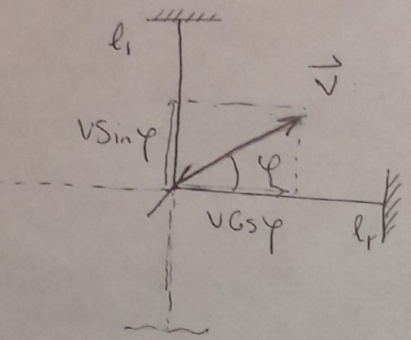
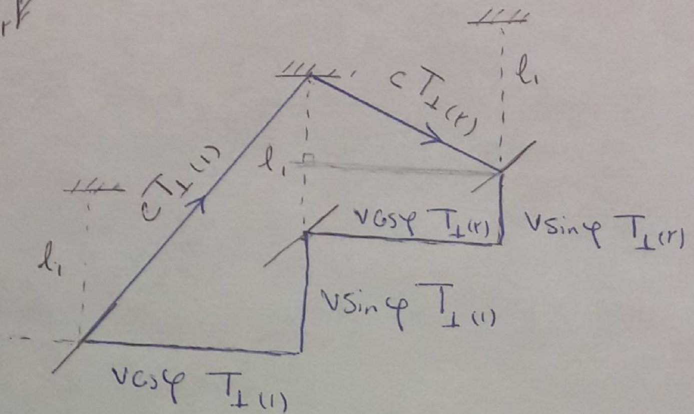
به نام خدا

پاسخ نامه تمرین سری اول نسبیت خاص

سؤال اول





l_1 در بازوی

که در
زمان رفت و برگشت $= T_{\perp(1)} + T_{\perp(2)} := T_\perp$

$T_{\perp(1)}:\quad (cT_{\perp(1)})^2 = (l_1 + v\sin\varphi\, T_{\perp(1)})^2 + (v\cos\varphi\, T_{\perp(1)})^2$

$$\longrightarrow T_{\perp(1)} = \frac{l_1}{\sqrt{c^2 - v^2\cos^2\varphi} - v\sin\varphi}$$

$T_{\perp(2)}:\quad (cT_{\perp(2)})^2 = (l_1 - v\sin\varphi\, T_{\perp(2)})^2 + (v\cos\varphi\, T_{\perp(2)})^2$

$$T_{\perp(2)} = \frac{l_1}{\sqrt{c^2 - v^2\cos^2\varphi} + v\sin\varphi}$$

$$T_\perp = \frac{l_1}{\sqrt{c^2 - v^2\cos^2\varphi} - v\sin\varphi} + \frac{l_1}{\sqrt{c^2 - v^2\cos^2\varphi} + v\sin\varphi} = \frac{l_1\, 2\sqrt{c^2 - v^2\cos^2\varphi}}{c^2 - v^2}$$

l_2 در بازوی $\longrightarrow \left(\varphi \to \frac{\pi}{2} - \varphi\right) \longrightarrow \begin{cases} \cos\varphi \to \sin\varphi \\ \sin\varphi \to \cos\varphi \end{cases}$

$$T_\parallel = \frac{l_2\, 2\sqrt{c^2 - v^2\sin^2\varphi}}{c^2 - v^2}$$

$$\rightarrow \Delta T = \frac{2l_1\sqrt{c^2-v^2\cos^2\varphi}}{c^2-v^2} - \frac{2l_2\sqrt{c^2-v^2\sin^2\varphi}}{c^2-v^2}$$

$$\Delta T = \frac{2l_1 c}{c^2-v^2}\left(1-\frac{v^2\cos^2\varphi}{2c^2}\right) - \frac{2l_2 c}{c^2-v^2}\left(1-\frac{v^2\sin^2\varphi}{2c^2}\right)$$

$$\Delta T' = \frac{2l_1 c}{c^2-v^2}\left(1-\frac{v^2\sin^2\varphi}{2c^2}\right) - \frac{2l_2 c}{c^2-v^2}\left(1-\frac{v^2\cos^2\varphi}{2c^2}\right)$$

به ازای ۹۰° چرخش
$$\begin{cases} \cos\varphi \rightarrow \sin\varphi \\ \sin\varphi \quad \cos\varphi \end{cases}$$

$$T = \frac{\lambda}{c} \qquad \Delta N = \frac{|\Delta T - \Delta T'|}{T} = \frac{c}{\lambda}|\Delta T - \Delta T'|$$

$$\Delta T - \Delta T' = \frac{-2l_1 c v^2}{(c^2-v^2)2c^2}(\cos^2\varphi - \sin^2\varphi) - \frac{2l_2 c v^2}{(c^2-v^2)2c^2}(\cos^2\varphi - \sin^2\varphi)$$

$$= -\frac{\cancel{2} c v^2}{\cancel{2}(c^2-v^2)c^2}(l_1+l_2)(\underbrace{\cos^2\varphi - \sin^2\varphi}_{\cos 2\varphi}) \underset{v \ll c}{\simeq} -\frac{v^2}{c^2}(l_1+l_2)\cos 2\varphi$$

$$\longrightarrow \Delta N = \frac{\frac{v^2}{c^2}(l_1+l_2)\cos 2\varphi}{\frac{\lambda}{c}} = \frac{v^2}{c^2}\frac{1}{\lambda}(l_1+l_2)\cos 2\varphi$$

سؤال دوم

$$\begin{cases} x \longrightarrow x+vt = x' \\ t \longrightarrow t+\delta t = t' \end{cases}$$

برحسب x, v خطی است

$$c^2t^2 - x^2 = c^2t'^2 - x'^2$$

$$= c^2(t+\delta t)^2 - (x+vt)^2$$

$$= \left[c^2t^2 + 2c^2t\,\delta t + O(v^2)\right] - \left[x^2 + 2xvt + O(v^2)\right]$$

$$= (c^2t^2 - x^2) + (\underbrace{2c^2t\,\delta t - 2xvt}_{=0})$$

$$\longrightarrow \boxed{\delta t = \frac{xv}{c^2}}$$

$$\begin{cases} x \longrightarrow x + vt + \delta x^{?} \\ t \longrightarrow t + \frac{v}{c^2}x + (\delta t) \end{cases}$$

از مرتبه $O(v^2)$
وخطی نسبت به t, x

$$c^2t'^2 - x'^2 = c^2\left(t^2 + 2t\frac{v}{c^2}x + \frac{x^2v^2}{c^4}\right) - \left(x^2 + 2xvt + v^2t^2 - 2x\,\delta x\right) + O(v^3)$$

$$= (c^2t^2 - x^2) + \left(\cancel{2xvt} + \frac{x^2v^2}{c^2} - \cancel{2xvt} - \cancel{v^2t^2} - 2x\,\delta x\right)$$

$$\longrightarrow \boxed{\delta x = \frac{xv^2}{2c^2}}$$

$$c^2t'^2 - x'^2 = c^2\left(t^2 + \frac{v^2x^2}{c^4} + 2t\frac{v}{c^2}x + \underbrace{2\delta t\left(t + \frac{v}{c^2}x\right)}_{2\delta t\, t + O(v^3)}\right) - \left(x^2 + v^2t^2 + 2xvt + 2x\frac{xv^2}{2c^2}\right)$$

$$= (c^2t^2 - x^2) + \left(\cancel{\frac{v^2x^2}{c^2}} + \cancel{2xvt} + 2\delta t\, tc^2 - v^2t^2 - \cancel{2xvt} - \cancel{\frac{x^2v^2}{c^2}}\right)$$

$$\longrightarrow \boxed{\delta t = \frac{v^2t}{2c^2}}$$

$$\begin{cases} x \longrightarrow x + vt + \frac{v^2}{2c^2}x \\ t \longrightarrow t + \frac{v}{c}x + \frac{v^2}{2c^2}t \end{cases}$$

$$x \longrightarrow \gamma(x+vt) = \frac{x+vt}{\sqrt{1 - v^2/c^2}} = (x+vt)\left(1 + \frac{v^2}{2c^2}\right) = x + vt + \frac{v^2x}{2c^2} + \cdots$$

$$t \longrightarrow \gamma\left(t + \frac{v}{c^2}x\right) = \frac{t + v/c^2\, x}{\sqrt{1 - v^2/c^2}} = \left(t + \frac{v}{c^2}x\right)\left(1 + \frac{v^2}{2c^2}\right) = t + \frac{v}{c^2}x + \frac{v^2t}{2c^2} + \cdots$$

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سؤال سوم

$$\begin{cases} t' = t \\ x' = x - vt \end{cases} \qquad \partial/\partial t = \partial/\partial x' \underbrace{\partial x'/\partial t}_{-v} + \partial/\partial t' \underbrace{\partial t'/\partial t}_{1}$$

$$= \partial/\partial t' - v\, \partial/\partial x'$$

$$\partial/\partial x = \partial/\partial x' \underbrace{\partial x'/\partial x}_{1} + \partial/\partial t' \underbrace{\partial t'/\partial x}_{0} = \partial/\partial x'$$

$$\longrightarrow \begin{cases} \partial/\partial t = \partial/\partial t' - v\,\partial/\partial x' \longrightarrow \partial^2/\partial t^2 = (\partial/\partial t' - v\,\partial/\partial x')(\partial/\partial t' - v\,\partial/\partial x') \\ \partial/\partial x = \partial/\partial x' \longrightarrow \nabla^2 = \nabla'^2 \end{cases}$$

معادله کلاین-گوردن : $(\Box + m^2)\phi(x,t) = 0 \longrightarrow \left(\frac{1}{c^2}\partial^2/\partial t^2 - \nabla^2 + m^2\right)\phi(x,t) = 0$

$$\longrightarrow \left[\frac{1}{c^2}\left(\partial^2/\partial t'^2 - 2v\frac{\partial^2}{\partial t'\partial x'} + v^2\,\partial^2/\partial x'^2\right) - \nabla'^2 + m^2\right]\phi(x',t') = 0$$

$$\longrightarrow \left(\frac{1}{c^2}\partial^2/\partial t'^2 - \nabla'^2 + m^2\right)\phi(x',t') + \left(\frac{v^2}{c^2}\nabla'^2 - \frac{2v}{c^2}\frac{\partial^2}{\partial t'\partial x'}\right)\phi(x',t') = 0$$

ناوردا نیست .

$$\begin{cases} t' = f(v)\left[t - \frac{v}{c^2}x\right] \\ x' = f(v)(x - vt) \end{cases} \longrightarrow \partial/\partial x = \partial/\partial x' \underbrace{\frac{\partial x'}{\partial x}}_{f(v)} + \partial/\partial t' \underbrace{\frac{\partial t'}{\partial x}}_{-v/c^2 f(v)}$$

$$= f(v)\left[\partial/\partial x' - \frac{v}{c^2}\,\partial/\partial t'\right]$$

$$\partial/\partial t = \partial/\partial x' \underbrace{\frac{\partial x'}{\partial t}}_{-v f(v)} + \partial/\partial t' \underbrace{\frac{\partial t'}{\partial t}}_{f(v)}$$

$$= f(v)\left[\partial/\partial t' - v\,\partial/\partial x'\right]$$

$$\left[\frac{1}{c^2} f^2(v)\left(\partial/\partial t' - v\,\partial/\partial x'\right)\left(\partial/\partial t' - v\,\partial/\partial x'\right) - f^2(v)\left(\partial/\partial x' - v/c^2\,\partial/\partial t'\right)\left(\partial/\partial x' - \frac{v}{c^2}\partial/\partial t'\right)\right.$$

$$\left. + m^2\right]\phi(x',t') = \left\{f^2(v)\left[\frac{1}{c^2}\partial^2/\partial t'^2 - \frac{2v}{c^2}\cancel{\frac{\partial^2}{\partial x'\partial t'}} + \frac{v^2}{c^2}\partial^2/\partial x'^2 - \partial^2/\partial x'^2 + \frac{2v}{c^2}\cancel{\frac{\partial^2}{\partial x\partial t'}}\right.\right.$$

$$\left.\left. - \frac{v^2}{c^4}\partial^2/\partial t'^2\right] + m^2\right\}\phi(x',t') = 0$$

$$\longrightarrow \left[f_{(v)}^2 \frac{1}{c^2}\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2}{\partial t'^2} + f_{(v)}^2\left(\frac{v^2}{c^2}-1\right)\frac{\partial^2}{\partial x'^2} + m^2 \right]\psi(x',t') = 0$$

$$\longrightarrow f_{(v)}^2\left(1-\frac{v^2}{c^2}\right) = 1$$

$$- f_{(v)}^2\left(1-\frac{v^2}{c^2}\right) = -1 \longrightarrow \boxed{f_{(v)} = \frac{1}{\sqrt{1-v^2/c^2}}}$$

کلاین-گوردن

معادله شرودینگر :

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right)\psi(x,t)$$

$$\longrightarrow i\hbar\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)\psi(x',t') = -\frac{\hbar^2}{2m}\left(\nabla'^2 + V(x')\right)\psi(x',t')$$

$$\longrightarrow i\hbar\frac{\partial}{\partial t'}\psi(x',t') = \left[-\frac{\hbar^2}{2m}\left(\nabla'^2 + V(x')\right) + \underline{i\hbar v\frac{\partial}{\partial x'}}\right]\psi(x',t')$$

صورت معادله ناوردا نمی ماند.

(اما با توجه به اینکه تابع موج کوانتومی تا اختلاف در یک فاز کلی قابل تشخیص است و ترم اضافه شده به معادله تنها یک فاز کلی به $\psi(x,t)$ اضافه می کند و بدین معنی معادله ناورداست) :

با یک مثال: موج تخت

$$\psi(x',t') = \int dx'dt'\,\alpha(x',t')\,e^{i(p'x'-E't')/\hbar}$$

$$e^{i(p'x'-E't')/\hbar} \qquad x' = x - vt \longrightarrow \dot{x}' = \dot{x} - v \longrightarrow p' = p - mv$$

$$p'x' - E't' = (p-mv)(x-vt) - \frac{(p-mv)^2}{2m}t = px - \cancel{pvt} - mvx + mv^2t - \frac{p^2}{2m}t - \frac{mv^2}{2}t + \cancel{pvt}$$

$$= \left(px - \frac{p^2}{2m}t\right) \underbrace{- mv\left(x - \frac{vt}{2}\right)}_{f(x,t)} = (px - Et) - f(x,t)$$

$$e^{i(p'x'-E't')/\hbar} = e^{i(px-Et)/\hbar}\, e^{-i f(x,t)/\hbar}$$

(فاز کلی وابسته به E,p نیست) .

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