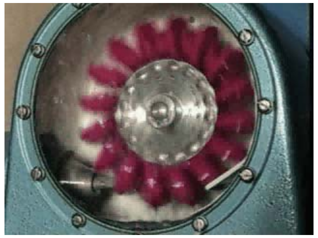
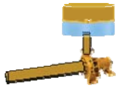
Finite Control Volume Analysis



Moving from a System to a Finite Control Volume



- Mass
- Linear Momentum
- Moment of Momentum
- Energy
- Putting it all together!

Conservation of Mass

B = Total amount of mass in the system
b = mass per unit mass = 1

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \, d\mathcal{V} + \int_{cs} \rho b \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$$ cv equation

$$\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V} + \int_{cs} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$$ But $DM_{sys}/Dt = 0$!

$$\int_{cs} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = -\frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V}$$ Continuity Equation

mass leaving - mass entering = - rate of increase of mass in cv

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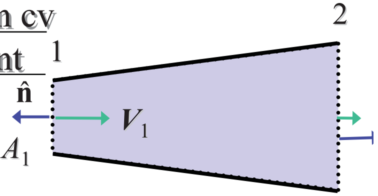
Conservation of Mass

$$\int_{cs} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \cancel{-\frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V}}$$ If mass in cv is constant



$$\int_{cs_1} \rho_1 \mathbf{V}_1\cdot\hat{\mathbf{n}}_1\, dA + \int_{cs_2} \rho_2 \mathbf{V}_2\cdot\hat{\mathbf{n}}_2\, dA = 0$$

Unit vector $\hat{\mathbf{n}}$ is normal to surface and pointed out of cv

$$\int_{cs} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \underline{\rho\bar{V}A} = \underline{\dot{m}}$$ [M/T]

We assumed uniform ρ on the control surface

$$\bar{V} = \frac{\int_{cs} \mathbf{V}\cdot\hat{\mathbf{n}}\, dA}{A}$$

$\bar{V}$ is the spatially averaged

Continuity Equation for Constant Density and Uniform Velocity

$$\int_{cs_1} \rho_1 \mathbf{V}_1 \cdot \hat{\mathbf{n}}_1 dA + \int_{cs_2} \rho_2 \mathbf{V}_2 \cdot \hat{\mathbf{n}}_2 dA = 0$$

$$-\rho_1 \overline{V}_1 A_1 + \rho_2 \overline{V}_2 A_2 = 0$$

Density is constant across cs

$$\overline{V}_1 A_1 = \overline{V}_2 A_2 = Q \qquad \underline{[L^3/T]}$$

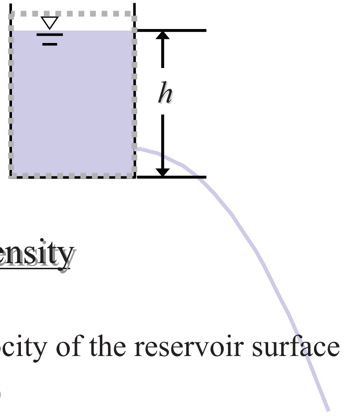
Density is the same at cs_1 and cs_2

$$V_1 A_1 = V_2 A_2 = Q$$

Simple version of the continuity equation for conditions of constant density. It is understood that the velocities are either uniform or spatially averaged.

Example: Conservation of Mass?

The flow out of a reservoir is 2 L/s.
The reservoir surface is 5 m x 5 m.
How fast is the reservoir surface dropping?



$$\int_{cs} \cancel{\rho} \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\frac{\partial}{\partial t} \int_{cv} \cancel{\rho} d\mathcal{V}$$

$$\int_{cs} \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\frac{\partial \mathcal{V}}{\partial t}$$

Constant density

$$Q_{out} - \cancel{Q}_{in} = -\frac{d\mathcal{V}}{dt}$$

$$Q_{out} = -\frac{A_{res}\, dh}{dt}$$

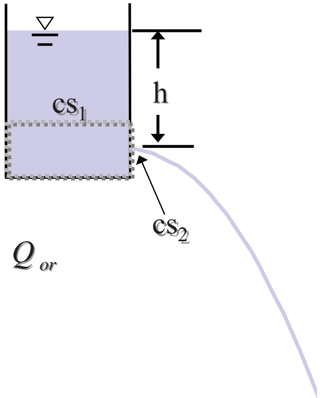
Velocity of the reservoir surface

$$\frac{dh}{dt} = -\frac{Q}{A_{res}}$$

Example: Conservation of Mass

- The flow through the orifice is a function of the depth of water in the reservoir $Q = CA_o\sqrt{2gh}$
- Find the time for the reservoir level to drop from 10 cm to 5 cm. The reservoir surface is 15 cm x 15 cm. The orifice is 2 mm in diameter and is 2 cm off the bottom of the reservoir. The orifice coefficient is 0.6.
- CV with constant or changing mass.
- Draw CV, label CS, solve using variables starting with $$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\frac{\partial}{\partial t} \int_{cv} \rho d\mathcal{V}$$

Example Conservation of Mass Constant Volume



$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \cancel{-\frac{\partial}{\partial t} \int_{cv} \rho d\mathcal{V}}$$

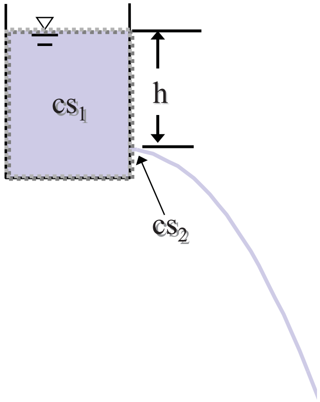
$$\int_{cs_1} \rho_1 \mathbf{V}_1 \cdot \hat{\mathbf{n}}_1 dA + \int_{cs_2} \rho_2 \mathbf{V}_2 \cdot \hat{\mathbf{n}}_2 dA = 0$$

$$-V_{res} A_{res} + V_{or} A_{or} = 0 \qquad V_{or} A_{or} = Q_{or}$$

$$V_{res} = \frac{-dh}{dt}$$

$$\frac{dh}{dt} A_{res} + CA_{or}\sqrt{2gh} = 0$$

Example Conservation of Mass Changing Volume



$$\int_{cs} \cancel{\rho}\mathbf{V}\cdot\hat{\mathbf{n}}\,dA = -\frac{\partial}{\partial t}\int_{cv} \cancel{\rho}\,d\rlap{-}V$$

$$V_{or}A_{or} = -\frac{\partial}{\partial t}\int_{cv} d\rlap{-}V$$

$$V_{or}A_{or} = -\frac{d\rlap{-}V}{dt} = -\frac{A_{res}dh}{dt}$$

$$V_{or}A_{or} = Q_{or}$$

$$\frac{dh}{}A_{res} + CA_{or}\sqrt{2gh} = 0$$

Example Conservation of Mass

$$\frac{-A_{res}}{CA_{or}\sqrt{2g}}\int_{h_0}^{h}\frac{dh}{\sqrt{h}} = \int_0^t dt$$

$$\frac{-A_{res}}{CA_{or}\sqrt{2g}}2\left(h^{1/2} - h_0^{1/2}\right) = t$$

$$\frac{-2\,(0.15m)^2}{(0.6)\left(\frac{\pi\,(0.002m)^2}{4}\right)\sqrt{2\,(9.8m/s^2)}}\left((0.03m)^{1/2} - (0.08m)^{1/2}\right) = t$$

$$t = 591\ s$$

Linear Momentum Equation

$$\underline{\sum F \neq 0}$$

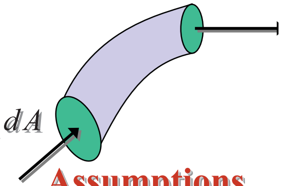
$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv}\rho b\,d\rlap{-}V + \int_{cs}\rho b\mathbf{V}\cdot\hat{\mathbf{n}}\,dA$$ R.T.T.

$\mathbf{B} = m\mathbf{V}$ momentum $\qquad \mathbf{b} = \frac{m\mathbf{V}}{m}$ momentum/unit mass

$$\frac{Dm\mathbf{V}}{Dt} = \frac{\partial}{\partial t}\int_{cv}\rho\mathbf{V}\,d\rlap{-}V + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\,dA$$

$$\frac{Dm\mathbf{V}}{Dt} = \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\,dA$$ Steady state

Linear Momentum Equation



$$\frac{Dm\mathbf{V}}{Dt} = \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\,dA$$

$$\frac{Dm\mathbf{V}}{Dt} = \int_{cs_1}\mathbf{V}_1\underbrace{\rho_1\mathbf{V}_1\cdot\hat{\mathbf{n}}_1 dA}_{} + \int_{cs_2}\mathbf{V}_2\rho_2\mathbf{V}_2\cdot\hat{\mathbf{n}}_2 dA$$

$$\frac{Dm\mathbf{V}}{Dt} = \overbrace{-(\rho_1 V_1 A_1)}^{}\underline{\mathbf{V}}_1 + (\rho_2 V_2 A_2)\underline{\mathbf{V}}_2$$

$$\underline{\mathbf{M}}_1 = -(\rho_1 V_1 A_1)\underline{\mathbf{V}}_1 = -(\rho Q)\underline{\mathbf{V}}_1$$

$$\underline{\mathbf{M}}_2 = (\rho_2 V_2 A_2)\underline{\mathbf{V}}_2 = (\rho Q)\underline{\mathbf{V}}_2$$

Vectors!!!

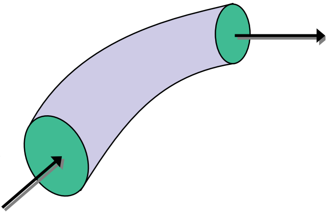
Assumptions

- Uniform density
- Uniform velocity
- **V ⊥ A**
- Steady
- **V** fluid velocity

Steady Control Volume Form of Newton's Second Law

$$\sum \mathbf{F} = \frac{D(m\mathbf{V})}{Dt} = \mathbf{M}_1 + \mathbf{M}_2$$

- What are the forces acting on the fluid in the control volume?
  - Gravity
  - Shear forces at the walls
  - Pressure forces at the walls
  - Pressure forces on the ends



$$\sum \mathbf{F} = \mathbf{M}_1 + \mathbf{M}_2$$

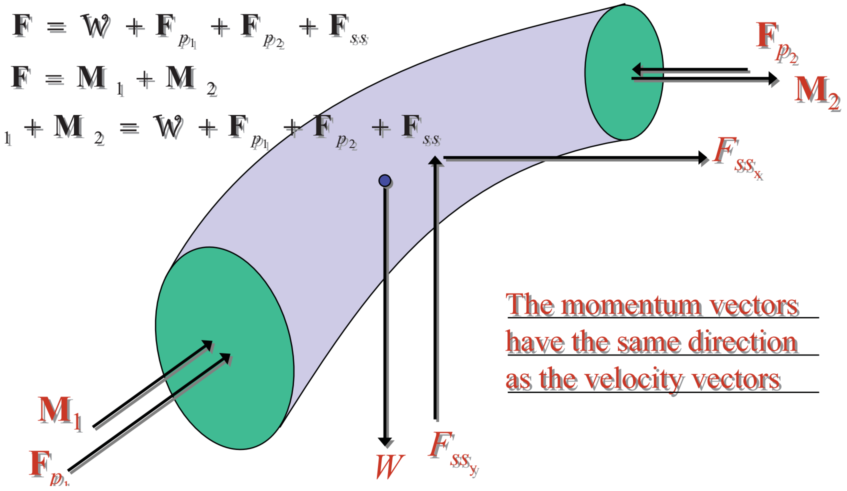
$$\sum \mathbf{F} = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{p_{wall}} + \mathbf{F}_{\tau_{wall}}$$

Linear Momentum Equation

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

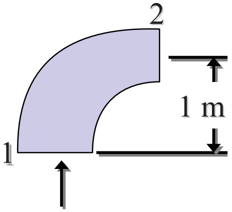
$$\sum \mathbf{F} = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M}_1 + \mathbf{M}_2 = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$



The momentum vectors have the same direction as the velocity vectors

Example: Reducing Elbow



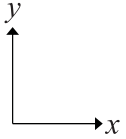
Reducing elbow in vertical plane with water flow of 300 L/s. The volume of water in the elbow is 200 L. Energy loss is negligible. P1=150 kPa

Calculate the force of the elbow on the fluid.

W = -1961 N ↑

$$\mathbf{M}_1 + \mathbf{M}_2 = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

	section 1	section 2	
D	50 cm	30 cm	
A	0.196 m^2	0.071 m^2	
V	1.53 m/s ↑	4.23 m/s →	
p	150 kPa	?	
M	-459 N ↑	1269 N →	Direction of V vectors
F_p	29,400 N ↑	?	



Example: What is p_2?

$$\frac{p_1}{\gamma_1} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma_2} + z_2 + \frac{V_2^2}{2g}$$

$$p_2 = p_1 + \gamma\left[z_1 - z_2 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right]$$

$$p_2 = \left(150 \text{ x } 10^3 \text{Pa}\right) + \left(9810 \text{ N/m}^3\right)\left[0 - 1 \text{ m} + \frac{(1.53 \text{ m/s})^2}{2\left(9.8 \text{ m/s}^2\right)} - \frac{(4.23 \text{ m/s})^2}{2\left(9.8 \text{ m/s}^2\right)}\right]$$

P_2 = 132 kPa

F_{p2} = -9400 N

Example: Reducing Elbow Horizontal Forces

$$\mathbf{M}_1 + \mathbf{M}_2 = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

$$\mathbf{F}_{ss} = \mathbf{M}_1 + \mathbf{M}_2 - \mathbf{W} - \mathbf{F}_{p_1} - \mathbf{F}_{p_2}$$

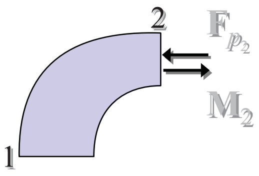
$$F_{ss_x} = \cancel{M}_{1_x} + M_{2_x} - \cancel{W}_x - \cancel{F}_{p_{1x}} - F_{p_{2x}}$$

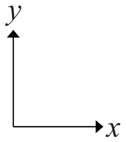
$$F_{ss_x} = M_{2_x} - F_{p_{2x}}$$

$$F_{ss_x} = (1269\,N) - (-9400\,N)$$

$$F_{ss_x} = 10.7\,\mathrm{kN}$$ Force of pipe on fluid

Pipe wants to move to the left





Example: Reducing Elbow Vertical Forces

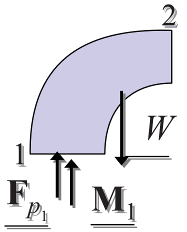
$$F_{ss_y} = M_{1_y} + \cancel{M}_{2_y} - W_y - F_{p_{1y}} - \cancel{F}_{p_{2y}}$$

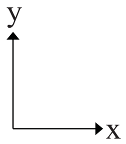
$$F_{ss_y} = M_{1_y} - W_y - F_{p_{1y}}$$

$$F_{ss_y} = -459\mathrm{N} - (-1{,}961\mathrm{N}) - (29{,}400\mathrm{N})$$

$$F_{ss_y} = -27.9\,k\mathrm{N}$$ 28 kN acting downward on fluid

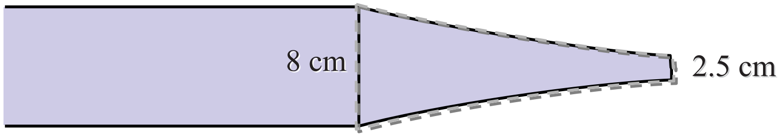
Pipe wants to move up





Example: Fire nozzle

A small fire nozzle is used to create a powerful jet to reach far into a blaze. Estimate the force that the water exerts on the fire nozzle. The pressure at section 1 is 1000 kPa (gage). Ignore frictional losses in the nozzle.

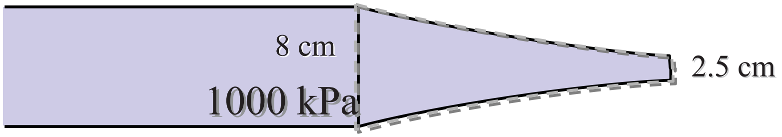


Fire nozzle

Identify what you need to know

Count your unknowns

Determine what equations you will use



Find the Velocities

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{V_2^2}{2g}$$

$$V_1D_1^2=V_2D_2^2$$

$$\frac{p_1}{\gamma}=\frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

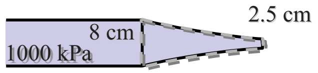
$$V_2^2\left(\frac{D_2}{D_1}\right)^4=V_1^2$$

$$p_1=\rho\frac{V_2^2}{2}\left(1-\left(\frac{D_2}{D_1}\right)^4\right)$$

$$V_2=\sqrt{\frac{2p_1}{\rho\left[1-\left(\frac{D_2}{D}\right)^4\right]}}$$

Fire nozzle: Solution

	section 1	section 2	
D	0.08	0.025	m
A	0.00503	0.00049	m^2
P	1000000	0	Pa
V	4.39	44.94	m/s
Fp	5027		N
M	-96.8	991.2	N
Fssx	-4132		N force applied by nozzle on water
Q	22.1		L/s



Which direction does the nozzle want to go? →

Is this the force that the firefighters need to brace against? NO!

$$F_{ss_x}=M_{1_x}+M_{2_x}-W_x-F_{p_{1x}}-F_{p_{2x}}$$

Example: Momentum with Complex Geometry

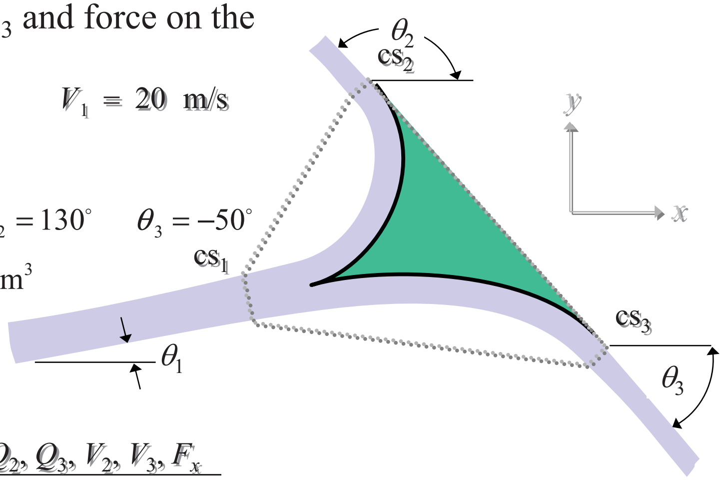
Find Q_2, Q_3 and force on the wedge.

$Q_1 = 10$ L/s $\quad V_1 = 20$ m/s

$F_y = 0$

$\theta_1 = 10^\circ \quad \theta_2 = 130^\circ \quad \theta_3 = -50^\circ$

$\rho = 1000\ \text{kg}/\text{m}^3$



Unknown: Q_2, Q_3, V_2, V_3, F_x

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5 Unknowns: Need 5 Equations

Identify the 5 equations!

Unknowns: Q_2, Q_3, V_2, V_3, F_x

Continuity $Q_1 = Q_2 + Q_3$

Bernoulli (2x)

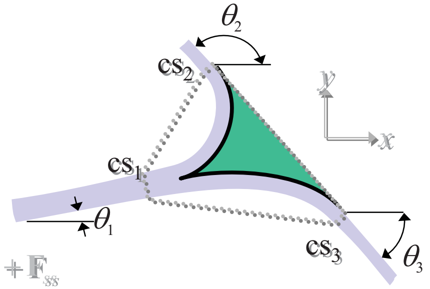
$$\cancel{\frac{p_1}{\gamma_1}}+\cancel{z_1}+\frac{V_1^2}{2g}=\cancel{\frac{p_2}{\gamma_2}}+\cancel{z_2}+\frac{V_2^2}{2g}$$

$$V_1=V_2$$

$$V_1=V_3$$

Momentum (in x and y)

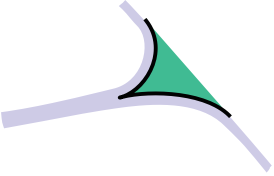
$$\mathbf{M}_1+\mathbf{M}_2+\mathbf{M}_3=\mathbf{W}+\mathbf{F}_{p_1}+\mathbf{F}_{p_2}+\mathbf{F}_{p_3}+\mathbf{F}_{ss}$$



Solve for Q_2 and Q_3

$$\mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 = \cancel{W} + \cancel{\mathbf{F}_{p_1}} + \cancel{\mathbf{F}_{p_2}} + \cancel{\mathbf{F}_{p_3}} + \mathbf{F}_{ss}$$

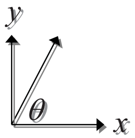
atmospheric pressure



$$F_y = 0 = M_{1y} + M_{2y} + M_{3y}$$

$$0 = -\rho Q_1 V_1 \sin\theta_1 + \rho Q_2 V_2 \sin\theta_2 + \rho Q_3 V_3 \sin\theta_3$$

$V \sin\theta =$ Component of velocity in y direction



$$Q_1 = Q_2 + Q_3$$

Mass conservation

$$V_1 = V_2 = V_3$$

Negligible losses – apply Bernoulli

Solve for Q_2 and Q_3

$$0 = -\cancel{\rho} Q_1 \cancel{V_1} \sin\theta_1 + \cancel{\rho} Q_2 \cancel{V_2} \sin\theta_2 + \cancel{\rho} Q_3 \cancel{V_3} \sin\theta_3$$

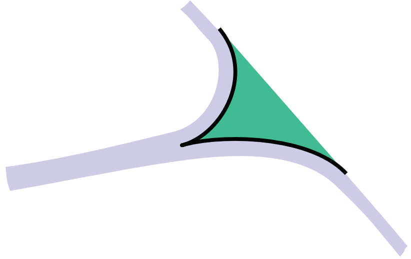
$$0 = -Q_1 \sin\theta_1 + Q_2 \sin\theta_2 + Q_3 \sin\theta_3$$

$$Q_3 = Q_1 - Q_2$$

$$Q_2 = Q_1 \frac{(-\sin\theta_1 + \sin\theta_3)}{(-\sin\theta_2 + \sin\theta_3)}$$

$$Q_2 = Q_1 \frac{[-\sin(10) + \sin(-50)]}{[-\sin(130) + \sin(-50)]}$$

$Q_2 = 6.133\ \mathrm{L/s}$



$Q_3 = 3.867\ \mathrm{L/s}$

Solve for F_x

$$F_x = M_{1x} + M_{2x} + M_{3x}$$

$$F_x = -\rho Q_1 V_1 \cos\theta_1 + \rho Q_2 V_1 \cos\theta_2 + \rho Q_3 V_1 \cos\theta_3$$

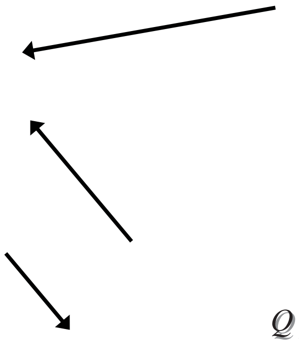
$$F_x = \rho V_1 [-Q_1 \cos\theta_1 + Q_2 \cos\theta_2 + Q_3 \cos\theta_3]$$

$$F_x = (1000\ \mathrm{kg/m^3})(20\ \mathrm{m/s}) \begin{bmatrix} -(0.01\ \mathrm{m^3/s})\cos(10) \\ +(0.006133\ \mathrm{m^3/s})\cos(130) \\ +(0.003867\ \mathrm{m^3/s})\cos(-50) \end{bmatrix}$$

$F_x = -226N$ Force of wedge on fluid

Vector solution

$$\mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 = \mathbf{F}_{ss}$$



$$|\mathbf{M}_1| = |-\rho Q_1 V_1| = 200\mathrm{N}$$

$$|\mathbf{M}_2| = |\rho Q_2 V_2| = 122.66\mathrm{N}$$

$$|\mathbf{M}_3| = |\rho Q_3 V_3| = 77.34\mathrm{N}$$

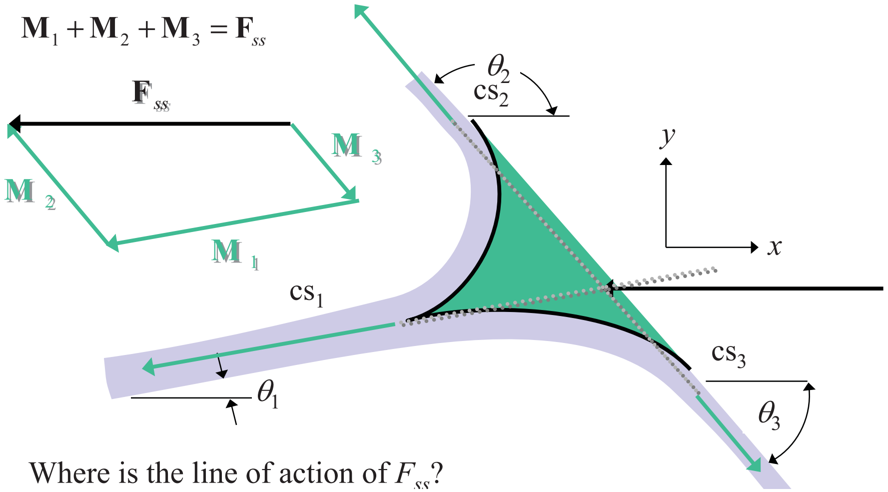
$Q_2 = 10\ L/s$

$Q_2 = 6.133\ L/s$

$Q_3 = 3.867\ L/s$

Vector Addition

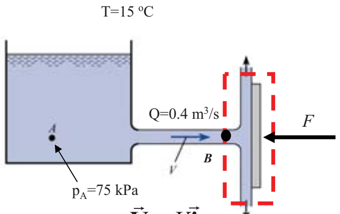
$$\mathbf{M}_1+\mathbf{M}_2+\mathbf{M}_3=\mathbf{F}_{ss}$$



Where is the line of action of F_{ss}?

Example

- **Given:** Figure
- **Find:** Horizontal force required to hold plate in position
- **Solution:**



$$\vec{\mathbf{V}}=V\vec{\mathbf{i}}$$

$$\frac{p_A}{\gamma}+z_A+\frac{V_A^2}{2g}=\frac{p_B}{\gamma}+z_B+\frac{V_B^2}{2g}$$

$$\frac{p_A}{\gamma}=\frac{V_B^2}{2g}$$

$$V_B=\sqrt{2\frac{p_A}{\rho}}=\sqrt{2*75000/999}=12.3m/s$$

$$\sum F_x=\frac{d}{dt}\int_{CV}u\rho d\forall+\int_{CS}u\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

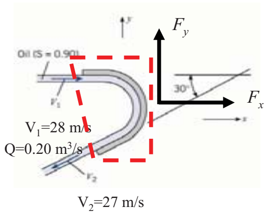
$$-F=\sum_{CS}u\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$=V\rho(-VA)$$

$$F=\rho QV=999*0.4*12.3=4.9kN$$

Example

- **Given:** Figure
- **Find:** External reactions in x and y directions needed to hold fixed vane.
- **Solution:**



$$\sum F_x=\frac{d}{dt}\int_{CV}u\rho d\forall+\int_{CS}u\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$F=\sum_{CS}u\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$=V_1\rho(-V_1A)+(-V_2\cos 30^o)\rho(V_2A_2)$$

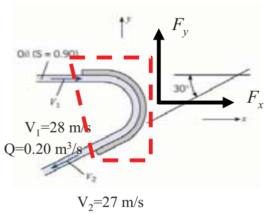
$$F=-V_1\ \rho(V_1\ A_1)-V_2\ \cos 30^o\ \rho(V_2\ A_2)$$

$$=-\rho Q(V_1\ +V_2\ \cos 30^o)$$

$$=-0.9*1000*0.2(28+27\cos 30^o)$$

$$F_x=-9.25kN\text{ (to the left)}$$

Example



$$\sum F_y=\frac{d}{dt}\int_{CV}v\rho d\forall+\int_{CS}v\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$F_y=\sum_{CS}v\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$=(-V_2\sin 30^o)\rho(V_2A_2)$$

$$F=-V_2\ \sin 30^o\ \rho(V_2\ A_2)$$

$$=-\rho Q(V_2\ \sin 30^o)$$

$$=-0.9*1000*0.2(27\sin 30^o)$$

$$F_y=-2.43kN\text{ (down)}$$

Example

- **Given:** Figure
- **Find:** Force applied to flanges to hold pipe in place
- **Solution:** Continuity equation

$$Q = V_1 A_1 = V_2 A_2$$

$$V = Q / A = 0.6/(\pi * 0.3^2 / 4) = 8.49 m/s$$

- Momentum

$$\sum F_x = \frac{d}{dt} \int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$F_x + p_1 A_1 + p_2 A_2 = V_1 \rho(-V_1 A_1) - V_2 \rho(V_2 A_2)$$

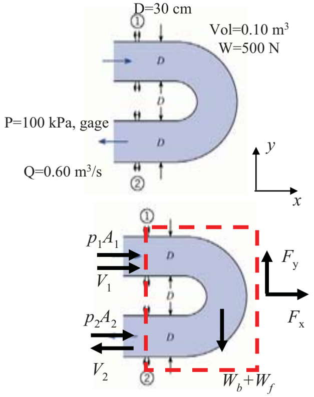
$$F_x = -2(100{,}000)(\pi * 0.3^2 / 4) - 2 * 8.49 * 1000 * 0.6$$

$$F_x = -24{,}325\ N$$

$$\sum F_y = \frac{d}{dt} \int_{CV} v\rho d\forall + \int_{CS} v\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

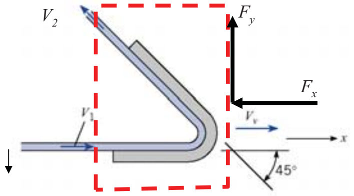
$$F_y - W_b - W_f = 0$$

$$F_y = 500 + 0.1 * 9810 = 1481 N$$



Example

- **Given:** Water jet, 6 cm diameter, with velocity 20 m/s hits vane moving at 7 m/s.
- **Find:** Find force on vane by water.
- **Solution:** Select CV moving with the vane at constant velocity. The magnitude of the velocity along the vane is constant



$$V_1 = V_2 = V - V_v$$

$$V_1 A_1 = V_2 A_2 = (V - V_v) A$$

$$\sum F_x = \frac{d}{dt} \int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$-F_x = (V - V_v)\rho[-(V - V_v)A_1] + (V - V_v)\cos 45^o \rho[(V - V_v)A_2]$$

$$= -(V - V_v)^2 \rho A(1 + \cos 45^o)$$

$$= -(20 - 7)^2 (1000)(\pi * 0.06^2 / 4)(1 + \cos 45^o)$$

$$F_x = 815.7\,N$$

$$\sum F_y = \frac{d}{dt} \int_{CV} v\rho d\forall + \int_{CS} v\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$F_y = (V - V_v)\sin 45^o \rho[(V - V_v)A_2]$$

$$= (V - V_v)^2 \rho A \sin 45^o$$

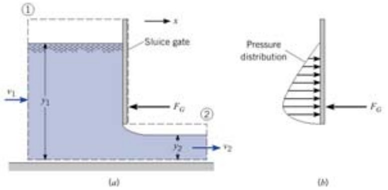
$$= (20 - 7)^2 (1000)(\pi * 0.06^2 / 4)(\sin 45^o)$$

$$F_y = 337.9\,N$$

Sluice Gate

- **Find:** Force due to pressure on face of gate
- **Solution:**

  Assume: v_1 and v_2 are uniform (so pressure is hydrostatic)



$$\sum F_x = \frac{d}{dt} \int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$\sum F_x = v_1 \rho[-v_1 A_1] + v_2 \rho[v_2 A_2]$$

$$\bar{p}_1 A_1 - \bar{p}_2 A_2 - F_G = \rho Q(v_2 - v_1)$$

$$(\gamma \frac{y_1}{2} y_1 b) - (\gamma \frac{y_2}{2} y_2 b) - F_G = \rho Q(v_2 - v_1)$$

$$F_G = \rho Q(v_1 - v_2) + \frac{\gamma b}{2}(y_1^2 - y_2^2)$$

Exercise:

5.14, 5.29, 5.31, 5.50, 5.52, 5.56

Continuity Equation

- Reynolds Transport Theorem

$$B = M_{sys} \text{ (extensive)}$$

$$b = \frac{dB}{dm} = \frac{dM_{sys}}{dm} = 1 \text{(intensive)}$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho d\forall + \sum_{CS} b\rho \vec{V}\cdot\vec{A}$$

$$\boxed{0 = \frac{d}{dt}\int_{CV} \rho d\forall + \sum_{CS} \rho \vec{V}\cdot\vec{A}} \qquad \boxed{0 = \sum_{CS} \rho \vec{V}\cdot\vec{A}}$$

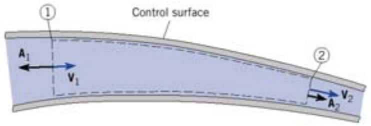
Unsteady Case

Steady Case

1-D Flow in a Conduit

- Continuity Eq.

$$0 = \frac{d}{dt}\int_{CV} \rho d\forall + \sum_{CS} \rho \vec{V}\cdot\vec{A}$$



- Steady flow

$$0 = \sum_{CS} \rho \vec{V}\cdot\vec{A}$$

- Incompressible fluid

$$0 = -V_1 A_1 + V_2 A_2$$

$$V_1 A_1 = V_2 A_2$$

$$Q_1 = Q_2$$

Momentum Equation

- Reynolds Transport Theorem

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho d\forall + \int_{CS} b\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

- b = velocity; B_{sys} = system momentum

$$\boxed{\sum\vec{\mathbf{F}} = \frac{d}{dt}\int_{CV} \vec{\mathbf{V}}\rho d\forall + \int_{CS} \vec{\mathbf{V}}\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}}$$

- Vector equation -- 3 components, e.g., x

$$\sum F_x = \frac{d}{dt}\int_{CV} u\rho d\forall + \int_{CS} u\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$\frac{dB_{sys}}{dt} = \frac{d\vec{\mathbf{M}}_{sys}}{dt} = \sum\vec{\mathbf{F}}$$

$$\vec{\mathbf{V}} = u\vec{\mathbf{i}} + v\vec{\mathbf{j}} + w\vec{\mathbf{k}}$$