1

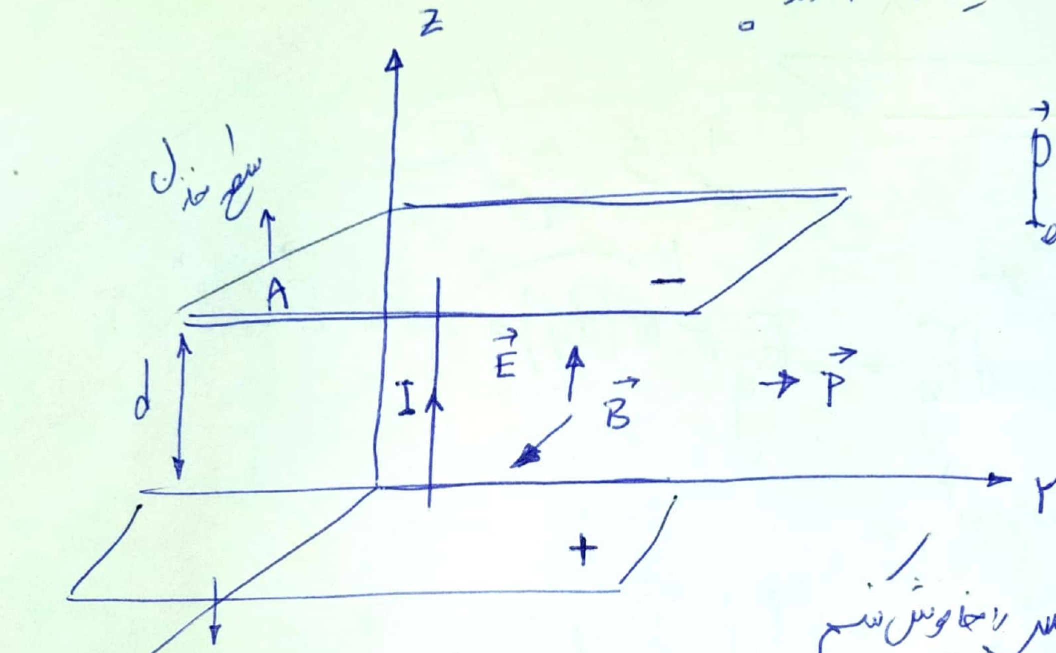
الکترومغناطیس ۲ گرفتیم

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$ بردار پوینتینگ

$$\vec{g} = \varepsilon_0 \mu_0 \vec{S} = \varepsilon_0 \vec{E} \times \vec{B}$$

• حامل میدان الکترومغناطیسی : موج الکترومغناطیسی

• میدان الکتریکی و مغناطیسی استاتیک تکانه دارند!



$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$\vec{P}_{em} = \varepsilon_0 E B A d\, \hat{y}$$

• اگر میدان الکتریکی یا مغناطیسی را خاموش کنیم

Capacitor حرکت خواهد کرد.

$$I_{impulse} = \int F\, dt = dB \int_{Q}^{0} \left(-\frac{dq}{dt}\right) dt = dBQ$$ Lorentz force law!

فاصله ۲ خازن

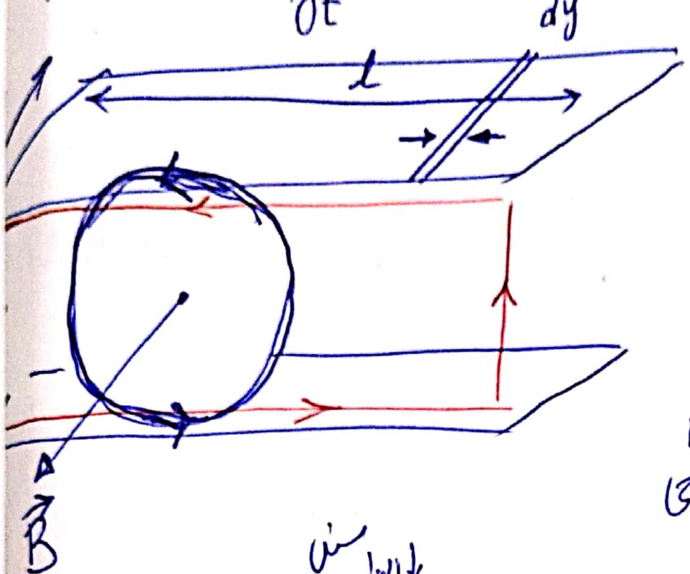
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$$I_{impulse} = \varepsilon_0 E B A d\, \hat{y}$$

quasi-static. no radiation!

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$



Faraday's law.

چگالی سطحی

عرض

$\vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$

$$dF_y = -E_y\,\sigma\,w(dy) = -\sigma w\,\vec{E}\cdot d\vec{l}$$

نیرو در راستای y

$$F_y = \sigma w \oint \vec{E}\cdot d\vec{l} = -\sigma w d l \left(\frac{dB}{dt}\right)$$

$$I = \int F dt = -\sigma \underbrace{w\,d\,l}_{\text{area}}\left(\frac{dB}{dt}\right)$$

$$= \int F dt = -\sigma w d l \int_B^0 \left(\frac{dB}{dt}\right) dt\,\hat{y} = \sigma w d l B\,\hat{y}$$

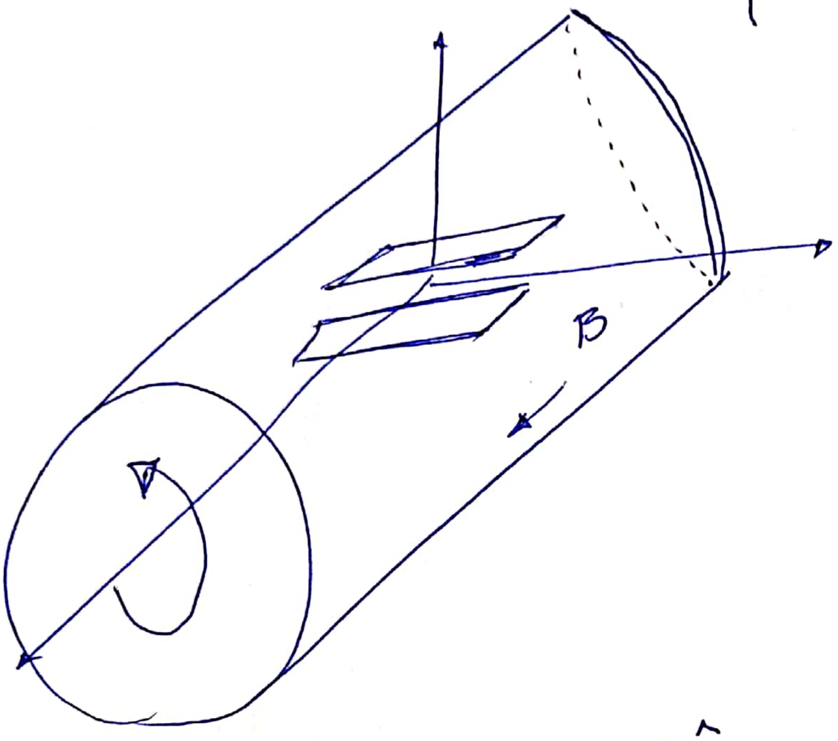
$$= dBQ\,\hat{y}$$

3

North Carolina student: Babson's

long Solonoied

$$\left\{\begin{array}{l} E(2\pi r) = -\pi r^2 \frac{dB}{dt} \\ \vec{E} = -\frac{r}{2}\frac{dB}{dt}\hat{\phi} \end{array}\right.$$



$$\vec{E}\cdot d\vec{l} = -\frac{r}{2}\frac{dB}{dt}\underbrace{(-\sin\phi\,\hat{y} + \cos\phi\,\hat{z})}_{\hat{\phi}}\cdot(dx\hat{x} + dy\hat{y} + dz\hat{z})$$

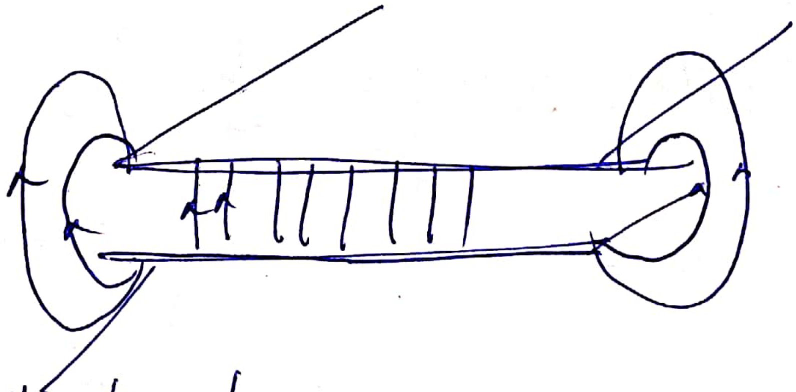
integrat- large

$$= \frac{1}{2}\frac{dB}{dt}(r\sin\phi\, dy - r\cos\phi\, dz) = \frac{1}{2}\frac{dB}{dt}(z\,dy - y$$

$$F = \sigma w\left(-\int_{top}\vec{E}\cdot d\vec{l} + \int_{bottom}\vec{E}\cdot d\vec{l}\right)$$

$$= \sigma w\left[-\frac{1}{2}\frac{dB}{dt}\frac{d}{2}l + \frac{1}{2}\frac{dB}{dt}\left(-\frac{d}{2}\right)l\right] = -\frac{1}{2}\sigma w d \times \frac{dB}{dt}$$

Fringing field :



superposition of the two !

$$P_{em} = \varepsilon_0 (\vec{E} \times \vec{B})$$

$\vec{E}$ is uniform
super-position,

$$P_{em} = \frac{1}{c^2} (\vec{E} \times \vec{m})$$

$$P_{em} = \frac{1}{2} (\vec{B} \times \vec{p})$$

الکترومغناطیس ۲

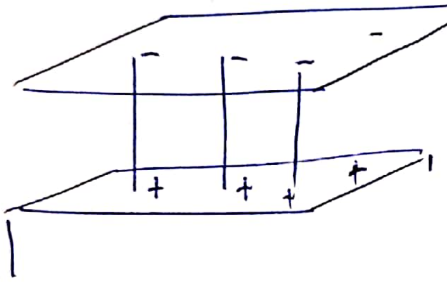
Momentum initially stored in fields $BQd\,\hat{y}$

Momentum delivered to capacitor discharge $BQd\,\hat{y}$

Momentum delivered to capacitor B decrease $\frac{1}{2}BQd\,\hat{y}$

- $P_{em} = \varepsilon_0\, \vec{E}\times\vec{B}$ does not satisfy superposition

  $P_{em}, \vec{L}, u$



$$P_{em} = \frac{1}{c^2}(\vec{E}\times\vec{m}) \quad \text{uniform electric f}$$

$$P_{em} = \frac{1}{2}(\vec{B}\times\vec{p}) \quad \text{uniform magnetic fie}$$

$$= \frac{1}{2}BQd\,\hat{y} \quad \checkmark$$

Kirk T. McDonald.

$C = \frac{A}{4\pi d}$ Gaussian units.

radiation & retardation neglected.

$$P_{EM} = \int \frac{\rho\vec{A}}{c}\,dV \overset{\text{Faraday\_Maxwell}}{=} \int \frac{E\times B}{4\pi c}\,dV = \int \frac{\Phi\vec{J}}{c^2}\,dV$$

Poyntings Furry

$$= \int \frac{\vec{J}\cdot\vec{E}}{c^2}\, r\,dV \quad \text{Aharanov}$$

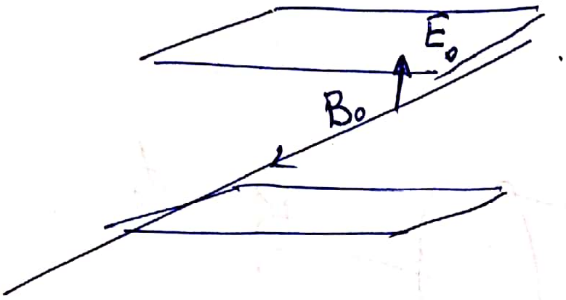
J. J. Thomson

Poincaré

$\vec{\nabla}\cdot\vec{A} = 0$ Coloumb gauge.

$\vec{A}$: magnetic vector , ρ : charge

1 Kirk T. McDonald



$C = \frac{A}{4\pi d}$ in Gaussian units.

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

$\vec{\mathcal{P}} = \varepsilon_0 \mu_0 \vec{S} = \varepsilon_0 \vec{E} \times \vec{B}$, $\vec{P} = \varepsilon_0 A d E B$

چگالی تکانه

$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A \varepsilon_0}$ $\vec{P} = Q d B \hat{j}$

$\varepsilon_0 \to \frac{1}{4\pi c}$

$\frac{1}{2} \quad 8 \int^{\omega}$

radiation & retardation is ignored.

$$P_{EM} = \int \frac{\rho \vec{A}}{c} dV = \int \frac{\vec{E} \times \vec{B}}{4\pi c} dV = \int \frac{\Phi \vec{J}}{c^2} dv = \int \frac{\vec{J}}{c^2}$$

1852 Faraday – 1865 Maxwell 1884 Poynting – 1891 J.J. Thomson Furry 1969 Ahar… 198…

Φ پتانسیل ρ چگالی بار Poincare 1900

A J چگالی جریان

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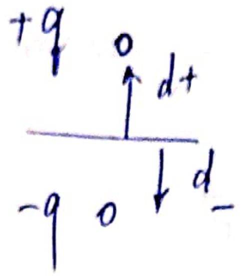
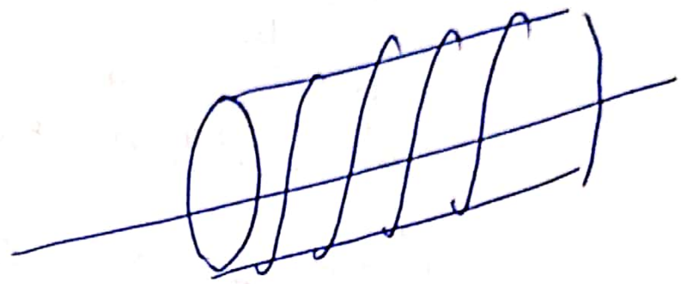
Classical Version. Momentum of photon

$p = q d$ electric_dipole $d = d^+ - d^-$



$A_\phi = \rho B_0/2$



$B_0 \equiv cte$

$$= \frac{\rho}{2} B_0 \hat{\phi} = \frac{B_0}{2}\left(-\rho \sin\phi \hat{x} + \rho \cos\phi \hat{y}\right)$$

$$= \frac{B_0}{2}\left(-y\hat{x} + x\hat{y}\right) = \frac{B_0}{2} \hat{z} \times \hat{r} = \frac{\vec{B}_0 \times \vec{r}}{2}$$

$\vec{r} = (x, y, z) = (\rho, \phi, z)$ cylindrical.

$$= \int \frac{\rho \vec{A}}{c} dV = \frac{q B_0}{2c} \times (d^+ - d^-) = \frac{\vec{B}_0 \times \vec{p}}{2c}$$

$$\nabla \times \vec{V} = \left[\frac{1}{s}\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right]\hat{\phi}$$

$$+ \frac{1}{s}\left[\frac{\partial}{\partial s}(s V_\phi) - \frac{\partial V_s}{\partial \phi}\right]\hat{z} \Bigg| \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{\rho}{2}\rho V_\phi\right) =$$

$$\begin{cases} P_{em} = \frac{1}{c^2}(\vec{E}\times\vec{m}) \\ P_{em} = \frac{1}{2}(\vec{B}\times\vec{p}) \end{cases}$$

Stored in field!

$$\vec{p} \to \underline{\underline{QD}}$$

$$: \underline{\underline{\frac{1}{2} BQd\,\hat{y}}}$$

Center of Energy theorem:

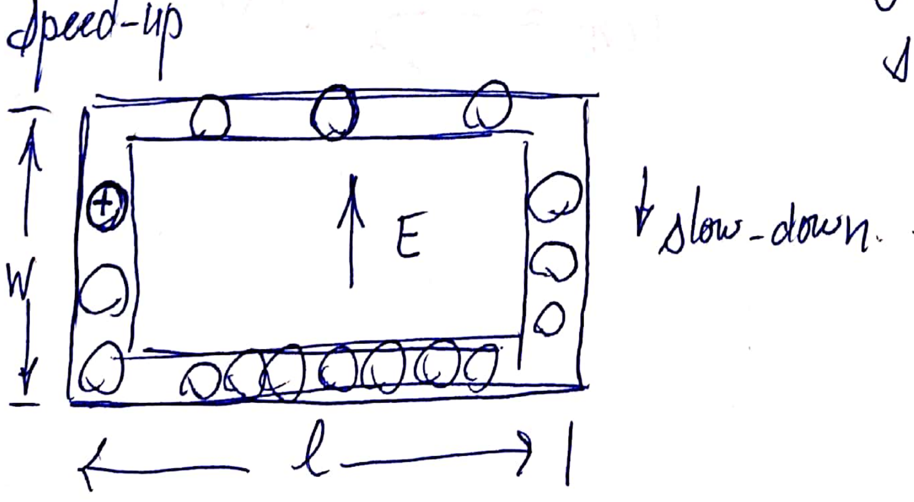
Closed system:

Total Momentum is @ rest.

Hidden-Momentum, (Motion in whole)

Toy-model

steady Current



$$I = \frac{qN_t}{l} V_t = \frac{qN_b}{l} V_b$$

Calkin Model

$$N_t v_t = N_b v_b = \frac{Il}{q}$$

$$P_{class} = m N_t v_t - m N_b v_b = m\frac{Il}{q} - m\frac{Il}{q} = 0$$

$$P_{rel} = \gamma_t m N_t v_t - \gamma_b m N_b v_b$$

$$= \frac{mIl}{q}(\gamma_t - \gamma_b)$$

$$\gamma_t - \gamma_b = \frac{qEw}{mc^2}$$

$$P_{rel} = \frac{\overbrace{IlEw}^{m}}{c^2}$$

$$E = \gamma mc^2$$

$$P_{Hid} = \frac{1}{c^2}(\vec{m} \times \vec{E})$$

$$P_{em} = -\frac{1}{c^2}(\vec{m} \times \vec{E})$$

energy

c

γ_{cm}

!

are

5

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma\left(t - \frac{vx}{c^2}\right) \\ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \end{cases}$$

$$\begin{cases} p^\mu \equiv \frac{dx^\mu}{d\tau} = \gamma m \frac{dx^\mu}{dt} \\ E = p^0 = \gamma \frac{dx^0}{dt} = \gamma c \end{cases}$$

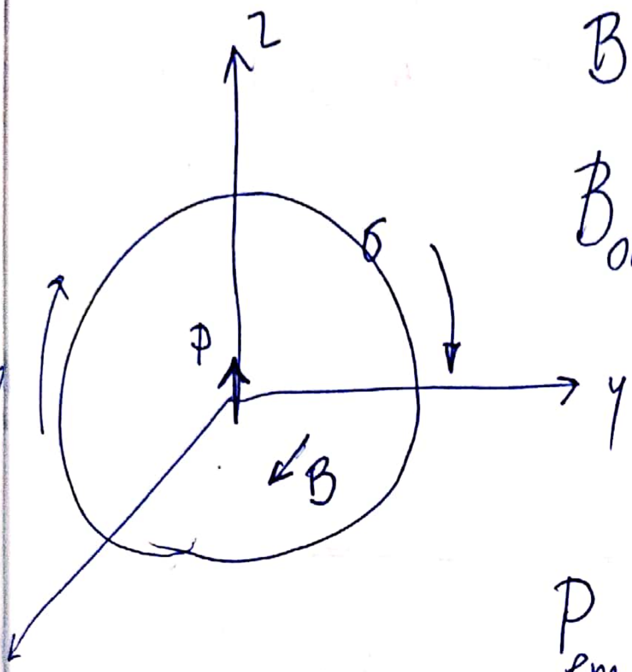
Current in Coil.

charges in cylinder !

presence of electric field!

fringing field ! (Nightmare

Make it easy!



$$B_{in} = \frac{2}{3}\mu_0 \sigma R \omega$$

$$B_{out} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - m\right]$$

$$m = \frac{4\pi}{3}\sigma\omega R^4$$

$$P_{em} = -\frac{1}{2}(\vec{p}\times\vec{B})$$ points to left

$$P_{hid} = \frac{1}{2}(\vec{p}\times\vec{B})$$

electric field

↑ نقش حامل‌ها

$$P_{total} \equiv 0$$

Turn off the electric field.

Lorentz-force!

$$J = \int F\,dt = \int I\,(\vec{dx}\times\vec{B})\,dt = \int_q^0 \left(\frac{dq}{dt}dt\right)\vec{dx}\times\vec{B}$$

$$= -\vec{p}\times\vec{B}$$

7,

$$\tau_{dip} = -(\vec{p} \times \vec{B})$$

$$\boxed{\tau_{sphere} = \frac{1}{2}(\vec{p} \times \vec{B})}$$

in the another direction

changing electric field ;

introduce a magnetic field

exert a force on charge (cur

Turn magnetic field off !

$$\tau_{dip} = -\frac{1}{2}(\vec{p} \times \vec{B})$$

Turn

Co

8

original $P_{em} = -\frac{1}{2}(\vec{p}\times\vec{B})$ $P_{hid} = \frac{1}{2}(\vec{p}\times\vec{B})$ $P_{tot} = 0$

dipole discharges $I_{dip} = -\vec{p}\times\vec{B}$. $I_{sphere} = \frac{1}{2}(\vec{p}\times\vec{B}) + P_{hid}$? 0

sphere slows $I_{dip} = -\frac{1}{2}\vec{p}\times\vec{B}$ $I_{sphere} = 0 + P_{hid}$? 0

$$\downarrow$$

$$? \equiv$$

Hidden-momentum !

- It is purely mechanical
- intrinsicly relativistics
- Current loops (internally moving parts.)
- Not associated motion of center of energy.