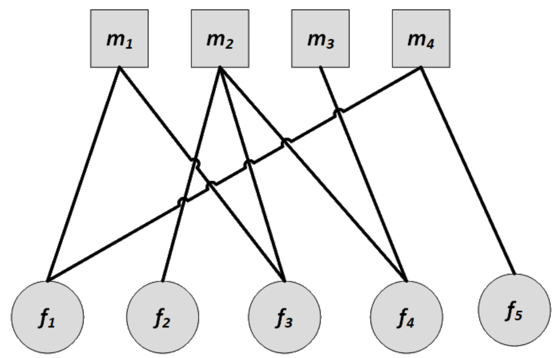

Fig. 1: A Graph for a Networked Market

Briefly speaking, we can consider firm f_i's profit as a combination of the aforementioned components:

$$\pi_i(q) = \sum_{m_k \in M_i} q_{ik} \cdot P_k(q) - C_i(q) \tag{3}$$

Given a set of network graph G, each f_i in competition with other firms solves the following optimization problems for computing its best response.

$$\begin{array}{lll} \underset{q_i}{\text{maximize}} & \pi_i\left(\boldsymbol{q}_i, \boldsymbol{q}_{-\boldsymbol{i}}\right) & \\ \text{subject to} & q_{ik} \geq 0 & \text{for } m_k \in M_i \\ & q_{ik} = 0 & \text{for } m_k \notin M_i \end{array} \tag{4}$$

where q_i and q_{-i} denotes the vector of production quantities of f_i and its competitors, respectively.

In [14], Ehsani et al. have focused on the equilibrium analysis of this model and their main result about existence and characterization of the unique Nash equilibrium of this game is adopted as the foundation of this research.

Theorem 1. *[Adopted from [14]] The unique Nash equilibrium of the game is given by*

$$q^* = [I + \gamma W]^{-1} \gamma \bar{\alpha}, \tag{5}$$

where $\gamma = \frac{1}{2(c+\beta)}$, $\bar{\alpha}$ is a $|E| \times 1$ vector such that for every edge $(i, k) \in E$ we have $\bar{\alpha}_{ik} = \alpha_k$ and W is an $|E| \times |E|$ matrix whose entries are

$$w_{i_1 k_1, i_2 k_2} = \begin{cases} 2c \text{ if } i_1 = i_2, k_1 \neq k_2 \\ \beta \text{ if } i_1 \neq i_2, k_1 = k_2 \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

The matrix $[I + \gamma W]^{-1}$ is called the *Leontief inverse*. We assume that the matrix $[I + \gamma W]$ is invertible. For a given economy, this assumption is shown to be true by [28],

[29]. In this paper, we propose and formalize the problem of governance of the aforementioned networked markets (networked Cournot competition) with the objective of maximizing the social welfare. Social welfare (SW) in Cournot competitions is defined as the sum of consumer surplus (CS) and firms' profits. The consumer surplus in the Nash equilibrium is computed by the following formula (see [30], [14]):

$$CS = \sum_{m_k \in M} \frac{(\alpha_k - P_k(q^*))^2}{2\beta_k}. \tag{7}$$

Therefore the social welfare's formula is:

$$\begin{aligned} SW &= \textstyle\sum_{f_i \in F} \pi_i(q^*) + CS \\ &= \textstyle\sum_{f_i \in F} [\sum_{m_k \in M_i} q^*_{ik} \cdot P_k(q^*) - C_i(q^*)] \\ &\quad + \textstyle\sum_{m_k \in M} \frac{1}{2\beta_k} (\alpha_k - P_k(q^*))^2 . \end{aligned} \tag{8}$$

Now, assume that an additional firm node is added to the network, such that it is owned by the government and has access to all the markets. This node's target is to maximize the weighted social welfare. It is provided with a budget B and can use this budget to provide some shocks to each market. In this paper, each shock to market m_k is defined as provisioning some quantity of the good ($\epsilon_k \leq q^t$) to this market alongside the competing firms. q^t is a threshold that is forced by external entities such as law or social pressure, and is the maximum amount that the government can intervene in a market. Firms compete until they reach an equilibrium. The equilibrium can be computed by the following theorem.

Theorem 2. *The unique Nash equilibrium of the game $\mathcal{G}$ in the presence of shocks $\{\epsilon_i\}_{i=1}^m$ is given by*

$$q^* = [I + \gamma W]^{-1} \gamma(\bar{\alpha} - \overline{\beta\epsilon}), \tag{9}$$

where W and $\overline{\alpha}$ are defined as in Theorem 1 and $\overline{\beta\epsilon}$ is a $|E| \times 1$ vector such that for every edge $(i,k) \in E$ we have $\overline{\beta\epsilon}_{ik} = \beta_k \cdot \epsilon_k$

Proof. In the presence of shocks we can rewrite the firms' utility functions:

$$\begin{aligned} \pi_i(q,\epsilon) &= \textstyle\sum_{m_k \in M_i} q_{ik} \cdot P_k(q) - C_i(q) \\ &= \textstyle\sum_{m_k \in M_i} q_{ik} [\alpha_k - \beta_k (\sum_{f_j \in F_k} q_{jk} + \epsilon_k)] \\ &\quad -c_i \cdot (\textstyle\sum_{m_k \in M_i} q_{ik})^2 \\ &= \textstyle\sum_{m_k \in M_i} q_{ik} [(\alpha_k - \beta_k \epsilon_k) + \beta_k \sum_{f_j \in F_k} q_{jk}] \\ &\quad -c_i \cdot (\textstyle\sum_{m_k \in M_i} q_{ik})^2 \end{aligned} . \tag{10}$$

Assume that we have a new game $\mathcal{G}'$ where everything is the same as the previous setting ($\mathcal{G}$) except that the values of α_ks have changed to $\alpha_k - \beta_k \epsilon_k$. By Theorem 1, we will have the following formula for the Nash equilibrium point:

$$q^* = [I + \gamma W]^{-1} \gamma(\bar{\alpha} - \overline{\beta\epsilon}) \tag{11}$$

Equilibria in $\mathcal{G}$ in the presence of shocks are equal to equilibria of $\mathcal{G}'$ because π_is computed by the above formula is exactly what must be for $\mathcal{G}'$. So by following the method used in [14] for proving Theorem 1, our desired target will be achieved.

□

Note that for each $m_k \in M$, we must have $\epsilon_k < \frac{\alpha}{\beta}$, because if not, the price function P_k will be negative which is not acceptable. Now, we can write the formulation of social welfare. The SW function in the presence of shocks can be recalculated as follows:

$$\begin{aligned} SW &= \textstyle\sum_{f_i \in F} \pi_i(q^*, \epsilon) + CS \\ &= \textstyle\sum_{f_i \in F} \left[\sum_{m_k \in M_i} q^*_{ik} \cdot P_k(q^*, \epsilon) - C_i(q^*) \right] \\ &\quad + \textstyle\sum_{m_k \in M} \frac{1}{2\beta_k} \cdot (\alpha_k - P_k(q^*, \epsilon))^2 \\ &= \textstyle\sum_{f_i \in F} \Big[\sum_{m_k \in M_i} q^*_{ik} (\alpha - \beta \sum_{f_j \in F_k} q^*_{jk} - \beta\epsilon_k) \\ &\qquad -c\big(\textstyle\sum_{m_k \in M_i} q^*_{ik}\big)^2 \Big] \\ &\quad + \textstyle\sum_{m_k \in M} \frac{1}{2\beta} \cdot \Big[\big(\beta \sum_{f_j \in F_k} q^*_{jk} + \beta\epsilon_k\big)^2 \Big] \\ &= \textstyle\sum_{m_k \in M} \Big(\sum_{f_i \in F_k} q^*_{ik} \alpha \Big) \\ &\quad -\beta/2 \cdot \textstyle\sum_{m_k \in M} \Big(\sum_{f_i \in F_k} q^*_{ik} \Big)^2 \\ &\quad -c \textstyle\sum_{f_i \in F} \Big(\sum_{m_k \in M_i} q^*_{ik} \Big)^2 \\ &\quad + \textstyle\sum_{m_k \in M} (\beta/2) \cdot \epsilon_k^2 \end{aligned} \tag{12}$$

Therefore, we have the following vectorized formula,

$$SW = {q^*}^T \alpha - (\beta/2 + c) {q^*}^T q^* - (1/2) {q^*}^T W q^* + (\beta/2) \bar{\epsilon}^T \bar{\epsilon}, \tag{13}$$

where $\bar{\epsilon}$ is a $m \times 1$ vector whose kth component is equal to ϵ_k and other variables are defined as before (see Theorem 1 and Theorem 2). By the above formulation the problem of governing social welfare with market shocks can be modeled by the optimization problem described in Definition 1.

Definition 1. *The problem of governing (maximizing) social welfare with shocks in a networked market $\mathcal{G}$ ($MaxSW(\mathcal{G})$) is defined as the following optimization problem:*

$$\begin{array}{lll} \textit{Maximize} & {q^*}^T \overline{\alpha} - (\beta/2 + c) {q^*}^T q^* - (1/2) {q^*}^T W q^* + (\beta/2) \bar{\epsilon}^T \bar{\epsilon} & \\ \textit{subject to} & q^* = [I + \gamma W]^{-1} \gamma (\bar{\alpha} - \overline{\beta\epsilon}) & \\ & c \cdot (\sum_{m_k \in M} \epsilon_k)^2 \leq B & \\ & 0 \leq \epsilon_k \leq q^t & \forall m_k \in M \end{array} \tag{14}$$

The above formula is not concave or convex, because both convex and concave expressions are appeared in it. This makes the convex optimization frameworks ineffective. In the next section, we devise a heuristic algorithm for this optimization problem. This is done by proposing a linear estimation for the social welfare and an optimization algorithm for maximizing it. Then, in Section 4, the good performance of this heuristic algorithm is demonstrated by experimentation on real and synthetic data.

3 Solution Estimation

In this section, we provide some insights into the social welfare function and by linearizing it with Taylor expansion, we propose an algorithm called the *Linear* heuristic for the $MaxSW(\mathcal{G})$ problem. More precisely, we propose a metric that can be computed using the network structure and we analytically show that picking the markets with larger amounts of this metric can (approximately) maximize the social welfare. In the next section, by running experiments on real and synthetic datasets, we will show the superiority of this approach over others.

The main idea is to use the first order multivariate Taylor expansion [31] to create a linear approximation for the social welfare function. This approximation leads to a linear combination of ϵ_is:

$$SW(\epsilon) = SW(0) + \zeta_1\epsilon_1 + \zeta_2\epsilon_2 + ... + \zeta_m\epsilon_m. \tag{15}$$

Keeping in mind that the government has a limited budget for its interventions, we can consider the shocks small ($\forall_{m_k \in M}\epsilon_k \leq q^t$), so that this approximation may be valid. The coefficient of each ϵ_i (ζ_i) can be considered as the aforementioned metric. Since SW is a differentiable function, we can write SW as follows:

$$\begin{aligned} SW(\epsilon) &\approx SW(0) + \epsilon \cdot \nabla SW(0) \\ &= SW(0) + \sum_{r=1}^{m} \epsilon_r \frac{\partial SW}{\partial \epsilon_r}|_{\epsilon=0} \end{aligned} \tag{16}$$

Thus, the amount of social welfare added by the shocks is a linear combination of ϵ_rs whose coefficients are $\zeta_r = \frac{\partial SW}{\partial \epsilon_r}|_{\epsilon=0}$. Therefore, to maximize social welfare, markets should be targeted for supplies in order of their ζ_rs. The details of this algorithm can be seen in Algorithm 1. Now, we discuss how to calculate the coefficients.

Using Theorem 2 and expanding the formula derived for q^*, we have:

$$\begin{aligned} q^*_{ik} &= \frac{\alpha - 2c\sum_{m_\ell \in M_i, m_\ell \neq m_k} q^*_{i\ell} - \beta\sum_{f_j \in F_k} q^*_{jk}}{2(\beta + c)} \\ &= \frac{\alpha}{2(\beta+c)} - \sum_{(j,\ell)\in E(\mathcal{G})} (\gamma W)_{ik,j\ell} q^*_{j\ell} \end{aligned} \tag{17}$$

If we define function $f(z) = \gamma\alpha - \gamma z$,

$$q^*_{ij} = f(\sum_{k,\,l} w_{ij,k\ell} q^*_{k\ell} + \gamma\beta\epsilon_j) \tag{18}$$

denotes the amount firm f_i supplies in market m_j in equilibrium under the presence of shock ϵ_j. Moreover, from Equation 13 with $h(x) = \alpha x - (\frac{\beta}{2} + c)x^2$ and $u(q_{ij}, q_{k\ell}) = w_{ij,k\ell} q_{ij} q_{k\ell}$,

$$SW = \sum_{i,\,j} h(q^*_{ij}) - \frac{1}{2}\sum_{i,\,j,\,k,\,\ell} u(q^*_{ij}, q^*_{k\ell}) + \sum_{m_k \in M} (\beta/2)\cdot\epsilon_k^2 \tag{19}$$

is social welfare under the circumstances discussed so far.

We start our analysis by calculating $\frac{\partial q^*_{ij}}{\partial \epsilon_r}$. Exploiting the idea used by [20],

$$\begin{aligned} \frac{\partial q^*_{ij}}{\partial \epsilon_r} = f'\left(\sum_{i,\,j,\,k,\,\ell} w_{ij,k\ell} q^*_{k\ell} + \gamma\beta\epsilon_j\right)\cdot \\ \left(\sum_{i,\,j,\,k,\,\ell} w_{ij,k\ell}\frac{\partial q^*_{k\ell}}{\partial \epsilon_r} + \gamma\beta\mathbf{1}\{r=j\}\right) \end{aligned} \tag{20}$$

Evaluating this equation using the matrix form at point $\epsilon = (\epsilon_1, \cdots, \epsilon_{|E|}) = 0$, which is the absence of the government, yields

$$\frac{\partial q^*}{\partial \epsilon_r}|_{\epsilon=0} = -\gamma\beta(I + \gamma W)^{-1} e_r, \tag{21}$$

where e_r is a $|E| \times 1$ vector, with ones for edges connecting to market m_r and zeros elsewhere. Thus, we have:

$$\frac{\partial q^*_{ij}}{\partial \epsilon_r}|_{\epsilon=0} = -\gamma\beta\sum_{k} \lambda_{ij,kr}\ , \tag{22}$$

where $\lambda_{ij,kr}$ is the corresponding element to edges ij and kr of matrix $(I + \gamma W)^{-1}$.

Setting our sights on social welfare, we have:

$$\frac{\partial\, \mathrm{SW}}{\partial \epsilon_r} = \sum_{i,j} h'\left(q^*_{ij}\right)\frac{\partial q^*_{ij}}{\partial \epsilon_r} - \frac{1}{2}\sum_{i,j,k,\ell}\left[\frac{\partial u}{\partial q^*_{ij}}\frac{\partial q^*_{ij}}{\partial \epsilon_r} + \frac{\partial u}{\partial q_{k\ell}}\frac{\partial q^*_{k\ell}}{\partial \epsilon_r}\right] + \beta\epsilon_r. \tag{23}$$

Considering $h'(x) = \alpha - (\beta + 2c)x$ and $\frac{\partial u(q_{ij}, q_{k\ell})}{\partial q_{ij}} = w_{ij,k\ell} q_{k\ell}$, we get:

$$\begin{aligned} \frac{\partial\, \mathrm{SW}}{\partial \epsilon_r} &= \sum_{i,j} (\alpha - (\beta + 2c) q^*_{ij}) \frac{\partial q^*_{ij}}{\partial \epsilon_r} \\ &- \frac{1}{2}\sum_{i,j,k,\ell}\left[w_{ij,k\ell} q^*_{k\ell}\frac{\partial q^*_{ij}}{\partial \epsilon_r} + w_{ij,k\ell} q^*_{ij}\frac{\partial q^*_{k\ell}}{\partial \epsilon_r}\right] \\ &+ \beta\epsilon_r \end{aligned} \tag{24}$$

Thus, we evaluate equation (24) at $\epsilon = 0$:

$$\begin{aligned} \frac{\partial\, \mathrm{SW}}{\partial \epsilon_r}|_{\epsilon=0} &= \sum_{i,j} (\alpha - (\beta + 2c) q^*_{ij}|_{\epsilon=0}) \frac{\partial q^*_{ij}}{\partial \epsilon_r}|_{\epsilon=0} \\ &- \frac{1}{2}\sum_{i,j,k,\ell}\Big[w_{ij,k\ell} q^*_{k\ell}|_{\epsilon=0}\frac{\partial q^*_{ij}}{\partial \epsilon_r}|_{\epsilon=0} \\ &+ w_{ij,k\ell} q^*_{ij}|_{\epsilon=0}\frac{\partial q^*_{k\ell}}{\partial \epsilon_r}|_{\epsilon=0}\Big]) \end{aligned} \tag{25}$$

$q^*_{ij}|_{\epsilon=0}$ has been studied in [14]. Based on their results, we can deduce the following in our setting:

$$q^*_{ij}|_{\epsilon=0} = \gamma\alpha\sum_{k\ell} \lambda_{ij,k\ell}\ . \tag{26}$$

Using equation (22) and (26), we get:

$$\begin{aligned} \frac{\partial\, \mathrm{SW}}{\partial \epsilon_r}|_{\epsilon=0} = -\gamma\beta\sum_{ij}(\alpha - \gamma\alpha(\beta + 2c)\sum_{k\ell}\lambda_{ij,k\ell})\sum_{k}\lambda_{ij,kr} - \\ \sum_{i,j}(\gamma\alpha\sum_{k\ell}\lambda_{ij,k\ell}(\sum_{k\ell} w_{ij,k\ell}\sum_{t}\gamma\beta\lambda_{k\ell,tr})) \end{aligned} \tag{27}$$

Since we have derived a formula for computing $\zeta_r = \frac{\partial SW}{\partial \epsilon_r}|_{\epsilon=0}$s, we can state the final algorithm. This algorithm is shown in Algorithm 1. In the first four lines of this algorithm ζ_rs are computed from the network structure and the Leontief matrix. After that, a set T is initialized to the set of all markets and a variable S is initialized to 0. In the next while loop, T is to be the set of markets that have not been supplied with shocks so far and S is defined as the total goods supplied by the government. Therefore, the loop execution will continue until either all markets are supplied with shocks or the cost of shock supplies exceeds the budget intended for this purpose (B).

In each iteration of the while loop, the market with maximum ζ_r is chosen from T and extracted from this set. Then the maximum possible shock is computed as ϵ_r and is added to S. Note that for computing ϵ_r, three upper bounds must be considered:

1) q^t is the upper bound defined in Definition 1.

2) $\frac{\beta}{\alpha}$ is the upper bound defined by the price function. If $\epsilon_r > \frac{\beta}{\alpha}$, the price will be negative at that market, which is not acceptable.
3) $\sqrt{\frac{B}{c_r}} - S$ is the amount of goods that can be provided by the remaining budget.

4 Empirical Study

We evaluate the performance of our proposed method on a synthetic and a real-world dataset of different pharmaceutical companies as our firms and different drugs as our markets. This dataset is collected by contacting 135 production companies which produce 603 drugs altogether, and after negotiation, we succeed in getting their data. Then we transform, clean, and integrate all of their data to generate our desired dataset. For example, Aspirin and its users define a market in which players are companies that produce this drug. Additionally, we use identical parameters α, β, c for all the firms and markets, as we are considering the symmetric case. Using Ordinary Linear Regression, these parameters are set in a way to be close to real-life values. The synthetic graph also has 603 markets and 135 firms, which has been chosen uniformly at random from all bipartite graphs with the same numbers of markets, firms, and firm-market pairs. The characteristics of this dataset are shown in Table 1. A subgraph of this network is shown in Figure 2. The data associated with this subgraph is shown in Table 2.

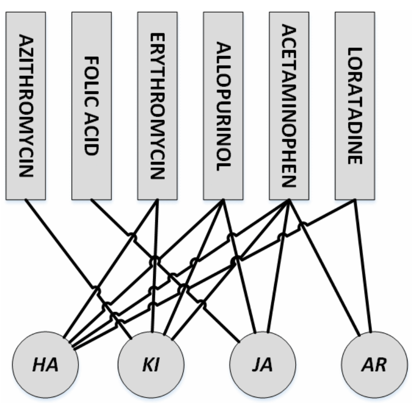

Fig. 2: A Subgraph of the Drug Company Dataset

4.1 Competitor Benchmark

The essence of competitors we consider is that the government takes on a measure to rank the markets. Next, it supplies goods to markets in that order, as much as possible and as long as permissible. Naively, it is possible to choose the markets at random. No measure, to the best of our knowledge, has been presented for picking the markets yet. Nevertheless, centrality measures are natural candidates for us to use as benchmarks. As for the centrality measures we consider, we use the followings:

- **Degree.** The simplest centrality measure is the degree of a node, which in our model, is the number of firms competing in a market:

$$d(m_i) = |F_i|. \tag{28}$$

- **Betweenness.** Generally speaking, betweenness centrality is a quantity for determining the impact of a node over the flow of information in a graph [32]. The betweenness of a node is an indicator for the fraction of shortest paths (with regard to the number of edges) in a graph that pass through this vertex. In our setting:

$$b(m_i) = \sum_{f_j, f_k \in F} \frac{\sigma_{jk}(m_i)}{\sigma_{jk}}, \tag{29}$$

where σ_{jk} is the total number of shortest paths from firm f_j to firm f_k and $\sigma_{jk}(m_i)$ is the total number of those paths that include market m_i.

- **Closeness.** Closeness centrality is an aggregate measure of a node's proximity to other nodes. More precisely, closeness of node v is defined as the inverse of sum of distances of node v from other nodes. Considering our model:

$$cl(m_i) = \sum_{f_j \in F} \frac{1}{dist(m_i, f_j)}, \tag{30}$$

where $dist(m_i, f_j)$ is the distance between market m_i and firm f_j in the graph.

Regardless of the measure one picks, it is possible to adopt the ascending or the descending order. Thus, we consider both choices, however, the ascending order empirically shows better performance with these benchmark centrality measures. In conclusion, we present our studied strategies in Table 3.

After having our graph, we use Theorem 2 to calculate the equilibrium of the game with our parameters α, β, and c. Subsequently, we pick a policy A for governance of social welfare from Table 3. Then, the government begins supplying the amount q^t of the commodity into the markets in the corresponding order, until the supplies violate the constraints in Definition 1. Thus, the government would supply up to $\left\lfloor \sqrt{\frac{B}{C}} \right\rfloor$, where B is the budget that innates to the $MaxSW(\mathcal{G})$ problem. Altering B gives us a trajectory for social welfare. Let $SW_A(B)$ be the social welfare obtained by applying strategy A on $MaxSW(\mathcal{G})$ with parameter B. In the next subsection, we compare trajectories $SW_A(B)$ for policies in Table 3.

4.2 General performance

Our empirical results are indicated in Figures 3a, 3b, 4a, 4b, 5a and 5b. For each policy A of Table 3, we plot the difference between the amount of social welfare which can be obtained by our Linear algorithm and the social welfare which can be obtained by the heuristic A, i.e. $SW_{Linear}(B) - SW_A(B)$. Moreover, in Figure 6, the social welfare obtained by implementing our algorithm with different budgets is depicted.

We observe that our proposed measure is strictly better than other mentioned quantities. For the random case, we use the average results over 50 different realizations to extract the expected performance. It is good to note that picking the markets according to ascending order of a centrality measure seems to be better than the reverse order. This is

Input: A network market $\mathcal{G}$ alongside with parameters α, β and γ
Output: The amount of shocks ϵ_1, ϵ_2, ..., ϵ_m which makes the maximum social welfare

Set $\lambda_{ij,kr}$ equal to the corresponding elements to edges ij and kr of matrix $(I + \gamma W)^{-1}$
for $r \leftarrow 1$ ***to*** m **do**
  $\zeta_r \leftarrow -\gamma\beta \sum_{ij}(\alpha - \gamma\alpha(\beta + 2c)\sum_{k\ell}\lambda_{ij,k\ell})\sum_k \lambda_{ij,kr} - \sum_{i,j}(\gamma\alpha\sum_{k\ell}\lambda_{ij,k\ell}(\sum_{k\ell} w_{ij,k\ell}\sum_t \gamma\beta\lambda_{k\ell,tr}))$
end
$T \leftarrow M$; $S \leftarrow 0$
while $T \neq \emptyset$ *and* $c \cdot S^2 < B$ **do**
  Set r to the index of the market $m_r \in T$ with maximum ζ_r
  $T \leftarrow T \setminus m_r$
  $\epsilon_r \leftarrow \min\{q^t, \frac{\alpha}{\beta}, \sqrt{\frac{B}{c}} - S\}$
  $S \leftarrow S + \epsilon_r$
end
return $\epsilon_r s$

Algorithm 1: The Linear Heuristic for solving $MaxSW(\mathcal{G})$

TABLE 1: Summary of Dataset's Characteristics

Drug Companies Dataset	
Characteristic	Value
#Markets	603
#Firms	135
#Firm-Market pairs (edges)	2049

TABLE 2: A Sample of the Dataset Related to Figure 2

Drug	Company	Sale ($)	Sale (#)
Azithromycin	KI	39,730	153,400
Folic Acid	JA	1,131,726	4,369,600
Erythromycin	HA	167,132	645,300
Erythromycin	KI	22,885,058	88,359,300
Alloporinol	HA	2,089,793	8,068,700
Alloporinol	KI	223,542	863,100
Alloporinol	JA	1,300,128	5,019,800
Acetaminophen	HA	45,079,908	174,053,700
Acetaminophen	KI	22,672,264	87,537,700
Acetaminophen	JA	12,809,311	49,456,800
Acetaminophen	AR	15,824,330	61,097,800
Loratadine	HA	3,447,200	6,894,400
Loratadine	AR	1,843,900	3,687,800

intuitive, since markets with lower centralities are generally more monopolized and government's interventions have more impact on them. This observation is in accordance with the general idea in economics that more competition leads to higher social welfare. In addition, the difference in social welfare obtained by the Linear heuristics and other methods tend to rise, until a certain point at which drops. Since the end point of the plots suggests the government to supply in almost all markets by each method, the last points have y coordinates close to zero. All the plots are strictly above the horizontal line $y = 0$, except beginning points which indicates the superiority of our proposed approach over other mentioned strategies.

TABLE 3: Summary of algorithms in the competitor benchmark

Heuristic	Description
$Linear$	Descending order of ζ_i
$AscDeg$	Ascending order of $d(m_i)$
$DescDeg$	Descending order of $d(m_i)$
$AscBet$	Ascending order of $b(m_i)$
$DescBet$	Descending order of $b(m_i)$
$AscCL$	Ascending order of $cl(m_i)$
$DescCL$	Descending order of $cl(m_i)$
$Random$	Random order of m_i

The superiority of the presented algorithm compared to the competing algorithms is primarily due to the fact that the algorithm is really based on the optimization of the objective function, and its output is a metric that cannot be intuitively described based on structural features. In fact, the evaluation carried out in this article shows that the structural features of the network are not very suitable for choosing the best markets for intervention, and it is better to focus directly on the optimization of the objective function and designing better algorithms in this direction. Anyway, among the competing heuristics, the DescDegree algorithm has performed better. In fact, it can be concluded that if we are going to act only based on the structure, markets closer to other markets (in terms of distance on the graph) are more suitable for intervention.

In Figure 6, we considered different budgets for the government and and implemented the Linear huristic. Then, we plotted the social welfare obtained. We observe that the social welfare increases in the beginning, which is a result of increased consumers' surplus. However, the social welfare peakes at a certain level and decreases from that point onwards. This observation is due to the fact that the supply by the government decreases the producers' surplus while it increases the consumers' surplus. In the second half of the trajectory, the decrease in producers' surplus dominates the increase in the consumers' surplus, which results in lower levels of social welfare.

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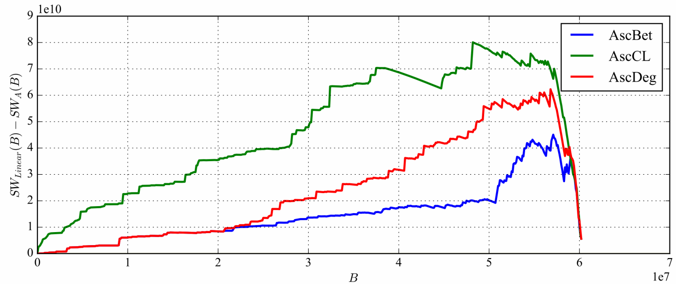
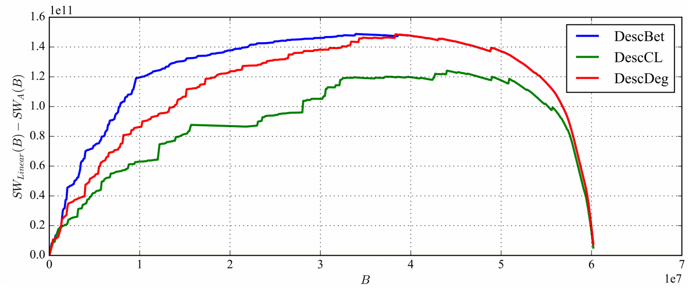
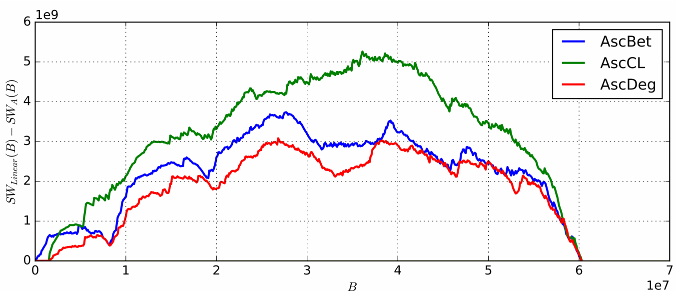


(a) Ascending order

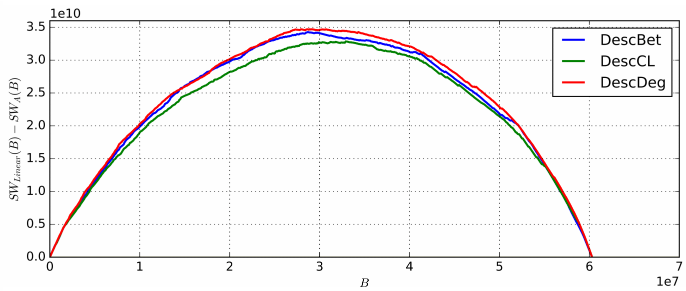


(b) Descending order

Fig. 3: The difference between the social welfare obtained by the Linear heuristic and competitor structural heuristics with different orders in the dataset of drug companies

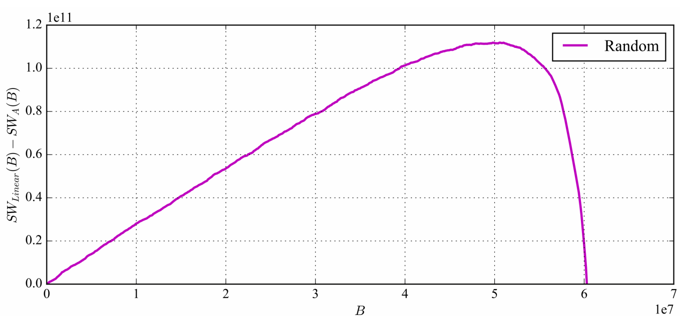


(a) Ascending order

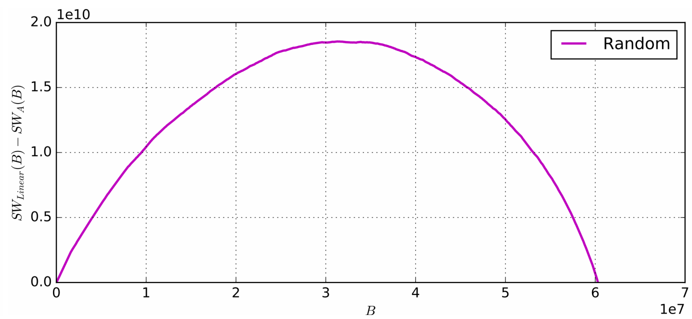


(b) Descending order

Fig. 4: The difference between the social welfare obtained by the Linear heuristic and competitor structural heuristics with different orders in the synthetic dataset



(a) Dataset of drug companies



(b) Synthetic dataset

Fig. 5: The difference between the social welfare obtained by the Linear heuristic and the random heuristic in different datasets

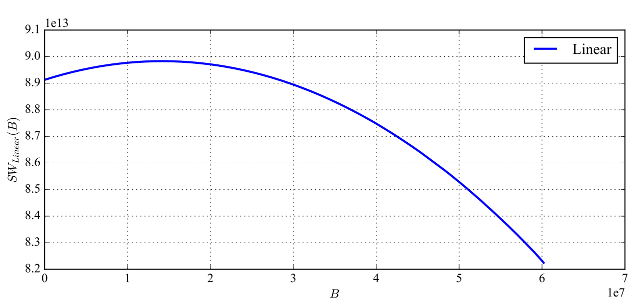


Fig. 6: The social welfare obtained by the Linear heuristic with different budgets in the dataset of drug companies

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