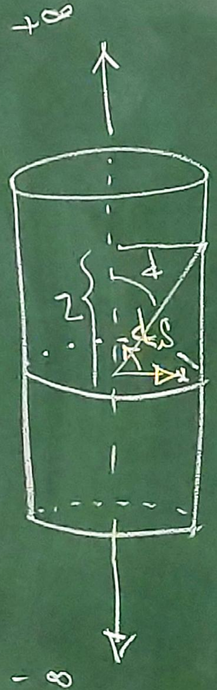
تمرین ۵: پوسته استوانه ای (بی نهایت بلند)، بر روی پوسته یک چگالی بار σ_0 قرار دارد!

سوال: پتانسیل در همه فضا را بدست آورید.

نکته مهم: $\sigma = \sigma(\phi)$



$$\nabla^2 \varphi = 0$$

شرایط مرزی: $\varphi_{inside} = \varphi_{out}\Big|_{s=R}$ + (این بی‌نهایت‌ها جذب نشدا

گسستگی

تقارن: استوانه‌ای، کروی

استوانه‌ای (s, ϕ, z) تقارن در جهت z !

ansatz:

$$\varphi(s,\phi) = S(s)\,\Phi(\phi)$$

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial \varphi}{\partial s}\right)+\frac{1}{s^2}\frac{\partial^2 \varphi}{\partial \phi^2}=0$$

$$\left[\frac{\Phi}{s}\frac{d}{ds}\left(s\frac{dS}{ds}\right)+\frac{S}{s^2}\frac{d^2\Phi}{d\phi^2}=0\right]s^2\varphi^{-1}$$

$$\frac{s}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right)+\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=0$$

$$0=C_1+C_2=0$$

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=-K^2$$

$$\Phi=A_K\sin K\phi+B_K\cos K\phi$$

$$\Phi(\phi+2\pi)=\Phi(\phi)$$

$$(K=0,1,2,\dots)\quad \Phi=\text{Const}$$

$$\frac{s}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right)=K^2$$

$$K=0 \rightarrow \begin{cases} S = cte \\ s\dfrac{dS}{ds} = C \rightarrow S = \overset{cte}{F}\ln s + \overset{cte}{E} \end{cases}$$

$$S = C s^{+K} + D s^{-K}$$

$K \neq 0$ integer

ansatz

$$S = s^n$$

$$\frac{s}{s^n}\frac{d}{ds}\Big(\underbrace{s n s^{n-1}}_{s^n}\Big) = \frac{s}{s^n} n^2 s^{n-1} = n^2 = K^2$$

$$n = \pm K$$

$$\varphi(s,\phi) = a_0 + b_0 \ln s + \sum_{k=1}^{\infty}\Big[s^{k}\left(a_k \cos k\phi + b_k \sin k\phi\right)$$

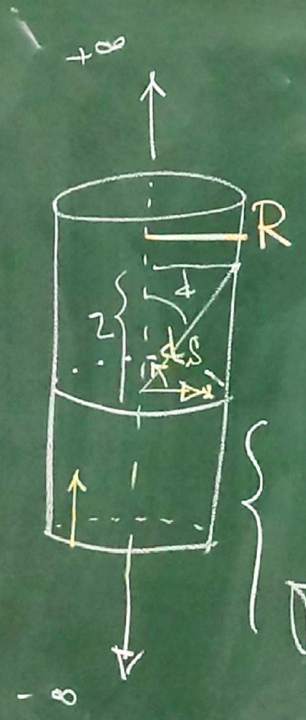
$$+ s^{-k}\left(c_k \cos(k\phi) + d_k \sin(k\phi)\right)\Big]$$

$$b_0 = 0 \quad \begin{cases} \varphi \xrightarrow[s\to\infty]{} 0 \\ \varphi(s=0) \text{ finite} \end{cases}$$

Griffith,

$$\sigma(\phi)\Big|_{s=R} = a \sin 5\phi$$

5)



$$\varphi_{inside}(s,\phi) = a_0 + \sum_{K=1}^{\infty} s^K \left(a_K \cos K\phi + b_K \sin K\phi \right)$$

$$\varphi_{outside}(s,\phi) = a_0' + \sum_{K=1}^{\infty} s^{-K} \left(c_K \cos K\phi + d_K \sin k\phi \right)$$

$$\begin{cases} \varphi_{inside}(s=R,\phi) = \varphi_{outside}(s=R,\phi) \\ \sigma(\phi) = a \sin 5\phi = -\varepsilon_0 \left(\dfrac{\partial \varphi_{out}}{\partial s} - \dfrac{\partial \varphi_{ins}}{\partial s} \right) \end{cases}$$

$$a\sin 5\phi = -\varepsilon_0 \sum_{k=1}^{\infty} \left\{ \frac{-k}{R^{k+1}} \left(c_k \cos k\phi + d_k \sin k\phi \right) - k R^{k-1} \left(a_k \cos k\phi + b_k \sin k\phi \right) \right\}$$

$$a_k = c_k = 0$$

$$k=5 : \rightarrow a = 5\varepsilon_0 \left(\frac{1}{R^6} d_5 + R^4 b_5 \right) \rightarrow b_5 = \frac{a}{10\varepsilon_0 R^4} , \quad d_5 = \frac{aR^6}{10\varepsilon_0}$$

$$\boxed{a_0 = a_0' = 0}$$

$$\left. \begin{array}{l} s = R \checkmark \\ k = 5 \checkmark \end{array} \right\} R^5 b_5 = R^{-5} d_5 \rightarrow d_5 = R^{10} b_5$$

[illegible] پتانسیل

$$\varphi(s,\phi) = \frac{a \sin 5\phi}{10\,\varepsilon_0} \times \begin{cases} s^5 / R^4 & s < R \\ R^6 / s^5 & s > R \end{cases}$$