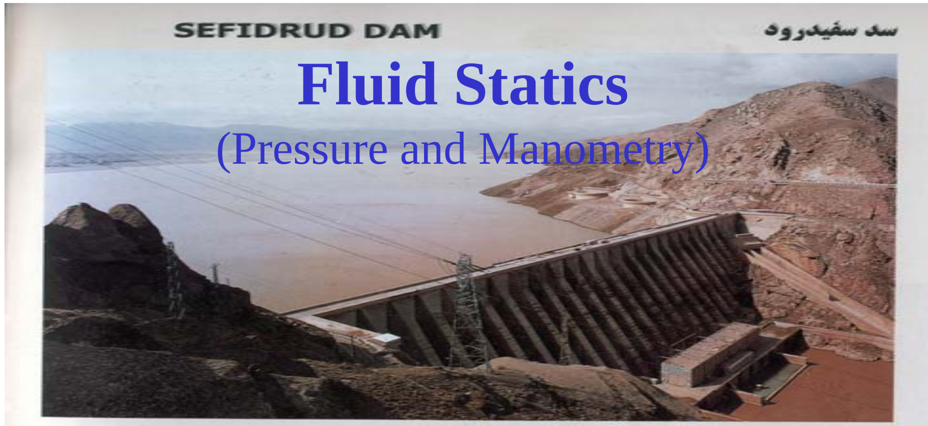
# Fluid Statics

## (Pressure and Manometry)




Owner: Gilan regional water authority
Name of dam: Sefidrud
Objective: water supply, irrigation and Hydroelectricity

1- Technical data:

General
Province: Gilan
Nearest city: Manjil
River: Sefidrud
Year of completion: 1962

Dam:
Type: Concrete Buttress
Height above foundation (m):106

Crest length (m): 425
Volume (million $m^3$): 840000

Appurtenant structures:
spillway type:Morning glory and chute
Max. discharge capacity ($m^3$/s): 7500

Reservoir:
Gross capacity (million $m^3$): 1650
Effective capacity (million $m^3$): 1100
Regulated annual water (million $m^3$): 1650
Installed capacity (MW): 87.5

Engineer: E.T.C.O. Edfer consulting engineers
Contractor: Saser Co.

# Definitions and Applications

- Statics: no relative motion between adjacent fluid layers.
  - Shear stress is zero
  - Only pressure can be acting on fluid surfaces
- Gravity force acts on the fluid (body force)
- Applications:
  - Pressure variation within a reservoir
  - Forces on submerged surfaces
  - Tensile stress on pipe walls
  - Buoyant forces

# What do we need to know?

- Pressure variation with direction
- Pressure variation with location
- How can we calculate the total force on a submerged surface?

# Pressure Variation with Direction (Pascal's law)

Equation of Motion

$\mathbf{F} = m\mathbf{a}$

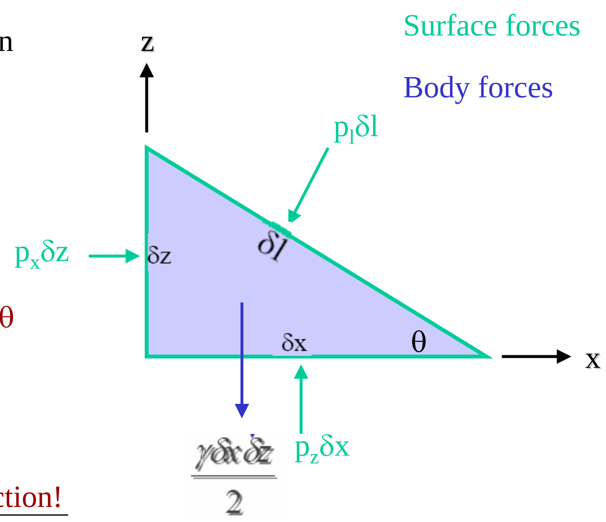
$$ma_x = \rho \frac{\delta x \delta z}{2} a_x = 0$$

$$\sum F_x = p_x \delta z - p_l \delta l \sin\theta$$

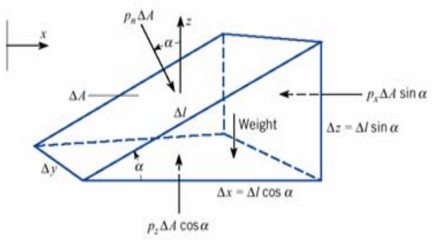
$$\delta s \sin\theta = \delta z$$

$$p_x \cancel{\delta z} - p_s \cancel{\delta z} = 0$$

Independent of direction!



# Pressure




$$\sum F_x = 0 = p_x \Delta y \Delta l \sin\alpha - p_n \Delta y \Delta l \sin\alpha \quad \Rightarrow \quad p_n = p_x$$

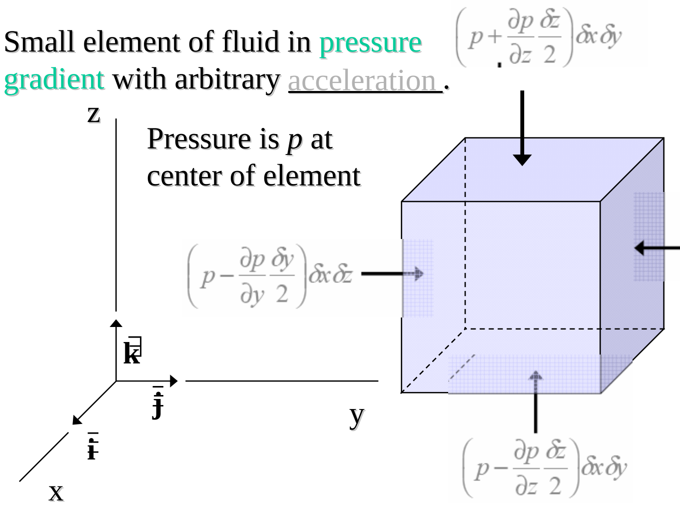
$$\sum F_z = 0 = p_z \Delta y \Delta l \cos\alpha - p_n \Delta y \Delta l \cos\alpha - \frac{1}{2}\gamma \Delta l \cos\alpha \Delta l \sin\alpha \Delta y$$

$$\Rightarrow \quad p_n = p_z$$

# Pressure Field

Small element of fluid in pressure gradient with arbitrary acceleration.

Forces acting on surfaces of element




Mass…

$$\delta m = \rho \delta x \delta y \delta z$$

# Simplify the expression for the force acting on the element

$$\delta F_y = \left(\cancel{p} - \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x \delta z - \left(\cancel{p} + \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x \delta z$$

Same in x, z!

$$\delta F_y = -\frac{\partial p}{\partial y}\delta x \delta y \delta z \qquad \delta \mathbf{F} = -\nabla p \, \delta x \delta y \delta z \qquad \delta F_z = -\frac{\partial p}{\partial z}\delta x \delta y \delta z$$

vector notation

$$\delta \mathbf{F} = -\left(\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}}\right)\delta x \delta y \delta z \qquad \frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}} = \nabla p$$

$$\delta F_x = -\frac{\partial p}{\partial x}\delta x \delta y \delta z$$

Forces acting on element of fluid due to pressure gradient

# Apply Newton's Second Law

$$\delta \mathbf{F} = \delta m \mathbf{a}$$
$$\delta \mathbf{F} = -\nabla p \, \delta x \delta y \delta z$$

Obtain a general vector expression relating pressure gradient to acceleration and write the 3 component equations.

$\delta m = \rho \delta x \delta y \delta z$ Mass of element of fluid

$-\nabla p \, \delta x \delta y \delta z = \rho \delta x \delta y \delta z \mathbf{a}$ Substitute into Newton's 2nd Law

$-\nabla p = \rho \mathbf{a}$ $\qquad -\nabla p = \rho \mathbf{a} + \gamma \hat{\mathbf{k}}$ $\qquad$ δF= Fs-W

$-\frac{\partial p}{\partial x} = \rho a_x \qquad -\frac{\partial p}{\partial y} = \rho a_y \qquad -\frac{\partial p}{\partial z} = \rho a_z$ $\quad$ 3 component equations

$-\frac{dp}{dz} = \rho g = \gamma$ At rest (independent of x and y)

# Pressure Variation When the Specific Weight is Constant (fluid at rest)

- What are the two things that could make specific weight ($\gamma$) vary in a fluid?

$\gamma = \rho g$ — Compressible fluid - changing density

Changing gravity

$$\frac{dp}{dz} = -\gamma$$

$\gamma$ is constant

$$p = -\gamma z + \text{constant}$$

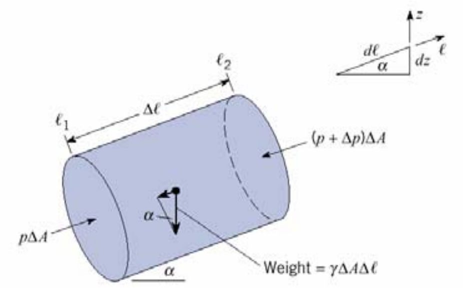
$$\frac{p}{\gamma} + z = \text{constant}$$

Piezometric head

$$\overbrace{\frac{p_1}{\gamma} + z_1}^{} = \frac{p_2}{\gamma} + z_2$$

# Pressure Variation with Elevation

- **Static fluid** – pressure varies ***only*** with elevation in the fluid.




$$\Sigma F_l = 0$$
$$= F_{left} - F_{right} - F_{weight}$$
$$= p\Delta A - (p+\Delta p)\Delta A - \gamma\Delta A\Delta l \sin\alpha$$

$$\frac{\Delta p}{\Delta l} = -\gamma \sin\alpha = -\gamma\frac{\Delta z}{\Delta l} \quad \text{or} \quad \frac{dp}{dl} = -\gamma\frac{dz}{dl}$$

$$\boxed{\frac{dp}{dz} = -\gamma}$$

# Pressure Variation with Elevation

- If $\gamma$ is a constant $\dfrac{dp}{dz} = -\gamma$

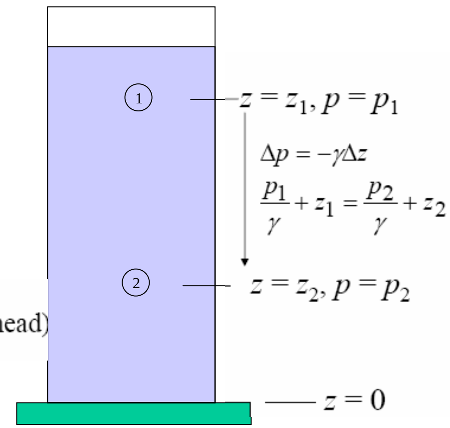
$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$
$$p_2 - p_1 = -\gamma(z_2 - z_1)$$
$$\Delta p = -\gamma\Delta z$$

$$h = \frac{p}{\gamma} + z = \text{constant (piezometric head)}$$

Elevation head

Pressure head

Piezometric head



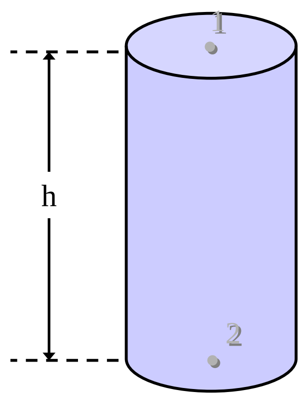

# Example: Pressure at the bottom of a Tank of Water?

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_2$$

$$p_2 = p_1 + \gamma h$$

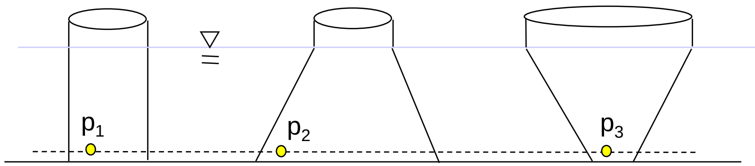
$\Delta p = \gamma h$




Does the pressure at the bottom of the tank increase if the diameter of the tank increases? NO!!!!

# Pressure Field

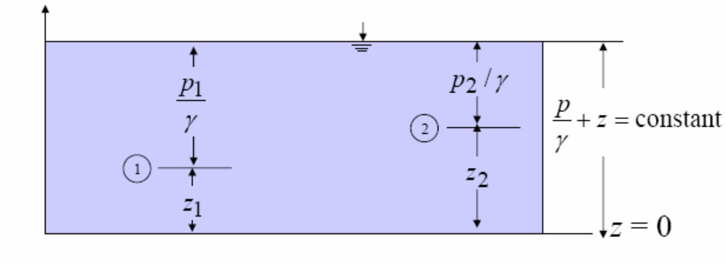
- In the absence of shearing forces (no relative motion between fluid particles) what causes pressure variation within a fluid?



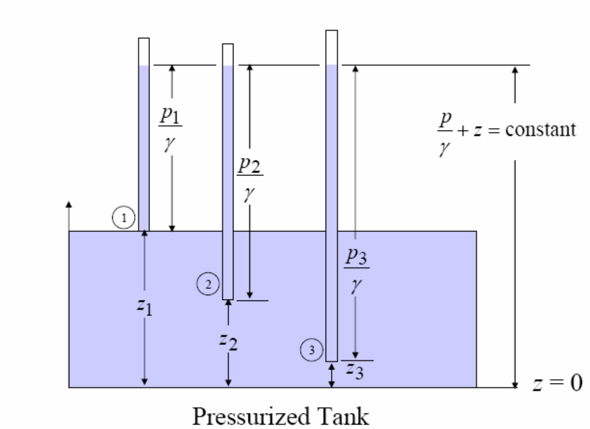

Which has the highest pressure?

# Piezometric Head

**Open Tank**



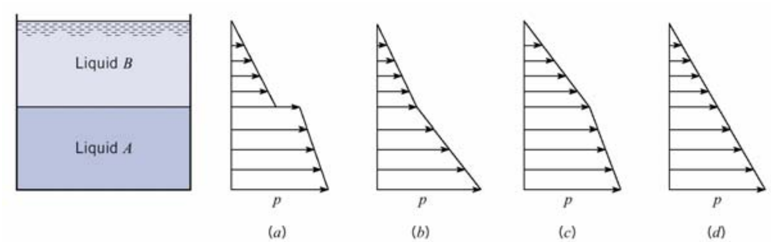

# Piezometric Head



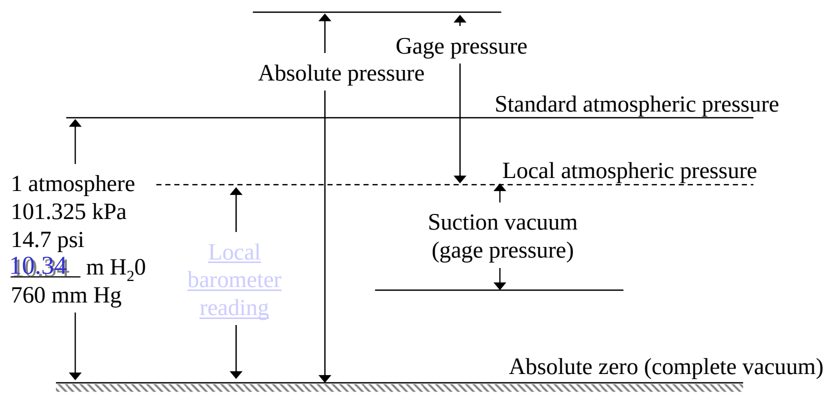

Pressurized Tank

# Example

- Tank contins fluid with $\gamma_A > \gamma_B$
- Which graph depicts the correct pressure distribution?



# Pressure Measurement (Absolute & Gage)




6894.76 Pa = 1 psi

# Mercury Barometer

What is the local atmospheric pressure (in kPa) when R is 750 mm Hg? $S_{Hg} = 13.6$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$$p_1 = \cancel{p_2} + \gamma_{Hg}(z_2 - z_1)$$

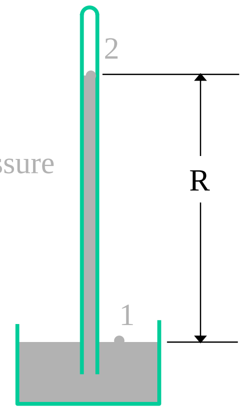
P$_2$ = Hg vapor pressure

$$p_1 = \gamma_{Hg} R$$

$$\gamma_{Hg} = S_{Hg}\gamma_{water}$$

$$p_1 = S_{Hg}\gamma R$$

$$p_1 = 13.6(9806 N/m^3)0.75m = 100{,}000 Pa$$




# Example

$F_1$ = 200 N

- What is maximum force $F_2$ that can be supported?

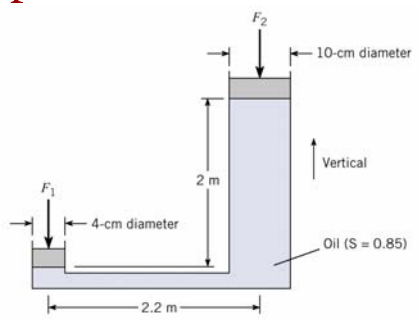
$$p_1 - p_2 = -\gamma_{oil}(z_1 - z_2)$$

$$p_2 = p_1 + \gamma_{oil}(z_1 - z_2)$$

$$= \frac{F_1}{A_1} + SG\gamma_w(z_1 - z_2)$$

$$= \frac{200}{\frac{\pi}{4}(0.04)^2} + (0.85)(9810)(0-2)$$

$$= 142{,}500\, N/m^2$$




$$F_2 = p_2 A_2 = 142{,}500(\frac{\pi}{4})(0.1)^2$$

$$\boxed{F_2 = 1{,}119\, N}$$

# Compressible Fluids

- Density is nearly proportional to pressure

$$\frac{dp}{dz} = -\gamma \quad \text{but} \quad \gamma = \rho g \quad \text{and} \quad \rho \neq \text{constant}$$

- For perfect gasses

$$p = \rho RT \quad \Rightarrow \quad \rho = \frac{p}{RT} \quad \Rightarrow \quad \frac{dp}{dz} = -\frac{p}{RT}g$$

- Need to know $T = T(z)$
  - Constant (Isothermal): $T = T_0 = const.$
  - Linear: $T = T_0 - \alpha(z - z_0)$

# Compressible Fluids

- Assume constant and integrate

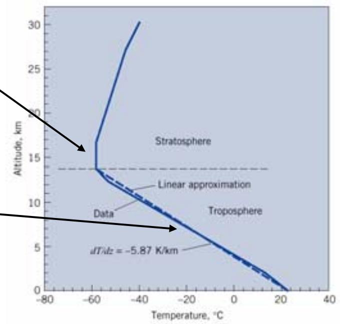
$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{g}{RT_0}\int_{z_0}^{z} dz$$

$$p(z) = p_0 e^{-g(z-z_0)/RT_0}$$

- Assume linear and integrate

$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{g}{R}\int_{z_0}^{z}\frac{dz}{T_0-\alpha(z-z_0)}$$

$$p(z) = p_0\left[\frac{T_0-\alpha(z-z_0)}{T_0}\right]^{g/\alpha R}$$



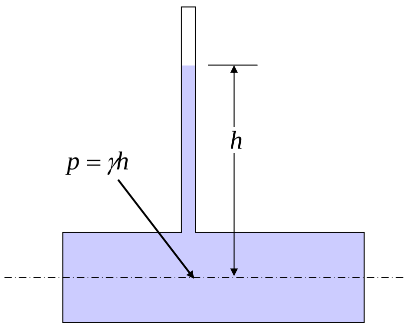

Temperature variation with altitude for the U.S. standard atmosphere

# Pressure Measurement

- Barometers
  Weight or pressure
- Manometers
  - Standard
  - Differential
- Mechanical devices and Pressure Transducers

# Manometry

- Pressure can be estimated by measuring fluid elevation




# Standard Manometers

What is the pressure at A in terms of h?

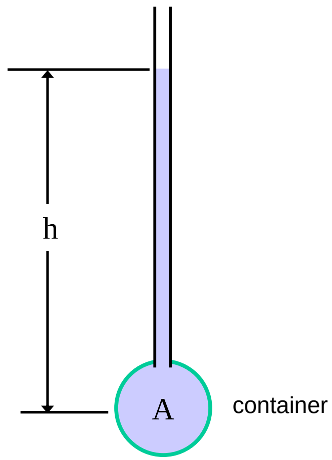
$p = \gamma h$ $\qquad \frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$

Pressure in water distribution systems commonly varies between 175 to 700 kPa. How high would the water rise in a manometer connected to a pipe containing water at 500 kPa?

$h = p/\gamma$

$h = 500{,}000\ \text{Pa}/9800\ \text{N/m}^3$

$h = 51\ \text{m}$ Not very practical!

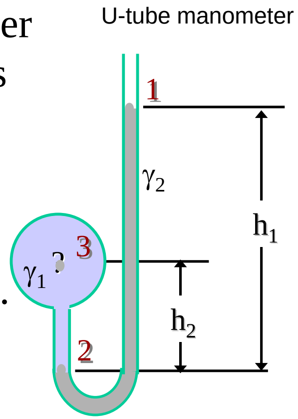


# Manometers for High Pressures

Find the **gage** pressure in the center of the sphere. The sphere contains fluid with $\gamma_1$ and the manometer contains fluid with $\gamma_2$.

What do you know? $P_1 = 0$

Use statics to find other pressures.




$P_1 + h_1\gamma_2 - h_2\gamma_1 = P_3$

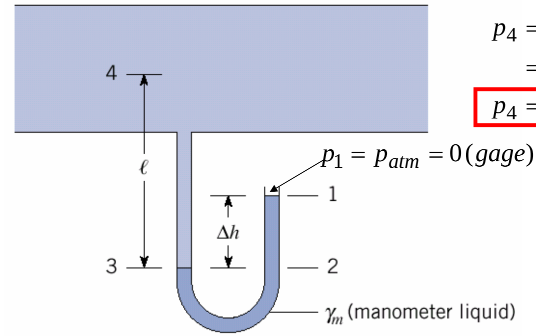
$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

For small $h_1$ use fluid with high density. Mercury!

# U-tube Manometer

$\Delta h$ = o.60 m , l=1.8 m

$$p_4 + l\gamma - \gamma_m \Delta h = p_1$$




$$p_4 = \gamma_m \Delta h - \gamma l$$
$$= 133{,}000(0.6) - 9810(1.8)$$
$$\boxed{p_4 = 62.1 kPa}$$

# Example

Find the location of the surface in the manometer

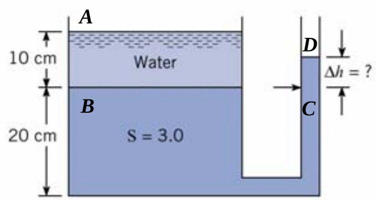
The distance $\Delta h$ is the height of the liquid in the manomoter above the heavier liquid in the tank.

$$p_A + 0.1 * \gamma_w - \Delta h * \gamma_m = p_D$$

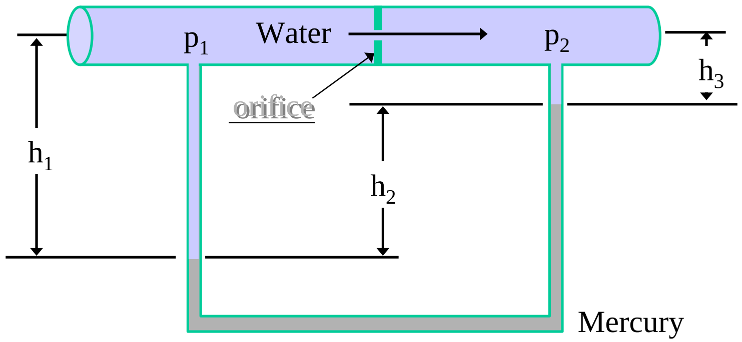
$$p_A = P_D = 0$$

$$\Delta h = 0.1 * \frac{\gamma_w}{\gamma_m}$$

$$\boxed{\Delta h = 0.1 * \frac{1}{3} = 3.33 cm}$$




# Differential Manometers



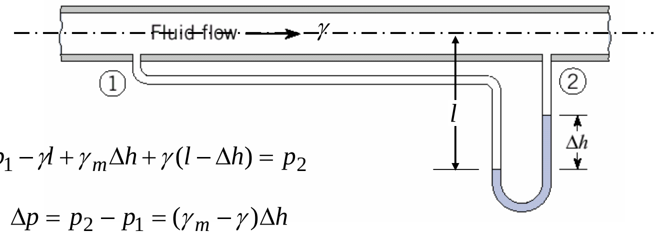

Find the drop in pressure between point 1 and point 2.

$p_1 + h_1\gamma_w - h_2\gamma_{Hg} - h_3\gamma_w = p_2$

$p_1 - p_2 = (h_3 - h_1)\gamma_w + h_2\gamma_{Hg}$

$p_1 - p_2 = h_2(\gamma_{Hg} - \gamma_w)$

# Differential Manometer




$$p_1 - \gamma l + \gamma_m \Delta h + \gamma(l - \Delta h) = p_2$$

$$\Delta p = p_2 - p_1 = (\gamma_m - \gamma)\Delta h$$

# Procedure to keep track of pressures

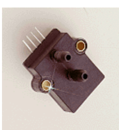
- Start at a known point or at one end of the system and write the pressure there using an appropriate symbol
- Add to this the change in pressure to the next meniscus (plus if the next meniscus is lower, and minus if higher)
- Continue until the other end of the gage is reached and equate the expression to the pressure at that point

$p_1 + \Delta p = p_2$

# Bourdon gage


# Pressure Transducers

- Can be monitored easily by computer
- Applications:
  - Volume of liquid in a tank
  - Flow rates
  - Process monitoring and control

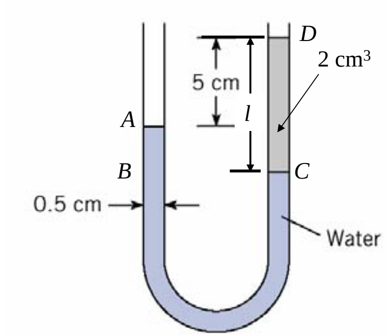


http://www.omega.com/pressure/psc.html

# Example

**Find:** Specific weight of fluid

Solution:




**Solution:**

$$V = \frac{\pi}{4} d^2 l$$

$$= \frac{\pi}{4}(0.5)^2 l = 2\,cm^3$$

$$l = 10.186\,cm$$

Manometer Equation

$$p_A + (l - 0.05)\gamma - l\gamma_{liq} = p_D$$

$$\gamma_{liq} = \frac{(l - 0.05)}{l}\gamma$$

$$= \frac{(0.10186 - 0.05)}{0.10186}(9810)$$

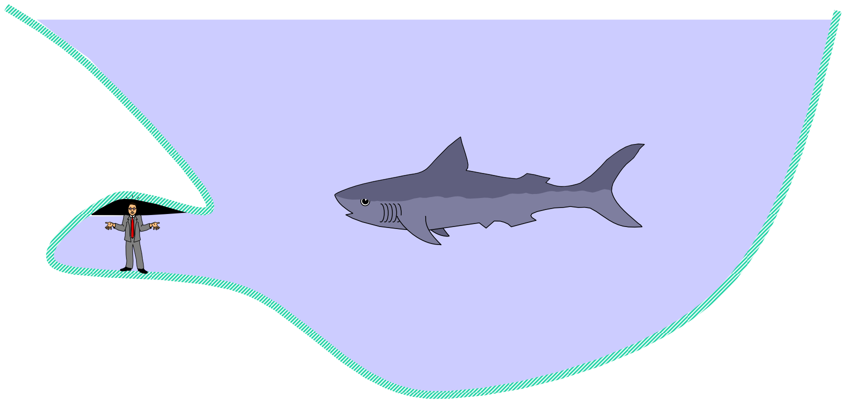
$$\boxed{\gamma_{liq} = 4{,}995\,N/m^3}$$

# Summary for Statics

- Pressure is independent of direction
- Pressure increases with depth
  - constant density $dp = -\gamma dz$ $p = \gamma h$
- Pressure scales
  - units
  - datum
- Pressure measurement

# Statics example

What is the air pressure in the cave air pocket?



**Exercise:**

**2.26, 2.28, 2.29, 2.30, 2.36, 2.45**