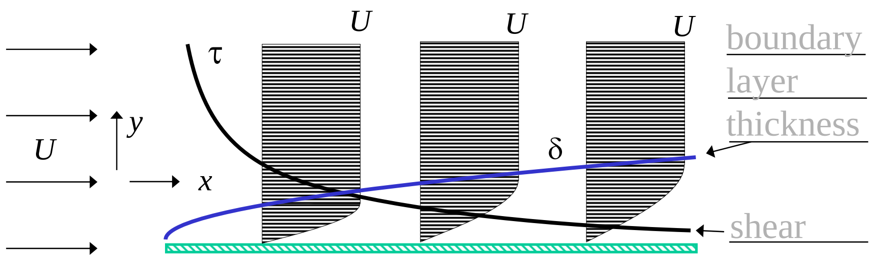
Flow over immersed bodies

External flow

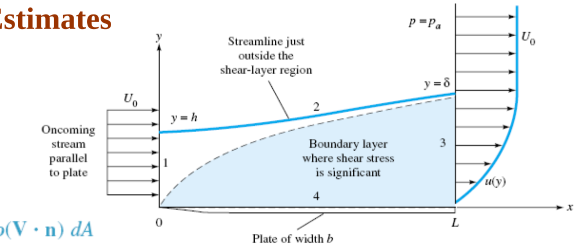
- "External" flows around bodies immersed in a fluid stream will have viscous (shear and no-slip) effects near the body surfaces and in its wake, but will typically be nearly inviscid far from the body.
- These are unconfined ***boundary-layer*** flows.
- External flows are unconfined, free to expand no matter how thick the viscous layers grow. In external flows the boundary layer is always growing and the flow is non-uniform
- Although boundary-layer theory is helpful in understanding external flows, complex body geometries usually require experimental data on the forces and moments caused by the flow.
- Such immersed-body flows are commonly encountered in engineering studies: *aerodynamics* (airplanes, rockets, projectiles), *hydrodynamics* (ships, submarines, torpedos), *transportation* (automobiles, trucks, cycles), *wind engineering* (buildings, bridges, water towers, wind turbines), and *ocean engineering* (buoys, breakwaters, pilings, cables, moored instruments).

Flat Plate: Parallel to Flow



Why is shear maximum at the leading edge of the plate? $\frac{du}{dy}$ is maximum

Momentum-Integral Estimates



$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho \, d\mathcal{V}\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V}\cdot\mathbf{n})\, dA$$

1. From (0, 0) to (0, h): a one-dimensional inlet, $\mathbf{V}\cdot\mathbf{n} = -U_0$
2. From (0, h) to (L, δ): a streamline, no shear, $\mathbf{V}\cdot\mathbf{n} \equiv 0$
3. From (L, δ) to (L, 0): a two-dimensional outlet, $\mathbf{V}\cdot\mathbf{n} = +u(y)$
4. From (L, 0) to (0, 0): a streamline just above the plate surface, $\mathbf{V}\cdot\mathbf{n} = 0$, shear forces summing to the drag force $-D\mathbf{i}$ acting from the plate onto the retarded fluid

$$\sum F_x = -D = \rho \int_1 u(\mathbf{V}\cdot\mathbf{n})\, dA + \rho\int_3 u(\mathbf{V}\cdot\mathbf{n})\, dA$$

$$= \rho \int_0^h U_0(-U_0)b\, dy + \rho\int_0^\delta u(+u)b\, dy$$

$$D = \rho U_0^2 bh - \rho b\int_0^\delta u^2 dy$$

$$\rho \int_{CS} (\mathbf{V}\cdot\mathbf{n})\, dA = 0 = \rho\int_0^h (-U_0)b\, dy + \rho\int_0^\delta ub\, dy$$

$$U_0 h = \int_0^\delta u\, dy$$

$$D(x) = \rho b \int_0^{\delta(x)} u(U - u)\, dy$$

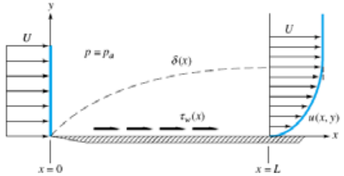
Theodore von Kármán in 1921

Kármán's Analysis of the Flat Plate

$$D(x) = \rho b U^2 \theta \qquad \theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad \textit{momentum thickness } \theta$$

$$D(x) = b \int_0^x \tau_w(x)\, dx \qquad \frac{dD}{dx} = b\tau_w$$

$$\frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx} \qquad \tau_w = \rho U^2 \frac{d\theta}{dx}$$



To get a numerical result for laminar flow, Kármán assumed that the velocity profiles had an approximately parabolic shape

$$u(x, y) \approx U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \qquad 0 \le y \le \delta(x)$$

which makes it possible to estimate both momentum thickness and wall shear

$$\theta = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy \approx \frac{2}{15}\delta$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \approx \frac{2\mu U}{\delta}$$

$$\delta\, d\delta \approx 15 \frac{\nu}{U} dx \qquad \frac{\delta}{x} \approx 5.5\left(\frac{\nu}{Ux}\right)^{1/2} = \frac{5.5}{\mathrm{Re}_x^{1/2}} \qquad c_f = \frac{2\tau_w}{\rho U^2} \approx \left(\frac{\frac{8}{15}}{\mathrm{Re}_x}\right)^{1/2} = \frac{0.73}{\mathrm{Re}_x^{1/2}}$$

Shear Stress Coefficients

- Shear stress coefficient = ratio of shear stress at wall to dynamic pressure of free stream

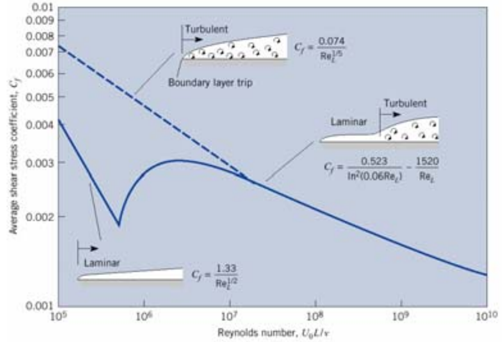
$$c_f = \frac{\tau_0}{\rho U_o^2 / 2}$$

- Total shearing force

$$F_s = \int_A \tau_0 dA$$

$$= \frac{\rho U_o^2}{2} \int_A c_f dA$$

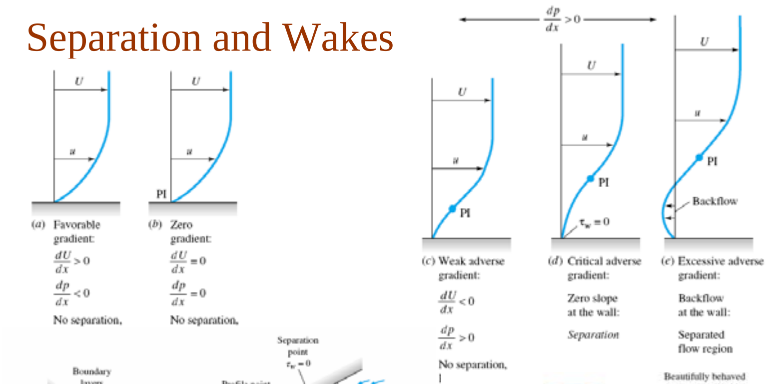
$$C_f = \frac{F_s}{\rho U_o^2 / 2}$$

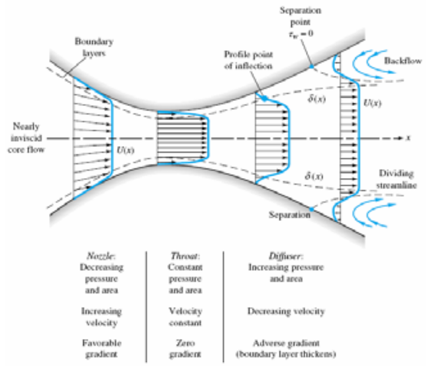


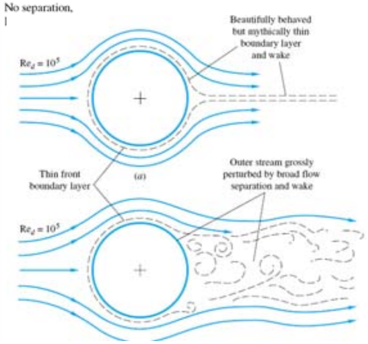
Separation and Wakes

- Separation often occurs at sharp corners
  - fluid can't accelerate to go around a sharp corner
- Velocities in the Wake are _small_ (relative to the free stream velocity)
- Pressure in the Wake is relatively _constant_ (determined by the pressure in the adjacent flow)

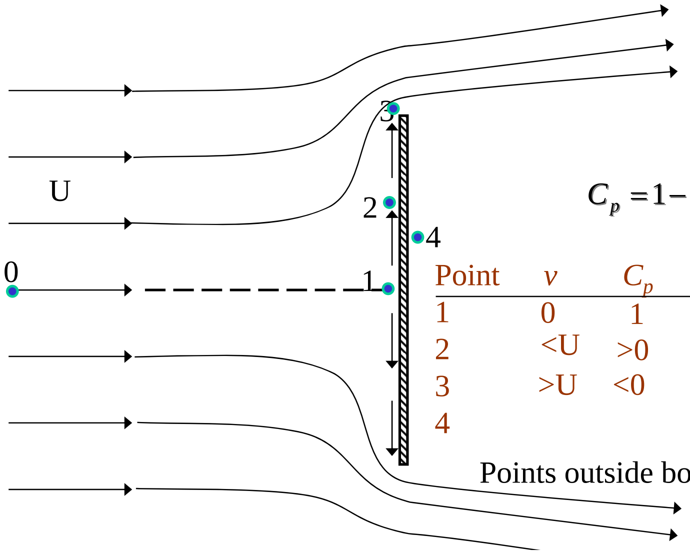
Separation and Wakes







Flat Plate: Streamlines



$$C_p = 1 - \frac{v^2}{U^2} = 2\left(\frac{p - p_0}{\rho U^2}\right)$$

Point	v	C_p	p
1	0	1	$>p_0$
2	$<U$	>0	$>p_0$
3	$>U$	<0	$<p_0$
4			$<p_0$

Points outside boundary layer!

Shear and Pressure Forces: Horizontal and Vertical Components

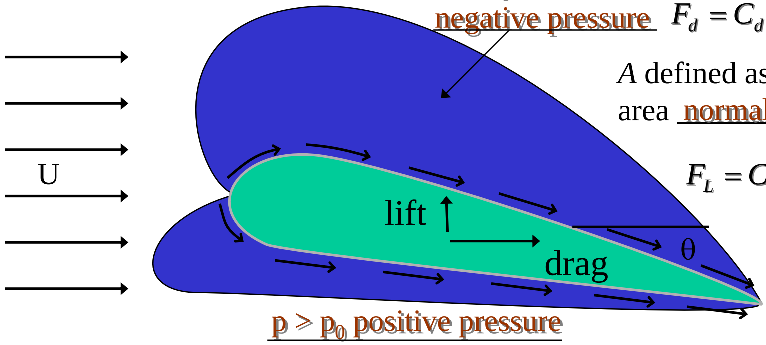
$$F_D = \int (p \sin\theta + \tau_0 \cos\theta)\, dA$$ drag: Parallel to the approach velocity

$$F_L = \int (p \cos\theta - \tau_0 \sin\theta)\, dA$$ lift: Normal to the approach velocity

$$F_d = C_d A \frac{\rho U^2}{2}$$

A defined as projected area normal to force!

$$F_L = C_L A \frac{\rho U^2}{2}$$



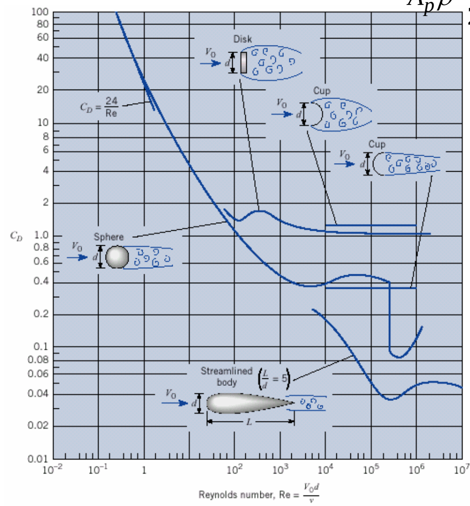
Shear and Pressure Forces

- Shear forces
  - viscous drag, frictional drag, or skin friction
  - caused by shear between the fluid and the solid surface
  - function of surface area and length of object
- Pressure forces
  - pressure drag or form drag
  - caused by flow separation from the body
  - function of area normal to the flow

Drag of Blunt Bodies and Streamlined Bodies

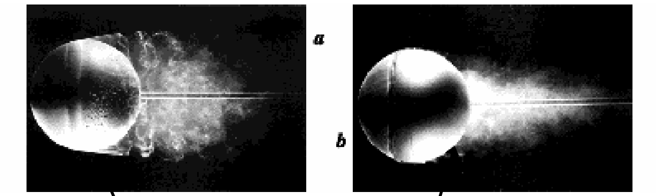
$$C_D = \frac{F_D}{A_p \rho \frac{V_o^2}{2}}$$

- Drag dominated by viscous drag, the body is streamlined.
- Drag dominated by pressure drag, the body is bluff.
- Whether the flow is viscous-drag dominated or pressure-drag dominated depends entirely on the shape of the body.



Effect of Turbulence Levels on Drag

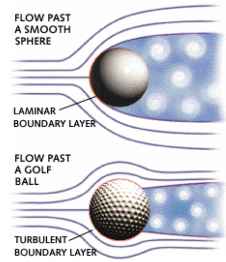
- Flow over a sphere: (a) Reynolds number = 15,000; (b) Reynolds number = 30,000, with trip wire. Causes boundary layer to become turbulent

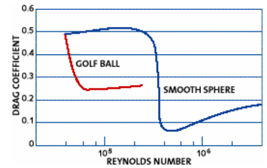


Point of separation

Drag on a Golf Ball

DRAG ON A GOLF BALL comes mainly from pressure drag. The only practical way of reducing pressure drag is to design the ball so that the point of separation moves back further on the ball. The golf ball's dimples increase the turbulence in the boundary layer, increase the inertia of the boundary layer, and delay the onset of separation. The effect is plotted in the chart, which shows that for Reynolds numbers achievable by hitting the ball with a club, the coefficient of drag is much lower for the dimpled ball.

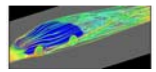




Exercise:

9.22, 9.24, 9.48, 9.67, 9.70, 9.72

Example: Beetle Power



$C_d = 0.38$
Height = 1.511 m
Width = 1.724 m
Length = 4.089 m
Ground clearance = 15 cm?
85 kW at 5200 rpm

Where does separation occur?

Calculate the power required to overcome drag at 60 mph and 120 mph.
Is the new beetle streamlined?

Solution: Power a New Beetle at 60 mph

$$C_d = \frac{2F_D}{\rho U^2 A} = f(\mathbf{R})$$

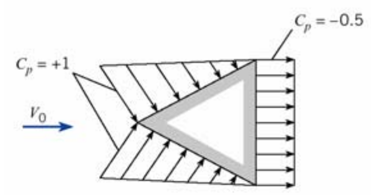
$$F_D = \frac{C_d \rho U^2 A}{2}$$

$$P = \frac{C_d \rho U^3 A}{2}$$

$$P = \frac{(0.38)(1.2kg/m^3)(26.82m/s)^3(2.0m^2)}{2}$$

P = 8.8 kW at 60 mph P = 70 kW at 120 mph

Example



- Given: Pressure distribution is shown, flow is left to right.
- Find: Find C_D
- Solution: C_D is based on the projected area of the block from the direction of flow. Force on downstream face is:

$$(F_D)|_{Drag} = C_p A_p \rho V^2/2 = 0.5 A_p \rho V^2/2$$

The total force on each side face is: $F_S = C_p A_p \rho V^2/2 = 0.5 A_p \rho V^2/2$

The drag force on one face is: $(F_S)|_{Drag} = F_S \sin\alpha = 0.5 A_p \rho V^2/2 * 0.5$

The total drag force is: $F_{Drag} = 2(F_S)|_{Drag} + (F_D)|_{Drag}$

$$= 2*(0.5 A_p \rho V^2/2*0.5) + 0.5 A_p \rho V^2/2 = C_D A_p \rho V^2/2$$

Coefficient of Drag is: C_D=1

Example

- Given: Flag pole, 35 m high, 10 cm diameter, in 25-m/s wind, P_{atm} = 100 kPa, T=20°C
- Find: Moment at bottom of flag pole
- Solution:

$$\nu = 1.51x10^{-5}\, m^2/s,\ \rho = 1.20\, kg/m^3$$

$$\text{Re} = \frac{VD}{\nu} = \frac{25*0.1}{1.51x10^{-5}} = 1.66x10^5$$

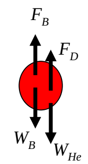
$$C_D = 0.95 \ (C_D \text{ diagram})$$

$$M = F_D \frac{H}{2} = C_D A_p \rho \frac{V_o^2}{2}\frac{H}{2}$$

$$= 0.95*0.10*35*1.2*\frac{25^2}{2}*\frac{35}{2}$$

$$= 21.8kN$$

Example



- Given: Spherical balloon 2-m diameter, filled with helium @std conditions. Empty weight = 3 N.
- Find: Velocity of ascent.
- Solution:

$$\sum F_y = 0 = F_B + F_D - W_B - W_{He}$$

$$= \gamma_{air}\frac{\pi}{6}D^3 + F_D - 3 - \gamma_{He}\frac{\pi}{6}D^3$$

$$F_D = 3 - (\gamma_{air} - \gamma_{He})\frac{\pi}{6}D^3$$

$$= 3 - \gamma_{air}(1 - \frac{287}{2{,}077})\frac{\pi}{6}2^3$$

$$= -1.422\,N$$

$$F_D = C_D A_p \rho \frac{V_o^2}{2}$$

$$V_o = \sqrt{\frac{2F_D}{C_D A_p \rho}} = \sqrt{\frac{2*1.422}{C_D(\pi/4)*2^2*1.225}} = \sqrt{\frac{0.739}{C_D}}$$

Iteration: Guess C_D=0.4

$$V_o = \sqrt{\frac{0.739}{0.4}} = 1.36\,m/s$$

Check Re

$$\text{Re} = \frac{VD}{\nu} = \frac{1.36*2}{1.46x10^{-5}} = 1.86x10^5$$

Iteration: C_D=0.42

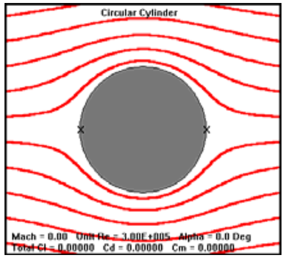
$$V_o = \sqrt{\frac{0.739}{0.42}} = 1.33\,m/s$$

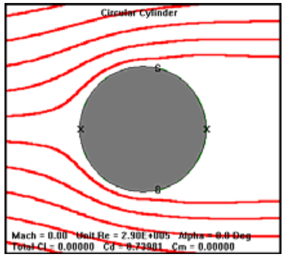
Effect of Boundary Layer Transition

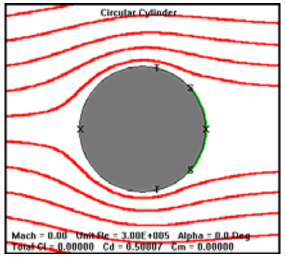
Ideal (non viscous) fluid

Real (viscous) fluid: laminar boundary layer

Real (viscous) fluid: turbulent boundary layer





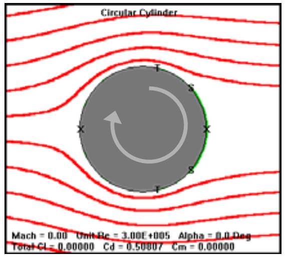


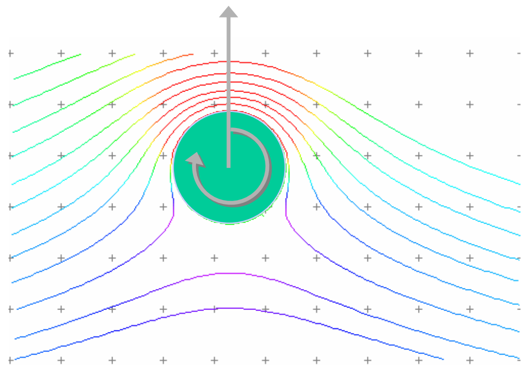
No shear!

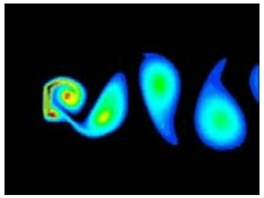
Spinning Spheres

- What happens to the separation points if we start spinning the sphere?

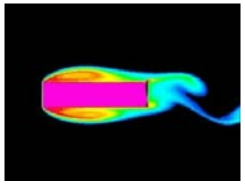
LIFT!





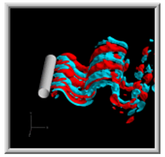


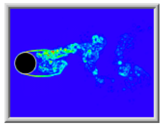
Vortex Shedding



- Vortices are shed alternately from each side of a cylinder
- The separation point and thus the resultant drag force oscillate
- Dimensionless frequency of shedding given by Strouhal number S
- S is approximately 0.2 over a wide range of Reynolds numbers (100 - 1,000,000)

$$S = \frac{nd}{U}$$





The Tacoma Narrows Bridge