Viscous Flow in Pipes

Pipe Flow Problems

- Piping systems are encountered in almost every engineering design and thus have been studied extensively. There is a small amount of theory plus a large amount of experimentation.

The basic piping problem is this:

- Given the pipe geometry and its added components (such as fittings, valves, bends, and diffusers) plus the desired flow rate and fluid properties, what pressure drop is needed to drive the flow?

Of course, it may be stated in alternate form:

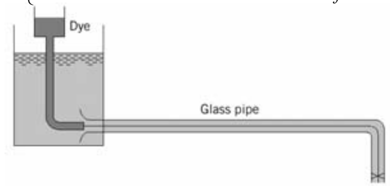
- Given the pressure drop available from a pump, what flow rate will result?

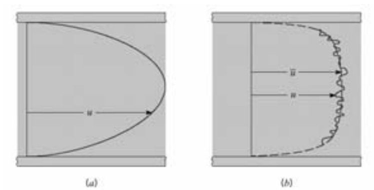
Reynolds Experiment

- Reynolds Number

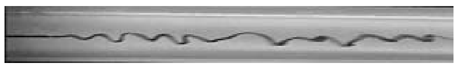
$$\mathrm{Re} = \frac{\rho V D}{\mu}\begin{cases} < 2000 & \text{Laminar flow} \quad h_f \propto V \\ 2000-4000 & \text{Transition flow} \\ > 4000 & \text{Turbulent flow} \quad h_f \propto V^2 \end{cases}$$

- Laminar flow: Fluid moves in smooth streamlines
- Turbulent flow: Violent mixing, fluid velocity at a point varies randomly with time





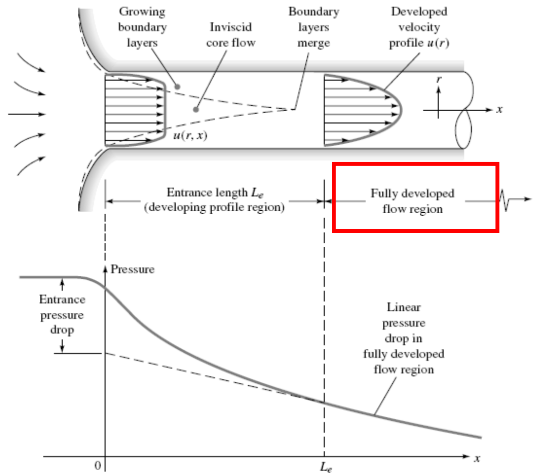
Laminar Turbulent



Pipe Entrance Region

- Developing flow
  - Includes boundary layer and core,
  - viscous effects grow inward from the wall
- Fully developed flow
  - Shape of velocity profile is same at all points along pipe

$$\frac{L_e}{D} \approx \begin{cases} 0.06\,\mathrm{Re} & \text{Laminar flow} \\ 4.4\mathrm{Re}^{1/6} & \text{Turbulent flow} \end{cases}$$



Shear Stress in Pipes

- Steady, uniform flow in a pipe: momentum flux is zero and pressure distribution across pipe is hydrostatic, equilibrium exists between pressure, gravity and shear forces

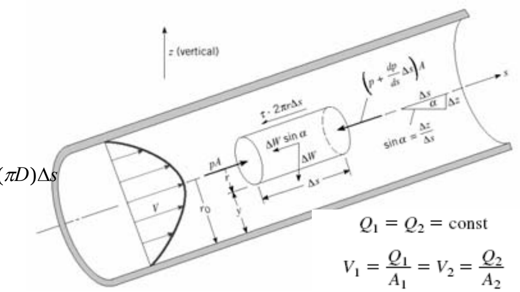
$$\sum F_s = 0 = pA - (p + \frac{dp}{ds}\Delta s)A - \Delta W \sin\alpha - \tau_0(\pi D)\Delta s$$

$$0 = -\frac{dp}{ds}\Delta s A - \gamma A \Delta s \frac{dz}{ds} - \tau_0(\pi D)\Delta s$$

$$\tau_0 = \frac{D}{4}[-\frac{d}{ds}\gamma(\frac{p}{\gamma} + z)]$$

$$\tau_0 = -\frac{D\gamma}{4}\frac{dh}{ds}$$

$$h_1 - h_2 = h_f = \frac{4L\tau_0}{\gamma D}$$



$$Q_1 = Q_2 = \text{const}$$

$$V_1 = \frac{Q_1}{A_1} = V_2 = \frac{Q_2}{A_2}$$

$$\frac{p_1}{\rho} + \frac{1}{2}\alpha_1 V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}\alpha_2 V_2^2 + gz_2 + gh_f$$

$$h_f = \left(z_1 + \frac{p_1}{\rho g}\right) - \left(z_2 + \frac{p_2}{\rho g}\right) = \Delta\left(z + \frac{p}{\rho g}\right) = \Delta z + \frac{\Delta p}{\rho g}$$

- Head loss is due to the shear stress.
- The shear stress will be zero at the center (r = 0) and increase linearly to a maximum at the wall.
- Applicable to either laminar or turbulent flow
- Now we need a relationship for the shear stress in terms of the Re and pipe roughness

Darcy-Weisbach Equation

τ_0	ρ	V	μ	D	ε
$ML^{-1}T^{-2}$	ML^{-3}	LT^{-1}	$ML^{-1}T^{-1}$	L	L

$$\tau_0 = F(\rho, V, \mu, D, \varepsilon)$$

$$\pi_4 = F(\pi_1, \pi_2)$$

Repeating variables: ρ, V, D

$$\pi_1 = \text{Re};\ \pi_2 = \frac{\varepsilon}{D};\ \pi_3 = \frac{\tau_0}{\rho V^2}$$

$$\frac{\tau_0}{\rho V^2} = F(\text{Re}, \frac{\varepsilon}{D})$$

$$\tau_0 = \rho V^2 F(\text{Re}, \frac{\varepsilon}{D})$$

$$h_f = \frac{4L}{\gamma D}\tau_0$$

$$= \frac{4L}{\gamma D}\rho V^2 F(\text{Re}, \frac{\varepsilon}{D})$$

$$= \frac{L}{D}\frac{V^2}{2g}\left[8F(\text{Re}, \frac{\varepsilon}{D})\right]$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Darcy-Weisbach Eq.

$$f = 8F(\text{Re}, \frac{\varepsilon}{D})$$

Friction factor

Laminar Flow in Pipes

- Laminar flow -- Newton's law of viscosity is valid:

$$\tau = \mu\frac{dV}{dy} = -\frac{r\gamma}{2}\frac{dh}{ds}$$

$$\frac{dV}{dy} = -\frac{dV}{dr}$$

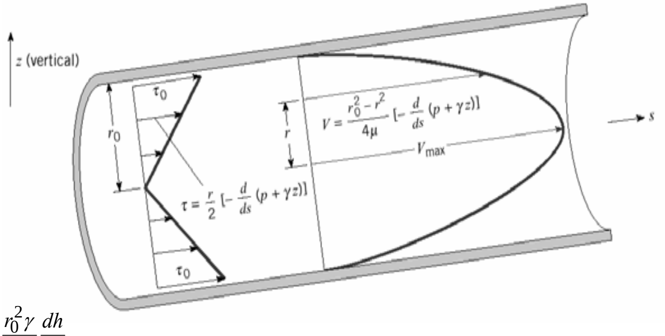
$$\frac{dV}{dr} = \frac{r\gamma}{2\mu}\frac{dh}{ds}$$

$$dV = \frac{r\gamma}{2\mu}\frac{dh}{ds}dr$$

$$V = \frac{r^2\gamma}{4\mu}\frac{dh}{ds} + C \qquad C = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}$$

$$V = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

$$V = V_{max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$



- Velocity distribution in a pipe (laminar flow) is parabolic with maximum at center.

Discharge in Laminar Flow

$$V = -\frac{\gamma}{4\mu}\frac{dh}{ds}(r_0^2 - r^2)$$

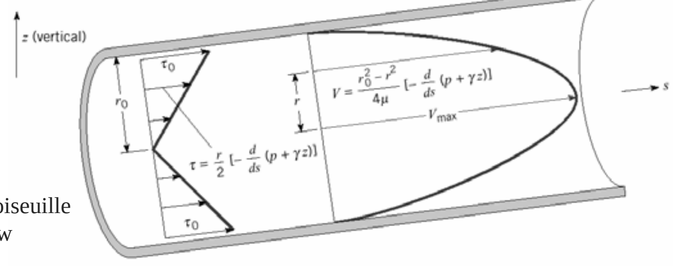
$$Q = \int V dA = \int_0^{r_0} -\frac{\gamma}{4\mu}\frac{dh}{ds}(r_0^2 - r^2)(2\pi r dr)$$

$$= \frac{\pi\gamma}{4\mu}\frac{dh}{ds}\frac{(r^2 - r_0^2)^2}{2}\Big|_0^{r_0}$$

$$Q = -\frac{\pi\gamma r_0^4}{8\mu}\frac{dh}{ds}$$

$$= -\frac{\pi\gamma D^4}{128\mu}\frac{dh}{ds}$$

Hagen-Poiseuille flow



$$\bar{V} = \frac{Q}{A}$$

$$\bar{V} = -\frac{\gamma D^2}{32\mu}\frac{dh}{ds}$$

Head Loss in Laminar Flow

$$\overline{V} = -\frac{\gamma D^2}{32\mu}\frac{dh}{ds}$$

$$\frac{dh}{ds} = -\overline{V}\frac{32\mu}{\gamma D^2}$$

$$dh = -\overline{V}\frac{32\mu}{\gamma D^2}ds$$

$$h_2 - h_1 = -\overline{V}\frac{32\mu}{\gamma D^2}(s_2 - s_1)$$

$$h_1 = h_2 + h_f$$

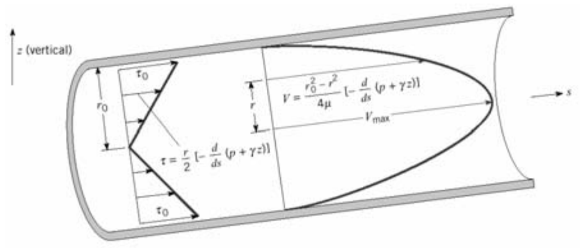
$$h_f = \frac{32\mu L\overline{V}}{\gamma D^2}$$

$$h_f = \frac{32\mu L\overline{V}}{\gamma D^2}$$

$$= 64(\frac{\mu}{\rho\overline{V}D})(\frac{L}{D})\overline{V}^2/2g$$

$$= \frac{64}{\mathrm{Re}}(\frac{L}{D})\rho\overline{V}^2/2$$

$$h_f = f\frac{L}{D}\frac{\overline{V}^2}{2g} \quad f = \frac{64}{\mathrm{Re}}$$

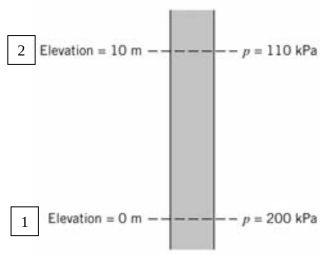


Example

Given: Liquid in pipe has γ= 8 kN/m^3. Acceleration = 0.
D = 1 cm, μ = 3x10^{-3} N-m/s^2.

Find: Is fluid stationary, moving up, or moving down?
What is the mean velocity?

Solution: Energy eq. from z = 0 to z = 10 m



$$\alpha_1\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_L = \alpha_2\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{200,000}{8000} - h_L = \frac{110,000}{8000} + 10$$

$$h_L = \frac{90}{8} - 10$$

$$h_L = 1.25\,m\ \text{(moving upward)}$$

$$h_L = \frac{32\mu LV}{\gamma D^2}$$

$$V = h_L\frac{\gamma D^2}{32\mu L}$$

$$V = 1.25\frac{8000*(0.01)^2}{32*3x10^{-3}*10}$$

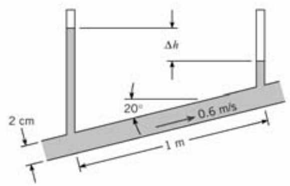
$$V = 1.04\,m/s$$

Example

Given: Glycerin@ 20^{o}C flows commercial steel pipe.

Find: Δh

Solution: $\gamma = 12,300\,N/m, \mu = 0.62\,Ns/m^2$



$$\alpha_1\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_L = \alpha_2\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{p_1}{\gamma} + z_1 - h_L = \frac{p_2}{\gamma} + z_2$$

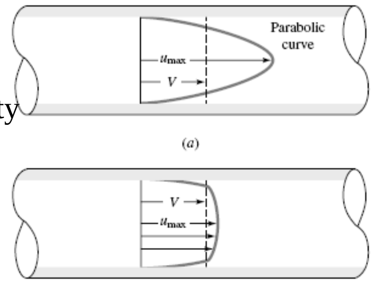
$$\Delta h = \frac{p_1}{\gamma} + z_1 - (\frac{p_2}{\gamma} + z_2) = h_L$$

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{VD}{\nu} = \frac{0.6*0.02}{5.1*10^{-4}} = 23.5\ \text{(laminar)}$$

$$\Delta h = h_L = \frac{32\mu LV}{\gamma D^2} = \frac{32(0.62)(1)(0.6)}{12,300*(0.02)^2} = 2.42\,m$$

Turbulent-Flow in Pipes

Laminar and turbulent pipe-flow velocity profiles for the same volume flow:



Two important parameters!
Re - Laminar or Turbulent
ε/d - Rough or Smooth

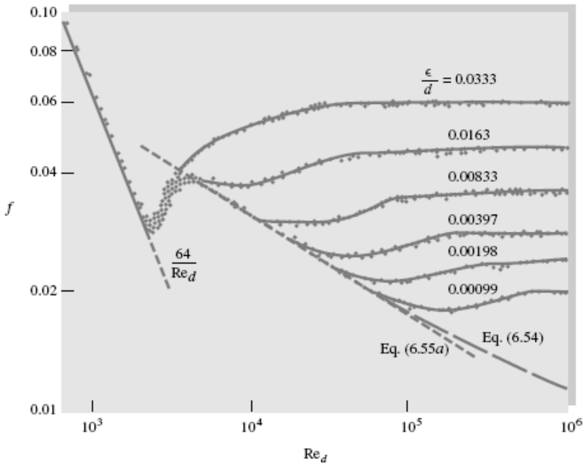
Turbulent-Flow in Pipes

Nikuradse's Experiments

Nikuradse simulated roughness by gluing uniform sand grains onto the inner walls of the pipes. He then measured the pressure drops and flow rates and correlated friction factor versus Reynolds number. at high Reynolds numbers.

- In general, friction factor
  $f = F(\mathrm{Re}, \frac{k}{D})$
  - Function of *Re* and *roughness*
- Laminar region
  $f = \frac{64}{\mathrm{Re}}$
  - Independent of roughness
- Turbulent region
  - Smooth pipe curve
    - All curves coincide @ ~Re=2300
  - Rough pipe zone
    - All rough pipe curves flatten out and become independent of Re

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$



Moody Diagram

Transition function for both smooth and rough pipe laws (Colebrook)

$$\frac{1}{f^{1/2}} = -2.0 \log\left(\frac{\epsilon/d}{3.7} + \frac{2.51}{\mathrm{Re}_d f^{1/2}}\right)$$

(used to draw the Moody diagram)

(Haaland)

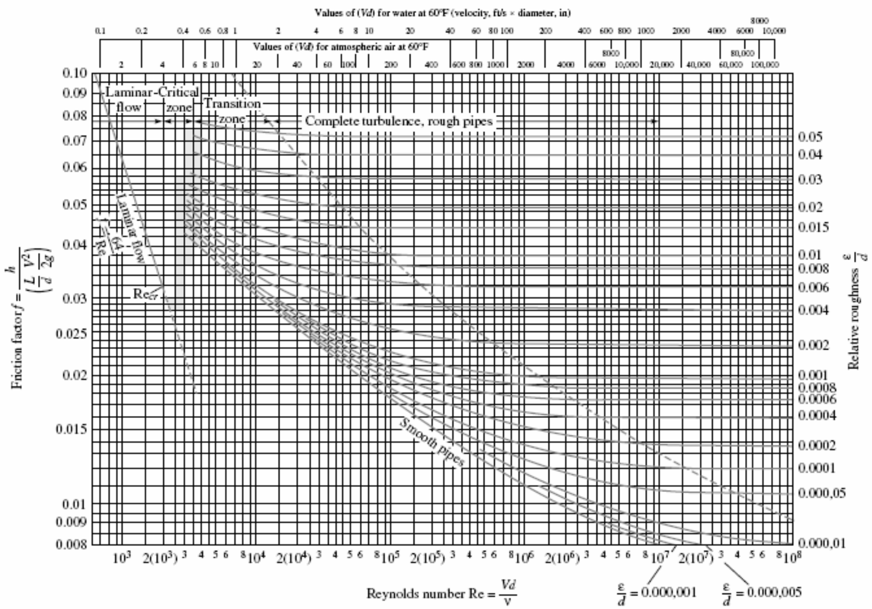
$$\frac{1}{f^{1/2}} \approx -1.8 \log\left[\frac{6.9}{\mathrm{Re}_d} + \left(\frac{\epsilon/d}{3.7}\right)^{1.11}\right]$$

varies less than 2 percent

Roughness Values

Material	Condition	ε ft	ε mm
Steel	Sheet metal, new	0.00016	0.05
	Stainless, new	0.000007	0.002
	Commercial, new	0.00015	0.046
	Riveted	0.01	3.0
	Rusted	0.007	2.0
Iron	Cast, new	0.00085	0.26
	Wrought, new	0.00015	0.046
	Galvanized, new	0.0005	0.15
	Asphalted cast	0.0004	0.12
Brass	Drawn, new	0.000007	0.002
Plastic	Drawn tubing	0.000005	0.0015
Glass	—	Smooth	Smooth
Concrete	Smoothed	0.00013	0.04
	Rough	0.007	2.0
Rubber	Smoothed	0.000033	0.01
Wood	Stave	0.0016	0.5

Moody Diagram



Example

Oil, with $\rho = 900$ kg/m^3 and $\nu = 0.00001$ m^2/s, flows at 0.2 m^3/s through 500 m of 200-mm-diameter cast-iron pipe. Determine (*a*) the head loss and (*b*) the pressure drop if the pipe slopes down at 10° in the flow direction.

First compute the velocity from the known flow rate

$$V = \frac{Q}{\pi R^2} = \frac{0.2 \text{ m}^3/\text{s}}{\pi(0.1 \text{ m})^2} = 6.4 \text{ m/s}$$

Then the Reynolds number is

$$\text{Re}_d = \frac{Vd}{\nu} = \frac{(6.4 \text{ m/s})(0.2 \text{ m})}{0.00001 \text{ m}^2/\text{s}} = 128{,}000$$

From Table 6.1, $\epsilon = 0.26$ mm for cast-iron pipe. Then

$$\frac{\epsilon}{d} = \frac{0.26 \text{ mm}}{200 \text{ mm}} = 0.0013$$

Enter the Moody chart on the right at $\epsilon/d = 0.0013$ (you will have to interpolate), and move to the left to intersect with Re = 128,000. Read $f \approx 0.0225$ [from Eq. (6.64) for these values we could compute $f = 0.0227$]. Then the head loss is

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.0225)\frac{500 \text{ m}}{0.2 \text{ m}}\frac{(6.4 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 117 \text{ m} \qquad \textit{Ans. (a)}$$

From Eq. (6.25) for the inclined pipe,

$$h_f = \frac{\Delta p}{\rho g} + z_1 - z_2 = \frac{\Delta p}{\rho g} + L \sin 10°$$

or

$$\Delta p = \rho g[h_f - (500 \text{ m}) \sin 10°] = \rho g(117 \text{ m} - 87 \text{ m})$$

$$= (900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(30 \text{ m}) = 265{,}000 \text{ kg/(m}\cdot\text{s}^2) = 265{,}000 \text{ Pa} \qquad \textit{Ans. (b)}$$

Three Types of Pipe-Flow Problems

Moody Diagram

1. Given *d*, *L*, and *V* or *Q*, ρ, μ , and *g*, compute the head loss h_f (head-loss problem).
2. Given *d*, *L*, h_f, ρ, μ and *g*, compute the velocity *V* or flow rate *Q* (flow-rate problem).
3. Given *Q*, *L*, h_f, ρ, μ and *g*, compute the diameter *d* of the pipe (sizing problem).

Example

Oil, with $\rho = 950$ kg/m^3 and $\nu = 2$ E-5 m^2/s, flows through a 30-cm-diameter pipe 100 m long with a head loss of 8 m. The roughness ratio is $\epsilon/d = 0.0002$. Find the average velocity and flow rate.

By definition, the friction factor is known except for *V*:

$$f = h_f\frac{d}{L}\frac{2g}{V^2} = (8 \text{ m})\left(\frac{0.3 \text{ m}}{100 \text{ m}}\right)\left[\frac{2(9.81 \text{ m/s}^2)}{V^2}\right] \quad \text{or} \quad fV^2 \approx 0.471 \quad \text{(SI units)}$$

To get started, we only need to guess *f*, compute $V = \sqrt{0.471/f}$, then get Re_d, compute a better *f* from the Moody chart, and repeat. The process converges fairly rapidly. A good first guess is the "fully rough" value for $\epsilon/d = 0.0002$, or $f \approx 0.014$ from Fig. 6.13. The iteration would be as follows:

Guess $f \approx 0.014$, then $V = \sqrt{0.471/0.014} = 5.80$ m/s and $\text{Re}_d = Vd/\nu \approx 87{,}000$. At $\text{Re}_d =$ 87,000 and $\epsilon/d = 0.0002$, compute $f_{\text{new}} \approx 0.0195$ [Eq. (6.64)].

New $f \approx 0.0195$, $V = \sqrt{0.481/0.0195} = 4.91$ m/s and $\text{Re}_d = Vd/\nu = 73{,}700$. At $\text{Re}_d =$ 73,700 and $\epsilon/d = 0.0002$, compute $f_{\text{new}} \approx 0.0201$ [Eq. (6.64)].

Better $f \approx 0.0201$, $V = \sqrt{0.471/0.0201} = 4.84$ m/s and $\text{Re}_d \approx 72{,}600$. At $\text{Re}_d = 72{,}600$ and $\epsilon/d = 0.0002$, compute $f_{\text{new}} \approx 0.0201$ [Eq. (6.64)].

We have converged to three significant figures. Thus our iterative solution is

$$V = 4.84 \text{ m/s}$$

$$Q = V\left(\frac{\pi}{4}\right)d^2 = (4.84)\left(\frac{\pi}{4}\right)(0.3)^2 \approx 0.342 \text{ m}^3/\text{s} \qquad \textit{Ans.}$$

The iterative approach is straightforward and not too onerous, so it is routinely used by engineers. Obviously this repetitive procedure is ideal for a personal computer.

Example

Given: The pressure at a water main is 300 kPa gage. What size pipe is needed to carry water from the main at a rate of 0.025 m3/s to a factory that is 140 m from the main? Assume galvanized-steel pipe is to be used and that the pressure required at the factory is 60 kPa gage at a point 10 m above the main connection.

Find: Size of pipe.

Solution:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{(Q/(\pi/4)D^2)^2}{2g}$$

$$D = \left(8\frac{fL}{h_f}\frac{Q^2}{\pi^2 g}\right)^{1/5}$$

$$\alpha_1\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_f = \alpha_2\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{300{,}000}{9810} - h_f = \frac{60{,}000}{9810} + 10$$

$$h_f = 14.45\,m$$

Assume $f = 0.020$

$$D = \left(8\frac{fL}{h_f}\frac{Q^2}{\pi^2 g}\right)^{1/5}$$

$$= \left(8\frac{0.02}{14.45}\frac{140}{}\frac{(0.025)^2}{\pi^2 9.81}\right)^{1/5} = 0.100\,m$$

Relative roughness: $\frac{\varepsilon}{D} = \frac{0.15}{100} = 0.0015$

Friction factor: $f = 0.022$

$$D = 0.100\left(\frac{0.022}{0.020}\right)^{1/5} = 0.102\,m$$

Use 12 cm pipe

Minor Losses in Pipe Systems

- For any pipe system, in addition to the Moody-type friction loss computed for the length of pipe, there are additional so-called *minor losses* due to:

1. Pipe entrance or exit
2. Sudden expansion or contraction
3. Bends, elbows, tees, and other fittings
4. Valves, open or partially closed
5. Gradual expansions or contractions

Minor Losses

- Since the flow pattern in fittings and valves is quite complex, the theory is very weak. The losses are commonly measured experimentally and correlated with the pipe flow parameters.
- The measured minor loss is usually given as a ratio of the head loss $h_m = \Delta p/(\rho g)$ through the device to the velocity head $V^2/(2g)$ of the associated piping system

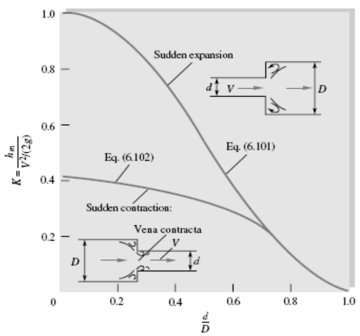
$$\text{Loss coefficient } K = \frac{h_m}{V^2/(2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

An alternate, and less desirable, procedure is to report the minor loss as if it were an *equivalent length* L_{eq} of pipe, satisfying the Darcy friction-factor relation

$$h_m = f\frac{L_{eq}}{d}\frac{V^2}{2g} = K\frac{V^2}{2g}$$

$$L_{eq} = \frac{Kd}{f}$$

Minor Loss in a Pipe



- A piping system may have many minor losses which are all correlated to $V^2/2g$
- Sum them up to a total system loss for pipes of the same diameter

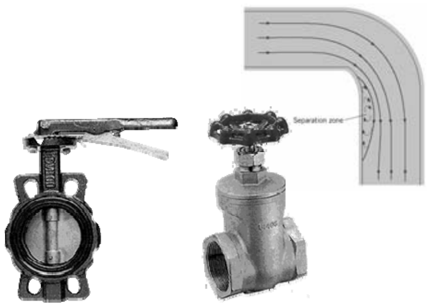
$$h_L = h_f + \sum_m h_m = \frac{V^2}{2g}\left[f\frac{L}{D} + \sum_m K_m\right]$$

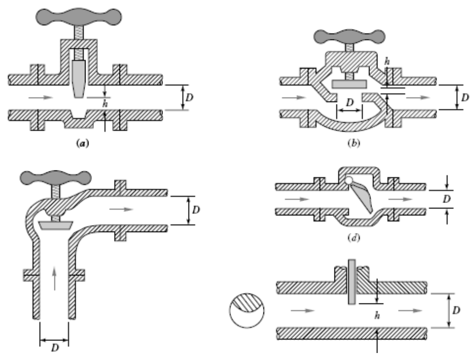
- Where,

h_L = Total head loss $\quad h_f$ = Frictional head loss

h_m = Minor head loss for fitting m

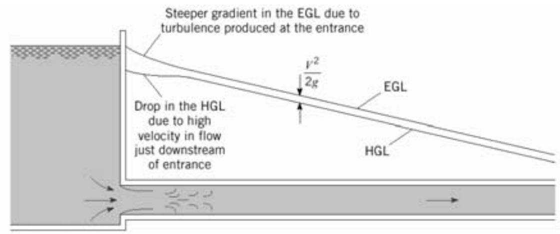
K_m = Minor head loss coefficient for fitting m

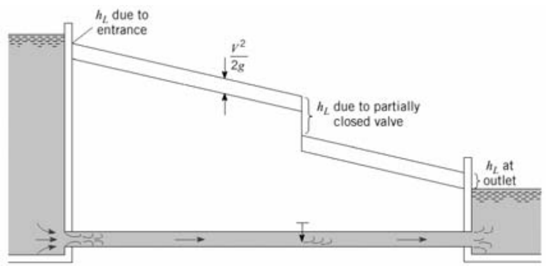




EGL & HGL for Losses in a Pipe

- Entrances, bends, and other flow transitions cause the EGL to drop an amount equal to the head loss produced by the transition.
- EGL is steeper at entrance than it is downstream of there where the slope is equal the frictional head loss in the pipe.
- The HGL also drops sharply downstream of an entrance

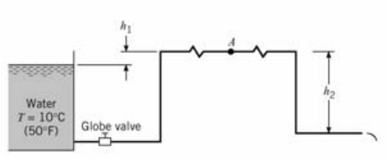




Example

Given: The 10-cm galvanized-steel pipe is 1000 m long and discharges water into the atmosphere. The pipeline has an open globe valve and 4 threaded elbows; h_1=3 m and h_2 = 15 m.

Find: What is the discharge, and what is the pressure at A, the midpoint of the line?



Solution:

$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - \sum h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$0 + 0 + 12 = (1 + K_e + K_v + 4K_b + f\frac{L}{D})\frac{V^2}{2g} + 0 + 0$$

D = 10-cm and assume $f = 0.025$

$$24g = (1 + 0.5 + 10 + 4*0.9 + 0.025\frac{1000}{0.1})V^2$$

$$V^2 = \frac{24g}{265.1}$$

$$V = 0.942\, m/s$$

$$Q = VA = 0.942(\pi/4)(0.10)^2 = 0.0074\, m^3/s$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{0.942*0.1}{1.31x10^{-6}} = 7x10^4$$

So $f = 0.025$

$$\alpha_A \frac{V_A^2}{2g} + \frac{p_A}{\gamma} + z_A - \sum h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{p_A}{\gamma} + 15 = (2K_b + f\frac{L}{D})\frac{V^2}{2g}$$

$$\frac{p_A}{\gamma} = (2*0.9 + 0.025\frac{500}{0.1})\frac{(0.942)^2}{2g} - 15 = -9.6\, m$$

$$p_A = 9810*(-9.26) = -90.8\, kPa$$

Near cavitation pressure, not good!

Multiple-Pipe Systems

- Pipes in series

$$Q_1 = Q_2 = Q_3 = \text{const}$$

$$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2$$

$$\Delta h_{A\to B} = \Delta h_1 + \Delta h_2 + \Delta h_3$$

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}\left(\frac{f_1 L_1}{d_1} + \sum K_1\right) + \frac{V_2^2}{2g}\left(\frac{f_2 L_2}{d_2} + \sum K_2\right) + \frac{V_3^2}{2g}\left(\frac{f_3 L_3}{d_3} + \sum K_3\right)$$

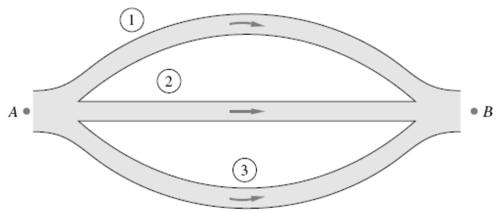
Since $V2$ and $V3$ are proportional to $V1$

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}(\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3)$$

The αi are dimensionless constants.

If the flow rate is given, we can evaluate the right-hand side and hence the total head loss. If the head loss is given, a little iteration is needed, since f1, f2, and f3 all depend upon V1 through the Reynolds number. Begin by calculating f1, f2, and f3, assuming fully rough flow, and the solution for V1 will converge with one or two iterations.

- Pipes in parallel



$$\Delta h_{A\to B} = \Delta h_1 = \Delta h_2 = \Delta h_3$$

$$Q = Q_1 + Q_2 + Q_3$$

If the total head loss is known, it is straightforward to solve for *Qi* in each pipe and sum them. The reverse problem, of determining Qi when hf is known, requires iteration.

$$h_f = f(L/d)(V^2/2g) = fQ^2/C, \text{ where } C = \pi^2 g d^5/8L$$

$$h_f = \frac{Q^2}{\left(\sum\sqrt{C_i/f_i}\right)^2} \qquad \text{where } C_i = \frac{\pi^2 g d_i^5}{8L_i}$$

Since the fi vary with Reynolds number and roughness ratio, one begins by guessing values of fi (fully rough values are recommended) and calculating a first estimate of hf. Then each pipe yields a flow-rate estimate Qi = $(C_i h_f / f_i)^{1/2}$ and hence a new Reynolds number and a better estimate of fi. Then repeat Eq. for h_f to convergence.

- Three-reservoir pipe junction

If all flows are considered positive toward the junction, which obviously implies that one or two of the flows must be away from the junction.

$$Q_1 + Q_2 + Q_3 = 0$$

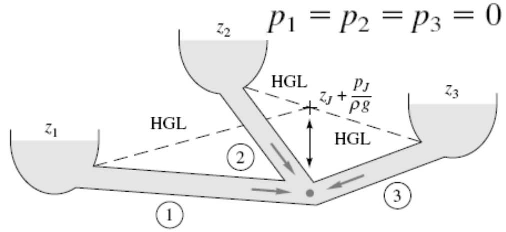
$$h_J = z_J + \frac{p_J}{\rho g}$$

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

$$p_1 = p_2 = p_3 = 0$$



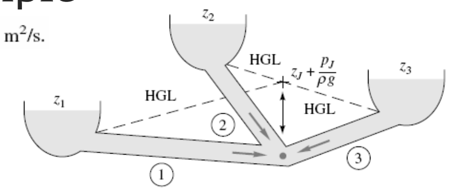
We guess the position hJ and solve Eqs. for V1, V2, and V3 and hence Q1, Q2, and Q3, iterating until the flow rates balance at the junction. If we guess hJ too high, the sum Q1+ Q2+ Q3 will be negative and the remedy is to reduce hJ, and vice versa.

Example

The fluid is water, $\rho = 1000\ \text{kg/m}^3$ and $\nu = 1.02 \times 10^{-6}\ \text{m}^2/\text{s}$.

$z_1 = 20$ m $z_2 = 100$ m $z_3 = 40$ m

Pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005



As a first guess, take h_J equal to the middle reservoir height, $z_3 = h_J = 40$ m. This saves one calculation ($Q_3 = 0$) and enables us to get the lay of the land:

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h	L_i/d_i
1	40	−20	0.0267	−3.43	−62.1	1250
2	40	60	0.0241	4.42	45.0	2500
3	40	0		0	0	2000
					$\sum Q = -17.1$	

Since the sum of the flow rates toward the junction is negative, we guessed h_J too high. Reduce h_J to 30 m and repeat:

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h
1	30	−10	0.0269	−2.42	−43.7
2	30	70	0.0241	4.78	48.6
3	30	10	0.0317	1.76	8.0
					$\sum Q = 12.9$

This is positive ΣQ, and so we can linearly interpolate to get an accurate guess: $h_J \approx 34.3$ m. Make one final list:

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h
1	34.3	−14.3	0.0268	−2.90	−52.4
2	34.3	65.7	0.0241	4.63	47.1
3	34.3	5.7	0.0321	1.32	6.0
					$\sum Q = 0.7$

This is close enough; hence we calculate that the flow rate is 52.4 m³/h toward reservoir 3, balanced by 47.1 m³/h away from reservoir 1 and 6.0 m³/h away from reservoir 3.

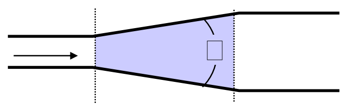
Exercise:

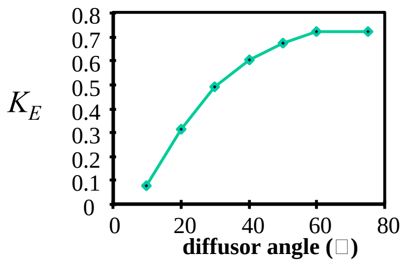
8.3, 8.17, 8.22, 8.24, 8.37
8.60, 8.72, 8.89, 8.100, 8.102

Head Loss due to Gradual Expansion (Diffusor)

$$h_E = K_E \frac{(V_1 - V_2)^2}{2g}$$

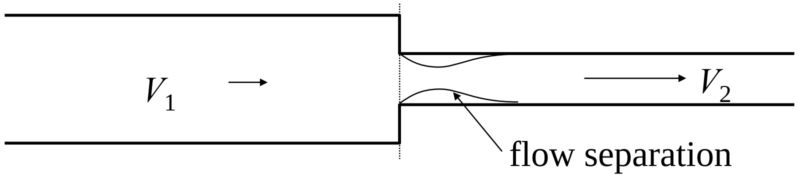
$$h_E = K_E \frac{V_2^2}{2g}\left(\frac{A_2}{A_1} - 1\right)^2$$





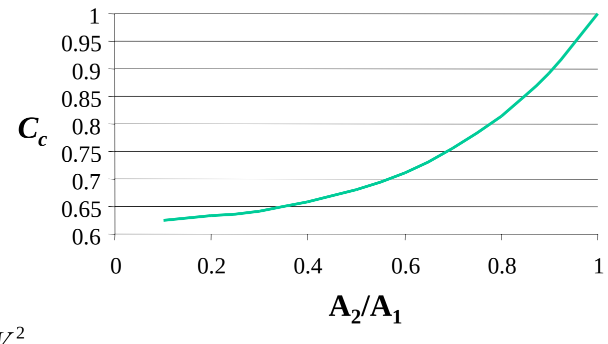
Sudden Contraction

$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$



- losses are reduced with a gradual contraction $C_c = \frac{A_c}{A_2}$

Sudden Contraction

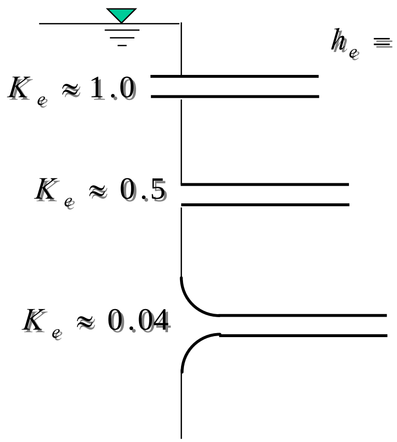


$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$

$Q_{orifice} = CA_{orifice}\sqrt{2gh}$

Entrance Losses

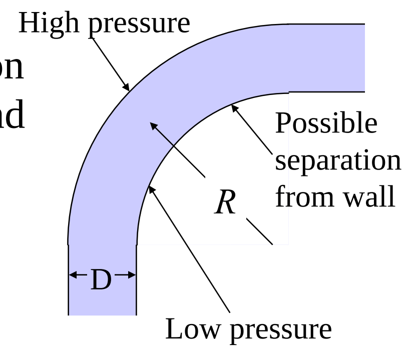
Losses can be reduced by accelerating the flow gradually and eliminating the vena contracta



$$h_e = K_e \frac{V^2}{2g}$$

Head Loss in Bends

- Head loss is a function of the ratio of the bend radius to the pipe diameter (R/D)
- Velocity distribution returns to normal several pipe diameters downstream



$$h_b = K_b \frac{V^2}{2g}$$

K_b varies from 0.6 - 0.9

Head Loss in Valves

- Function of valve type and valve position
- The complex flow path through valves can result in high head loss (of course, one of the purposes of a valve is to create head loss when it is not fully open)

$$h_v = K_v \frac{V^2}{2g}$$