FCS
Final Takehome Exam

Start: 9:00am, 97-11-1
End: 9:00am, 97-11-3

1. Develop a a **theory of computation** for *total functions*. Justify the fruitfulness of your theory. Is there any uncomputable function in your theory? why? (provide examples and different computation classes or discuss why such an effort is not a good idea!)

2. Consider the *deterministic PDA* model (or DPDA for short) and answer the following question.

   - How many different kinds of **languages** can you assign to a DPDA? (provide your justifications for the usefulness in each case)
   - Determine the equality or inequality of the four standard computation classes for each one of the languages for which the definitions of the classes are meaningful.
   - Is there any language assignment for which these classes are **not** meaningful? why?
     What sort of study is justified in these cases when you can not define the standard classes? (discuss and provide your analysis)

3. Assume that the language L has time complexity $o(\log n)$ on a deterministic decider Turing model. Can we deduce that L is a context-free language? Can we deduce that L is a regular language? (prove or disprove)

4. Prove that if a function is not computable in constant space, then any Turing machine computing it must take at least space proportional to $\log \log x$ for infinitely many input x.

5. Can we apply the diagonalization method to find a language between the classes of recursive and context-sensitive languages? why?

6. Prove that if a function is not computable in constant space, then any Turing machine computing it must take at least space proportional to $\log \log x$ for infinitely many input x.

7. A set is of type A if it is the range of a total computable function. Also a set is of type B if it is the range of a partial computable function. Does there exists a set which is of type B but is not of type A? (discuss)