Special Topics in Cosmology (Spring 2013)

Problem Set 5

1)Show that the tensor-to-scalar ratio of primordial perturbations in slow roll paradigm is equal to slow roll parameter as:

$$r = \frac{\mathcal{P}_t(k_*)}{\mathcal{P}_s(k_*)} \approx 16\epsilon \tag{1}$$

2)Prove the Lyth bound condition, which relates the inflaton field change with the number of e-fold N and the tensor-to-scaler ratio r:

$$\frac{\Delta\phi}{M_{pl}} \sim \frac{1}{\sqrt{8}} \int_0^\infty dN\sqrt{r}, \tag{2}$$

3)**Position of the acoustic peaks**

a)Calculate the sound horizon at decoupling,

$$r_s = (1+z)^{-1} \int_0^{t_{dec}} dt \frac{c_s}{a}, \tag{3}$$

in terms of z_{dec}, H_0, $\omega_m \equiv \Omega_m^0 h^2$ and $\omega_b \equiv \Omega_b^0 h^2$. Assume a constant speed of sound, $c_s = c_s(t_{dec})$, but include the effect of radiation and matter components in the expansion law. Neglect neutrino masses.

b)What is the separation l_A between the acoustic peaks in CMB angular spectrum C_l for the cases, $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 1 - \Omega_m$?

c)Give the numerical values of r_s and l_A for $z_{dec} = 1090$, $h = 0.7$, $\Omega_m 0 = 0.3$ and $\omega_b = 0.02$. Give also the numerical value of the comoving sound horizon.

4)**The effect of a varying sound speed**.
Same as the previous problem, but now take into account that the sound speed evolves.

5)Assume that the CMB multipole coefficients a_{lm}, $m = -l, ..., +l$ are independent Gaussian random variables for $m = 0, ..., l$, with variance $\langle |a_{lm}|^2 \rangle = C_l$ (same for all m). The coefficient a_{l0} is real but the other a_{lm} are complex, and $a_{l,-m} = a_{lm}^*$.
The observed angular power spectrum $\hat{C}_l$ is defined as:

$$\hat{C}_l \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2. \tag{4}$$

Clearly, $\langle \hat{C}_l \rangle = C_l$. Calculate the cosmic variance, defined as the expectation value

$$\langle (\hat{C}_l - C_l)^2 \rangle. \tag{5}$$