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Naturality of Network Creation Games, Measurement and Analysis

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***Abstract*—Modeling is one of the major research areas in social network analysis whose goal is to study networks structure and its evolution. Motivated by the intuition that members in social networks behave selfishly, network creation games have been introduced for modeling social networks. In this paper, our aim is to measure how much the output graphs of a given network creation game are compatible with a social network. We first show that the precise measurement is not possible in polynomial time. Then we propose a method for its approximation; finally, we show the usability of our method by conducting experiments on real network data.**

I. Introduction

Modeling social networks is an important research direction in the study of social networks. Its main goal is to study the structure of networks and their evolution in an attempt to generate graphs with similar structural properties to real networks. [1]

Network Creation Games presented up to now, fail to satisfy real networks' properties desirably. For the games defined in [2], [3] and [4] it is proved that they have diameters far from logarithmic diameter for their equilibria. In addition, most of network formation games yield only symmetric equilibrium networks [5]. In reality, however, asymmetry is noticed in social networks. The game-theoretic models to date have been primarily technological, rather than sociological [5]. On the other hand, Nash equilibria in most network creation games, are literally unrealistic.

The main idea of this paper is to introduce a general framework for measuring the naturality of network creation games, i.e., how much their output graphs match structurally to real social networks. Such measurements might be used for comparing two network creation models.

II. NCG Decidability

Before proposing a framework for comparing the compatibility of network creation games with a given network $\mathcal{N}$, we first study two fundamental relevant problems regarding the complexity of such models. We only focus on our fixed configuration (sequential dynamic + Nash equilibria).

Definition 1. *In* **NCG compatibility problem (NCG-Compat)***, a network creation game, $\mathcal{G}$, and two networks, N_0 and N_1, are given. Game $\mathcal{G}$ is started from N_0. Is there any sequential dynamic of the game which converges to N_1?*

The main result of this section is theorem 2 which shows that NCG-Compat problem can not be decided in polynomial time. Before declaration of our main result, we present a theorem about hardness of finding a Nash-equilibrium of a network creation..

Theorem 1. *Finding a Nash-equilibrium of a network creation game, even if computing players' best response can be accomplished in polynomial-time, is NP-Hard.*

Theorem 2. *NCG-Compat problem is NP-Complete.*

III. Framework

Assume that $\mathcal{G}$ is a network creation game and $\mathcal{N}$ is a network with n nodes. We use notation $comp(\mathcal{G}, \mathcal{N})$ for showing the amount of compatibility and we define it as follows:

$$comp(\mathcal{G}, \mathcal{N}) = \frac{1}{n} \sum_{i=1}^{n} \frac{current\_utility(v_i)}{best\_utility(v_i)}$$

$current\_utility(v_i)$ is the utility of v_i in $\mathcal{N}$ and $best\_utility(v_i)$ is the maximum utility that v_i can achieve assuming other players have their current strategies. Compatibility factor is a general form of smartness factor, which is introduced in an earlier work by the authors [6].

IV. Experimental Results

A. Case Studies

In this section, we use our framework to illustrate the compatibility of games with networks.

We consider Watts-Strogatz random graph [7] and Fabrikant et al.'s game [2]. For $\alpha = 1$, the current action of nodes is far from their best response. Increasing alpha brings the best response closer to the current action. As α rises even more, we will again get far from our current graph. Average compatibility factor vs α is illustrated in Figure 1. Observed results confirm our expectations discussed above.

In this case the undirected Epinion network [8] with two different games are considered. In the first game, the utility function is an exponential function and in the second one, it is

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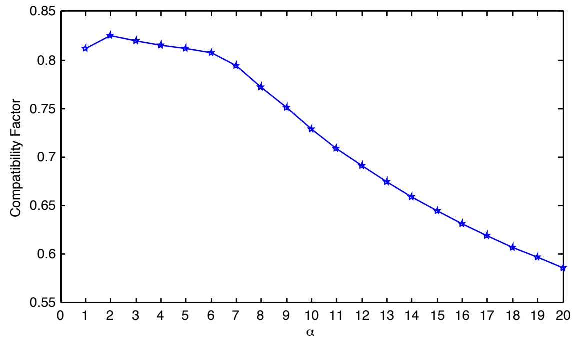

Fig. 1. Compatibility factor of Watts-Strogatz graphs [7] with Fabrikant et al.'s game [2] with different values of α.

a Gaussian function. The exponential utility function acts more like a power law. So, the first game seems more natural than the second one. Experimental results, as depicted in Figure 2, show that our intuition is true.

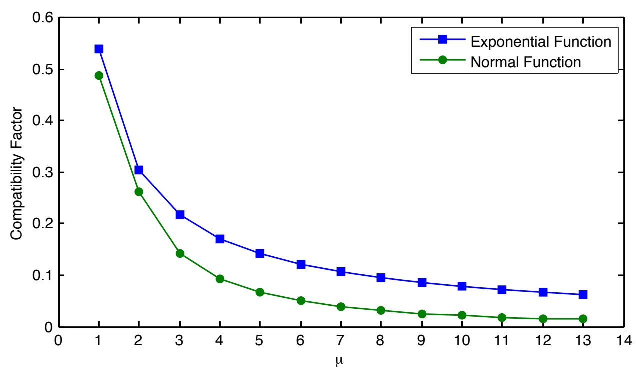

Fig. 2. Comparison between compatibility factor of two NCGs defined by exponential and Gaussian utility functions with Epinion network.

B. Natural Data[1]

In this section we measure the compatibility factor for real social networks. Signed network formation game is used for this purpose [6]. We consider three on-line social networks used by Leskovec [8], the trust network of the Epinions, the social network of the blog Slashdot, and the voting network of Wikipedia. For each node v_i, we calculate $current\_utility(v_i)$ and $best\_utility(v_i)$. Our results show that the former has a wide variety in its value, but the latter has almost a constant value. As shown in Figure 3, there is a big difference between these two values and thus the naturality of this game is very low.

[1]The detailed content of this subsection is also presented in a paper at SNA-KDD 2011.

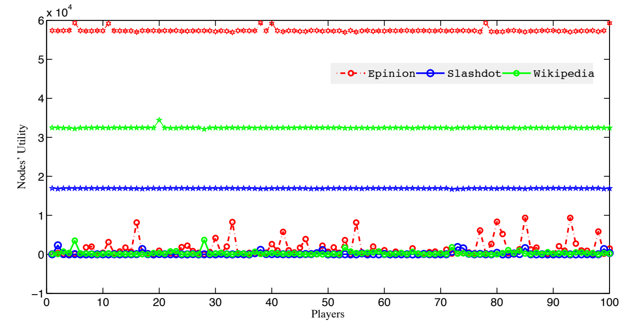

Fig. 3. Nodes' utility, below lines show current utility and above lines show best-response utility [6]

V. Concluding Remarks

Network creation games, in spite of their effort to represent a natural model for social networks, commonly result in models which significantly differ from real networks. There are also no accurate and practical measures for the compatibility of network creation games with a given social network.

Hence, it would be beneficial to have a suitable and applicable framework to reliably estimate and compare the naturality of network creation games. This paper attempts to introduce one such framework. We have tested the proposed method to measure the compatibility of a network creation game previously introduced in the literature, and have also applied the framework as a comparison measure between several games.

References

[1] J. Leskovec, D. Chakrabarti, J. Kleinberg, C. Faloutsos, and Z. Ghahramani, "Kronecker graphs: An approach to modeling networks," *The Journal of Machine Learning Research*, vol. 11, pp. 985–1042, 2010.

[2] A. Fabrikant, A. Luthra, E. Maneva, C. Papadimitriou, and S. Shenker, "On a network creation game," in *Proceedings of the twenty-second annual symposium on Principles of distributed computing*. ACM, 2003, pp. 347–351.

[3] E. D. Demaine, M. Hajiaghayi, H. Mahini, and M. Zadimoghaddam, "The price of anarchy in network creation games," in *Proceedings of the twenty-sixth annual ACM symposium on Principles of distributed computing*, ser. PODC '07. New York, NY, USA: ACM, 2007, pp. 292–298. [Online]. Available: http://doi.acm.org/10.1145/1281100.1281142

[4] N. Alon, E. D. Demaine, M. Hajiaghayi, and T. Leighton, "Basic network creation games," in *Proceedings of the 22nd ACM symposium on Parallelism in algorithms and architectures*, ser. SPAA '10. New York, NY, USA: ACM, 2010, pp. 106–113. [Online]. Available: http://doi.acm.org/10.1145/1810479.1810502

[5] M. Brautbar and M. Kearns, "A clustering coefficient network formation game," in *4th Symposium on Algorithmic Game Theory (SAGT)*, 2011. [Online]. Available: http://ssrn.com/abstract=1689219

[6] M. Malekzadeh, M. Fazli, P. Jalali Khalilabadi, H. Rabiee, and M. Safari, "Social balance and signed network formation games," in *Proceedings of KDD workshop on Social Network Analysis (SNA-KDD), to appear*, August 2011.

[7] D. Watts and S. Strogatz, "Collective dynamics of small-world networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[8] J. Leskovec, D. Huttenlocher, and J. Kleinberg, "Signed networks in social media," in *CHI '10: Proceedings of the 28th international conference on Human factors in computing systems*. New York, NY, USA: ACM, 2010, pp. 1361–1370.