1. Using the fact that the reduced mass of the electron-nucleus in the D (deuterium) atom is larger than in hydrogen, and the fact that the Lyman α ($n = 1 \to n = 2$) transition in H has a wavelength 1215.67 Å, find the wavelength of the photon emitted in the corresponding transition in D. Astronomers often define

$$v \equiv c\frac{\Delta\lambda}{\lambda}$$

to characterise the splitting of two nearby lines. What is v for H-D pair?
(Modern Cosmology, Scott Dodelson, Academic press, 2003, Chapter 1, Exercise 3)

2. Show that in non-reletivistic limit, $m \gg T$ both, the Fermi-Dirac and the Bose-Einstein distributions reduce to Maxwell-Boltzmann distribution and the number and energy density is given by

$$\begin{aligned} n &= \frac{2}{(2\pi)^3} exp(-(m-\mu)/T)(2\pi m T)^{3/2}, \\ \rho &= mn, \end{aligned}$$

where μ is the chemical potential.
(The Cosmic Microwave Background, Ruth Durrer, Cambridge university press, 2008, Chapter 1, Exercise 5)

3. Determine the angular diameter distance to the last scattering surface under the assumption $K = \Lambda = 0$. Under which angle do we presently see the causal horizon of this time, $a(t_{rec})$? How does this result change if one admits a cosmological constant so that $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$?
(The Cosmic Microwave Background, Ruth Durrer, Cambridge university press, 2008, Chapter 1, Exercise 8)

4. Consider the composition of the universe at a temperature slightly lower than 10^{12} K. We will assume that the universe is populated by electrons (e), positrons ($\bar{e}$), three species of massless neutrino (ν_e, ν_μ, ν_τ), neutrons (n), protons (p) and photons (γ).
(a) Estimate the number density of $p, n, e, \bar{e}, \nu_e, \nu_\mu$ and ν_τ relative to the photons γ at this temperature.
(b) Since neutrinos have no electric charge they have no direct, strong coupling with photons. Their interaction with baryons can be ignored because of the low density of baryons. So they are kept in equilibrium essentially through reactions like $\nu\bar{\nu} \leftrightarrow e\bar{e}$, $\nu e \leftrightarrow \nu e$, etc. The cross-section, $\sigma(E)$, for these weak interaction processes is of the order of ($\alpha^2 E^2/m_\chi$), where $\alpha \cong 2.8x10^{-2}$ and $m_\chi = 50$ GeV is the mass of the gauge vector boson mediating the weak interaction. Show that neutrinos decouple from the rest of the

matter at temperatures below $T_D \cong 1.4$ MeV.
(c) At the time of ν-decoupling, the photons, neutrinos and the rest of the matter had the same temperature. As long as the photon temperature decreases as a^{-1}, neutrinos and photons will continue to have the same temperature even though the neutrinos have decoupled. However, the photon temperature will decrease at a slightly lower rate if the g-factor is changing. In that case, T_γ will become higher than T_ν as the Universe cools. Such a change in the value of g occurs when the temperature of the Universe falls below $T = m_e$. The electron rest mass $m_e \cong 0.5$ MeV corresponds to a temperature of 5×10^9 K. When the temperature of the universe becomes lower than this value the mean energy of the photons will fall below the energy required to create ee-pairs. Thus the backward reaction in $e\bar{e} \leftrightarrow \gamma\gamma$ will be severely suppressed. The forward reaction will continue to occur, resulting in the disappearance of the ee pairs. Show that after e — $\bar{e}$ annihilation is complete, the temperature of the photons T_γ will be related to the temperature of the neutrinos T_ν by $T_\gamma = \frac{11}{4} T_\nu$. Use this to estimate the fraction of critical density Q_R contributed by relativistic particles today and show that $Q_R\, h^2 = 4.3 \times 10^{-5}$. Calculate the epoch at which non-relativistic and relativistic particles were contributing equal amounts of energy density and show that $(1 + z_{eq}) = 2.3 \times 10^4\, \Omega\, h^2$. What was the temperature T_{eq} and time t_{eq} at this epoch?
(Cosmology and Astrophysics through Problems, T. Padmanabhan, Cambridge university press, 1996, Exercise 6.7)

5. Explain how the Cold Dark Matter particles (WIMPs) decouple from the cosmic fluid in the early Universe. You can study the following reference: Cosmology, Steven Weinberg, Oxford university press, 2008, page 183