پاسخنامه تمرین سری سوم

سوال اول

$$x^2 - c^2t^2 = x_0^2 \longrightarrow 2xu - 2c^2t = 0 \longrightarrow u = \frac{c^2 t}{x}$$

$$\alpha = \gamma^3 \frac{du}{dt} = \frac{d}{dt}\left[\gamma(u)\, u\right] \longrightarrow \alpha t = \gamma(u)\, u = \frac{u}{\sqrt{1 - u^2/c^2}}$$

$$\longrightarrow \alpha t = \frac{c^2 t / x}{\sqrt{1 - c^2t^2/x^2}} = \frac{c^2 t/x}{\frac{1}{x}\sqrt{x_0^2}} = \frac{c^2 t}{x_0} \longrightarrow \alpha = \frac{c^2}{x_0}$$

یا

بنابراین

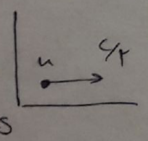
معادله جهان خط ذره‌ای با ویژه شتاب α

$$\longrightarrow x^2 - c^2t^2 = c^4/\alpha^2 = x_0^2 \longrightarrow x_0 = \frac{c^2}{\alpha}$$

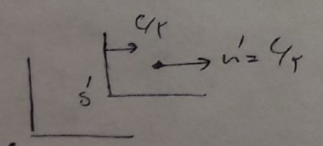
$$\alpha = \frac{c^2}{x_0}$$

سوال دوم:

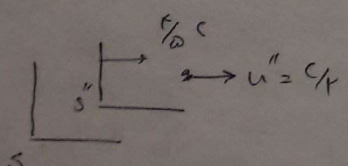
$$u' = \frac{u - v}{1 - uv/c^2}, \quad u = \frac{u' + v}{1 + u'v/c^2}$$



$u_1 = c/2$



$$u_2 = \frac{c/2 + c/2}{1 + (c/2)^2/c^2} = \frac{4}{5}c$$

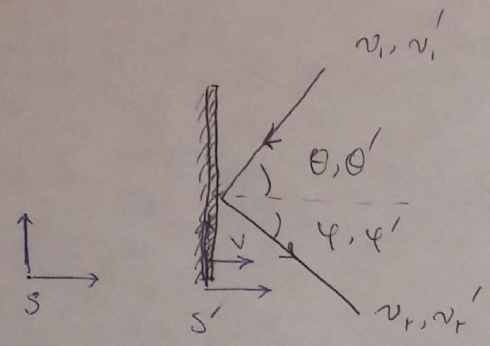


$$u_3 = \frac{c/2 + 4/5\, c}{1 + \frac{2}{5}} = \frac{\frac{13}{10}c}{\frac{7}{5}} = \frac{13}{14}c$$

در سه مرحله.

Scanned by CamScanner

سوال سوم :



S' دستگاه

$$\begin{cases} \theta' = \varphi' \\ \nu_1' = \nu_2' \end{cases}$$

از روابط ابراهی :

$$\tan\left(\frac{\theta'}{2}\right) = \sqrt{\frac{c-v}{c+v}}\ \tan\frac{\theta}{2}$$

$$\tan\left(\frac{\varphi'}{2}\right) = \sqrt{\frac{c+v}{c-v}}\ \tan\frac{\varphi}{2}$$

$$\theta' = \varphi' \longrightarrow \tan\left(\frac{\theta'}{2}\right) = \tan\left(\frac{\varphi'}{2}\right) \longrightarrow \sqrt{\frac{c-v}{c+v}}\ \tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{c+v}{c-v}}\ \tan\left(\frac{\varphi}{2}\right)$$

$$\longrightarrow \boxed{\frac{\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\varphi}{2}\right)} = \frac{c+v}{c-v}}$$

$$\begin{cases} \dfrac{\nu_2}{\nu_2'} = \dfrac{\sin\theta}{\sin\theta'} \\ \dfrac{\nu_1}{\nu_1'} = \dfrac{\sin\phi}{\sin\phi'} \end{cases}$$

$$\frac{\nu_2}{\nu_1} = \frac{\nu_2/\nu_2'}{\nu_1/\nu_1'} = \frac{\sin\theta/\sin\theta'}{\sin\varphi/\sin\varphi'} = \frac{\sin\theta}{\sin\varphi}$$

($\nu_2' = \nu_1'$, $\theta' = \varphi'$)

$$\longrightarrow \frac{\lambda_1}{\lambda_2} = \frac{\nu_2}{\nu_1} = \frac{\sin\theta}{\sin\varphi}$$

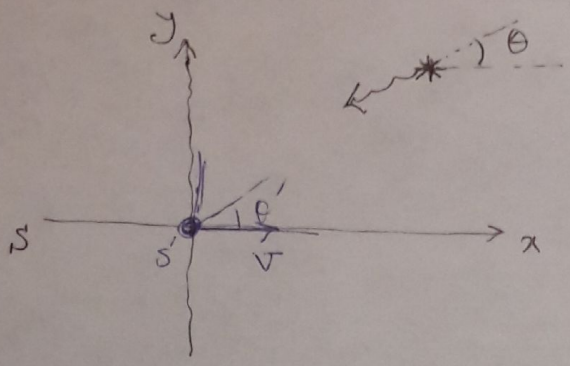
از روابط ابراهی

$$\longrightarrow \begin{cases} \sin\theta' = \dfrac{\sin\theta}{\gamma\left(1+\frac{v}{c}\cos\theta\right)} \\ \sin\varphi' = \dfrac{\sin\varphi}{\gamma\left(1-\frac{v}{c}\cos\varphi\right)} \end{cases} \longrightarrow \frac{\sin\theta}{\sin\varphi} = \frac{1+\frac{v}{c}\cos\theta}{1-\frac{v}{c}\cos\varphi} = \frac{c+v\cos\theta}{c-v\cos\varphi}$$

$$\begin{cases} \cos\theta' = \dfrac{\cos\theta + v/c}{1+v/c\cos\theta} \\ \cos\varphi' = \dfrac{\cos\varphi - v/c}{1-v/c\cos\varphi} \end{cases} \longrightarrow \frac{1+\frac{v}{c}\cos\theta}{1-v/c\cos\varphi} = \frac{\cos\theta+v/c}{\cos\varphi - v/c} = \frac{c\cos\theta + v}{c\cos\varphi - v}$$

$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{\sin\theta}{\sin\varphi} = \frac{c+v\cos\theta}{c-v\cos\varphi} = \frac{c\cos\theta+v}{c\cos\varphi - v}}$$

سوال چهارم :



$$\cos\theta' = \frac{\cos\theta + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta}$$

$$\longrightarrow \theta' = \cos^{-1}\left(\frac{\cos\theta + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta}\right)$$

$$\theta' < \frac{\pi}{2} \longrightarrow 0 < \cos\theta' < 1 \longrightarrow 0 < \frac{\cos\theta + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta} < 1$$

$$\left(1 + \frac{v}{c}\cos\theta > 0\right) \longrightarrow 0 < \cos\theta + \frac{v}{c} \longrightarrow \boxed{\cos\theta > -\frac{v}{c}}$$

$$\frac{\cos\theta + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta} < 1 \longrightarrow \left(1 - \frac{v}{c}\right)\cos\theta < \left(1 - \frac{v}{c}\right) \longrightarrow \cos\theta < 1 \quad \checkmark$$