Non Linear Structure Formation

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I. PRESS SCHECHTER FORMALISM

The Press-Schechter formalism [1] is used as a framework to obtain the number of structures in mass interval of M and $M + dM$. The idea is that the probability distribution function (PDF)of density contrast is related to the probability distribution of structures. The crucial point is how to relate these two PDFs. Assume that the PDF of density contrast is a Gaussian function:

$$\mathcal{P}_R(\delta) = \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left[-\frac{\delta^2}{2\sigma_R^2}\right], \tag{1}$$

where $\mathcal{P}$ is the PDF of density contrast, and R is the smoothing scale where mass M is enclosed, considering the background density of matter $R = (3M/4\pi\bar{\rho}_m)^{1/3}$, and the σ_R is the variance of perturbation in the linear regime defined as:

$$\sigma_R^2 = \int \frac{dk}{2\pi^2} k^2 P_L(k) W^2(kR), \tag{2}$$

where P_L and $W(kR)$ are linear matter power spectrum and the window function. The linear matter power spectrum defined as:

$$P_L(k, z) = A_{sf} T(k)^2 D^2(z) k^{n_s} \tag{3}$$

where A_{sf} is the perturbation amplitude, the subscript sf indicate that the amplitude is defined for structures for late time Universe. The $T(k)$ is the transfer function in which the physics of horizon crossing is imprinted for each scale. $D(z)$ is the growth function. (We will discuss all the physics of Transfer function and Growth function in linear theory). The window function for top-hat model is defined as:

$$W(x) = 3\frac{\sin(x) - x\cos(x)}{x^3}, \tag{4}$$

where $x \equiv kR$. Now coming back to the PDF of the density perturbations, the great idea! of Press and Schechter is that the fraction of collapsed objects with $M > M^* = 4\pi/3\bar{\rho}_m R^3$ is equal to the integral over PDF, from perturbations that pass the threshold of spherical collapse δ_c

$$F(> M) = \int_{\delta_c}^{\infty} \mathcal{P}_R(\delta) d\delta = \int_{\delta_c}^{\infty} \frac{d\delta}{\sqrt{2\pi}\sigma_R} \exp^{-\delta^2/2\sigma_R^2} = \frac{1}{2} erfc[\frac{\delta_c}{\sqrt{2}\sigma_R}], \tag{5}$$

where $F(> M)$ is the fraction of collapsed objects. An important point here to indicate is that the fraction of collapsed objects depend on the critical density of spherical collapse which is a local quantity and to the cosmological parameters, through the variance of matter perturbations. In the limit of $M \to 0$, the variance σ_R goes to Infinity and the $erfc(0) = 1$. This limit shows that only half of the structures in the Universe are in structures. The missing factor 2 is caused from the cloud-in-cloud effect. In the next section we discuss the excursion set theory and we find the factor 2 naturally. Now we can calculate the fraction of the collapsed objects $f(M)$ in the mass range of M and $M + dM$ as below:

$$F(>M) - F(>M + dM) = \frac{\partial F}{\partial M} dM = f(M)dM, \tag{6}$$

Now the comoving number density of collapsed objects in the mass range of M and $M+dM$ is as below:

$$n(M)dM = \frac{N_s \times f(M)}{V_t} dM = \frac{V_t/(M/\bar{\rho}) \times f(M)}{V_t} dM, \tag{7}$$

where N_s is the number of smoothed patches, V_t is the total volume and the $\bar{\rho}$ is the aveage density of matter. Now the comoving number density of structures is obtained as:

$$n(M) = -2\frac{\bar{\rho}}{M}\frac{\partial F}{\partial M} dM, \tag{8}$$

where we add the factor 2. In order to go further we should find the mass derivative F, where we define a new parameter $x \equiv \delta/\sigma_R$:

$$F = \int_{\delta_c}^{\infty} \frac{dx}{\sqrt{2\pi}} \exp^{-x^2/2}, \tag{9}$$

so the mass derivative of F can be expanded as:

$$\frac{\partial F}{\partial M} = \frac{1}{\sqrt{2\pi}} \delta_c \frac{\partial \sigma_m^{-1}}{\partial M} e^{-x^2/2}, \tag{10}$$

where we use $\partial F/\partial M \equiv (\partial F/\partial x)\delta_c(\partial\sigma^{-1}/\partial M)$ Notice that we change the subscript R to M for variance as they are interchangeable. Now by defining a very importent quantity ν as the height parameter:

$$\nu \equiv \frac{\delta_c}{\sigma_M}, \tag{11}$$

Now $\partial F/\partial M$ will be:

$$\frac{\partial F}{\partial M} = -f(\nu)\frac{d\ln\sigma_M}{d\ln M}.\frac{1}{M}, \tag{12}$$

where $f(\nu)$ is the Press-Schechter function obtained as:

$$f(\nu) = \frac{1}{\sqrt{2\pi}} \nu \exp^{-\nu^2/2}. \tag{13}$$

Now the comoving density of collapsed objects are obtained as:

$$n_{ps}(M) = -2\frac{\bar{\rho}}{M^2} f(\nu)\frac{d\ln\sigma_R}{d\ln M}, \tag{14}$$

where the primary mass function is obtained by Press-Schechter function defined in Eq.(13).

[1] W. H. Press and P. Schechter, Astrophys. J. **187**, 425 (1974).