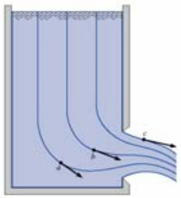
Fluid Kinematics

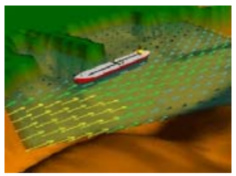
Fluid Flow Concepts and Reynolds Transport Theorem

- Descriptions of:
  - fluid motion
  - fluid flows
  - temporal and spatial classifications
- Analysis Approaches
  - Lagrangian vs. Eulerian
- Moving from a system to a control volume
  - Reynolds Transport Theorem

Fluid Motion

- Velocity field $\boldsymbol{V} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$ $\quad \boldsymbol{V} = u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k}$
- Two ways to describe fluid motion
  - Lagrangian
    - Follow particles around
  - Eularian
    - Watch fluid pass by a point or an entire region
  - Flow pattern
    - Streamlines – velocity is tangent to them





Analysis Approaches

- Lagrangian (system approach)
  - Describes a defined <u>mass</u> (position, velocity, acceleration, pressure, temperature, etc.) as functions of time
  - Track the location of a migrating bird`
- Eulerian
  - Describes the flow <u>field</u> (velocity, acceleration, pressure, temperature, etc.) as functions of position and time
  - Count the birds passing a particular location

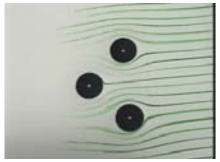
If you were going to study water flowing in a pipeline, which approach would you use? <u>Eulerian</u>

Descriptions of Fluid Motion

– streamline — Defined instantaneously
  - has the direction of the velocity vector at each point
  - no flow across the streamline
  - steady flow streamlines are fixed in space
  - unsteady flow streamlines move

– pathline — Defined as particle moves (over time)
  - path of a particle
  - same as streamline for steady flow

– streakline
  - tracer injected continuously into a flow
  - same as pathline and streamline for steady flow

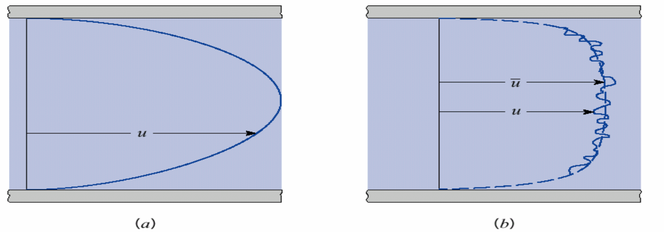


Descriptors of Fluid Flows

Laminar vs Turbulent Flow

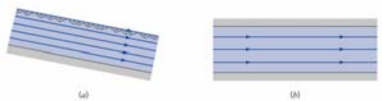
– Laminar flow
  - fluid moves along smooth paths
  - viscosity damps any tendency to swirl or mix

– Turbulent flow
  - fluid moves in very irregular paths
  - efficient mixing
  - velocity at a point fluctuates

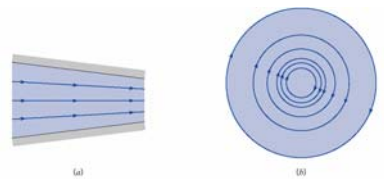


Flow Patterns Temporal/Spatial Classifications

- Uniform flow $\frac{\partial \mathbf{V}}{\partial s} = 0$



- Non-uniform flow $\frac{\partial \mathbf{V}}{\partial s} \neq 0$



- Steady flow $\frac{\partial \mathbf{V}}{\partial t} = 0$
- Unsteady flow $\frac{\partial \mathbf{V}}{\partial t} \neq 0$

Can turbulent flow be steady?

If averaged over a suitable time

Acceleration

- Cartesian coordinates $\vec{\mathbf{V}} = u\vec{\mathbf{i}} + v\vec{\mathbf{j}} + w\vec{\mathbf{k}}$ $\vec{\mathbf{a}} = a_x\vec{\mathbf{i}} + a_y\vec{\mathbf{j}} + a_z\vec{\mathbf{k}}$

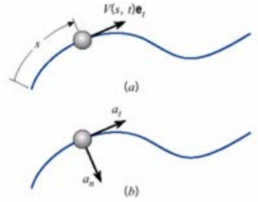
$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t} = .\frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial v}{\partial z}\frac{dz}{dt} + \frac{\partial v}{\partial t} = .\frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} + \frac{\partial w}{\partial t} = .\underbrace{\frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w}_{\text{Convective}} + \underbrace{\frac{\partial w}{\partial t}}_{\text{Local}}$$

Acceleration

- Acceleration = rate of change of velocity
- Components:
  - Normal – changing direction
  - Tangential – changing speed



$$\vec{V} = V(s,t)\hat{e}_t$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{dV}{dt}\hat{e}_t + V\frac{d\hat{e}_t}{dt}$$

$$\frac{dV}{dt} = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$$

$$\frac{d\hat{e}_t}{dt} = \frac{V}{r}\hat{e}_n$$

$$\vec{a} = (V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t})\hat{e}_t + \frac{V^2}{r}\hat{e}_n$$

The Dilemma

- The laws of physics in their simplest forms describe systems (the Lagrangian approach)
  - Conservation of Mass, Momentum, Energy
- It is impossible to keep track of the system in many fluids problems
- The laws of physics must still hold in a Eulerian world!
- We need some tools to bridge the gap

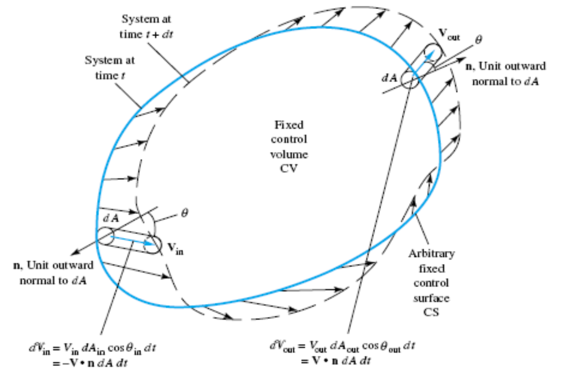
Reynolds Transport Theorem

- A moving system flows through the fixed control volume.
- The moving system transports extensive properties across the control volume surfaces.
- We need a bookkeeping method to keep track of the properties that are being transported into and out of the control volume

Control Volume Conservation Equation

B = Total amount of some properties in the system

b = Amount of the property per unit mass



$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{CV} \beta\rho\, d\mathcal{V}\right) + \int_{CV} \beta\rho(\mathbf{V}\cdot\mathbf{n})\, dA$$

Rate of increase of the property in the system = Rate of increase of the property in the control volume + Rate of efflux of the property across the control volume boundary

Application of Reynolds Transport Theorem

- Conservation of mass (for all species)
- Newton's 2nd law of motion (momentum)
  $F = ma$
- First law of thermodynamics (energy)

continuity, momentum, and energy equations

Summary

- Reynolds Transport Theorem can be applied to a control volume of finite size
  - We don't need to know the flow details within the control volume!
  - We do need to know what is happening at the control surfaces.
- We will use Reynolds Transport Theorem to solve many practical fluids problems

Exercise:

4.34, 4.43, 4.49, 4.67

Flow Rate

- Volume rate of flow
  - Constant velocity over cross-section

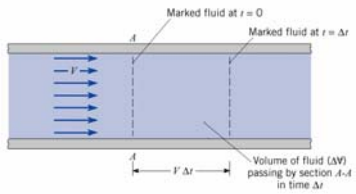
    $Q = VA$

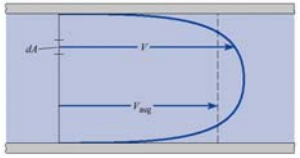
  - Variable velocity

    $$Q = \int_A V dA$$

- Mass flow rate

  $$\dot{m} = \int_A \rho V dA = \rho \int_A V dA = \rho Q$$





Flow Rate

- Only *x*-direction component of velocity (*u*) contributes to flow through cross-section

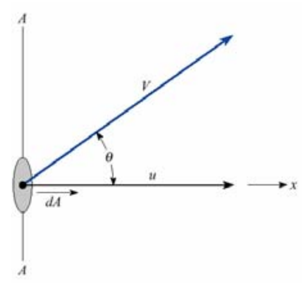
$$Q = \int_A V dA = \int_A u dA = \int_A V \cos\theta dA$$

or

$$Q = \int_A \mathbf{V} \cdot dA$$

or

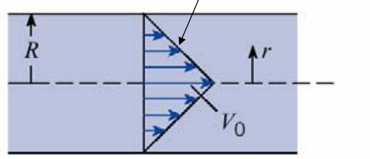
$$Q = \mathbf{V} \cdot \mathbf{A}$$



Example

- Find: $\frac{\bar{V}}{V_o}$

$$v(r) = V_o(1 - \frac{r}{R})$$



$$Q = \int_A V dA = \int_0^R V_o(1 - r/R) 2\pi r dr$$

$$= 2\pi V_o (\frac{r^2}{2} - \frac{r^3}{3R})\Big|_0^R = 2\pi V_o (\frac{R^2}{2} - \frac{R^2}{3})$$

$$= \frac{1}{3}\pi V_o R^2$$

$$\frac{\bar{V}}{V_o} = \frac{Q}{A} V_o = \frac{\frac{1}{3}\pi V_o R^2}{\pi R^2 V_o} = \frac{1}{3}$$

Example

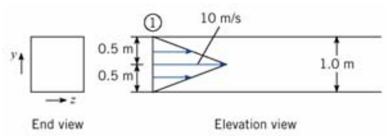
- Find: $Q, \bar{V}, \dot{m}$

$$Q = 2\int_0^{0.5} V dA = 2\int_0^{0.5} 20 y dy$$

$$= 40\frac{y^2}{2}\Big|_0^{0.5} = 5 m^3/s$$

$$\bar{V} = \frac{Q}{A} = \frac{5}{1} = 5 m/s$$

$$\dot{m} = \rho Q = 1.2 * 5 = 6 kg/s$$



Example

$$\text{Given}: \vec{V} = 3t\vec{i} + xz\vec{j} + ty^2\vec{k}$$

$$\text{Find}: \text{Acceleration}, \vec{a}$$

$$u = 3t; \quad v = xz; \quad w = ty^2$$

$$a_x = \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w + \frac{\partial u}{\partial t} = 0(3t) + 0(xz) + 0(ty^2) + 3 = 3$$

$$a_y = \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w + \frac{\partial v}{\partial t} = z(3t) + 0(xz) + x(ty^2) + 0 = 3zt + xy^2 t$$

$$a_z = \frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w + \frac{\partial w}{\partial t} = 0(3t) + 2ty(xz) + 0(ty^2) + y^2 = 2xyzt + y^2$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} = 3\vec{i} + (3tz + txy^2)\vec{j} + (2xyzt + y^2)\vec{k}$$