Special Topics in Cosmology

Take Home

Last Updated: 21 May 2013
Due to: 1 July 2013

Some comments and hints will be added during our discussions.

The aim is to investigate the probable effect of primordial anisotropic power spectrum from inflationary models in the CMB lensing observations. Recent Planck Observation confirms the previous seen CMB anomalies detected by WMAP. So it is a worth to think on the observations in CMB and Large Scale Structure (LSS) to study the probable effect of anomalies.
First of all, use the geodesics equations in a perturbed FRW Universe and show that the angle of an image θ_s is related to the midway gravitational potential as:

$$\theta^i_S = \theta^i + 2\int_0^{\chi_*} d\chi' \Phi_{,i}(\vec{x}(\chi'))\left(1 - \frac{\chi'}{\chi}\right), \tag{1}$$

where χ_* is the comoving distance to the source, and i is a spatial indices. Now by defining a 2×2 symmetric transformation function as

$$A_{ij} \equiv \frac{\partial \theta^i_S}{\partial \theta^j} \equiv \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \tag{2}$$

where κ is the convergence and γ_1 and γ_2 is two components of shear, defining

$$\psi_{ij} \equiv A_{ij} - \delta_{ij} \tag{3}$$

show that

$$\psi_{ij}(\vec{\theta}) = \int_0^{\chi_\infty} d\chi \Phi_{,ij}(\vec{x}(\chi))g(\chi) \tag{4}$$

where g is related to the distribution of structures $W(\chi)$ as below:

$$g(\chi) \equiv 2\chi \int_\chi^{\chi_\infty} d\chi'(1 - \frac{\chi}{\chi'})W(\chi'), \tag{5}$$

Now find the power spectrum of lensing components which is defined as:

$$\langle \psi_{ij}(\vec{l})\psi_{lm}(\vec{l'})\rangle = (2\pi)^3\delta^2(\vec{l} - \vec{l'})P^\Psi_{ijlm} \tag{6}$$

Show that the power spectrum P^Ψ_{ijlm} is related to matter power spectrum.
Now discuss that how the lensing power spectrum is related to transfer function $T(k)$ and growth function $D(z)$ from late time Universe and also investigate its relation to the spectral index and COBE normalization from Inflationary models. Then find the Power

spectrum of shear and convergence.

After this part consider the CMB lensing. Lensing re-maps the temperature according to

$$\tilde{\Theta}(\mathbf{x}) = \Theta(\mathbf{x}') = \Theta(\mathbf{x} + \vec{\nabla}\psi) \tag{7}$$

where ψ is lensing potential defined as:

$$\psi(\hat{n}) \equiv -2 \int_0^{\chi_*} d\chi \frac{(\chi_* - \chi)}{\chi_* \chi} \Phi(\chi\hat{n}; \eta_0 - \chi) \tag{8}$$

where η_0 is the present conformal time.
Taylor expand the Eq.(7) and fourier transform the lensed temperature in flat sky and show how the lensed C_l^{lens} is related to unlensed C_l and the angular power spectrum of lensing potential which is defined as:

$$\langle \psi_{lm} \psi^*_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l^{\psi} \tag{9}$$

Discuss the relation between the angular power spectrum and $P^{\Psi}_{lml'm'}$.
In the next step find the angular cross power spectrum of lensed CMB temperature Θ^{lens} and the lensing potential ψ, and relate it to the matter power spectrum.
Now we are ready to implant the anisotropic power spectrum, first show how the matter power spectrum in the cross lensed CMB temperature - lensing potential is related to the primordial curvature perturbation. Then modify the primordial power spectrum as:

$$\mathcal{P}_{\mathcal{R}}(k; \hat{n}) = \mathcal{P}^{iso}_{\mathcal{R}}(k) \left[1 + \sum_{lm} \sum_{l'm'} A(k) g(k) Y_{lm}(\gamma_{\hat{n}.\hat{p}}) Y_{l'm'}(\gamma_{\hat{n}.\hat{k}}) \right] \tag{10}$$

where $\mathcal{P}^{iso}$ is the isotropic dimensionless power spectrum, $\gamma_{\hat{n}.\hat{p}}$ is the angle between the observation angle $\hat{n}$ and a special angle in real space $\hat{p}$ and $\gamma_{\hat{n}.\hat{k}}$ is a special angle between observation angle $\hat{n}$ and specific direction in Fourier space $\hat{k}$.
Find the difference between cross power spectrum in two conditions, with isotropic and anisotropic primordial power spectrum.

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