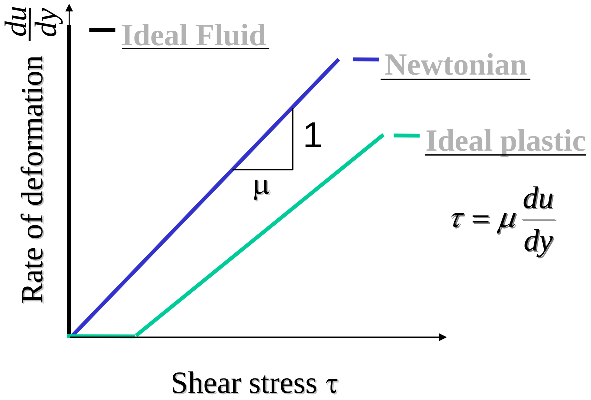
Fluid classification by response to shear stress

- Newtonian
- Ideal Fluid
- Ideal plastic

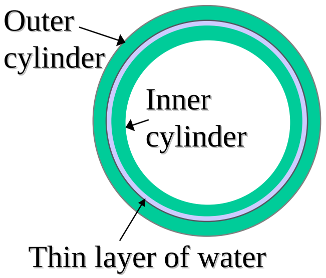


Fluid Viscosity

- Examples of highly viscous fluids
  - molasses, tar, 20w-50 oil
- Fundamental mechanisms
  - Gases - transfer of molecular momentum
    - Viscosity **increases** as temperature increases.
    - Viscosity **increases** as pressure increases.
  - Liquids - cohesion and momentum transfer
    - Viscosity **decreases** as temperature increases.
    - Relatively independent of pressure (incompressible)

Example: Measure the viscosity of water

The inner cylinder is 10 cm in diameter and rotates at 10 rpm. The fluid layer is 2 mm thick and 10 cm high. The power required to turn the inner cylinder is 50×10^{-6} watts. What is the dynamic viscosity of the fluid?



Solution Scheme

- Restate the goal
- Identify the given parameters and represent the parameters using symbols
- Outline your solution including the equations describing the physical constraints and any simplifying assumptions
- Solve for the unknown symbolically
- Substitute numerical values with units and do the arithmetic
  - Check your units!
  - Check the reasonableness of your answer

Solution

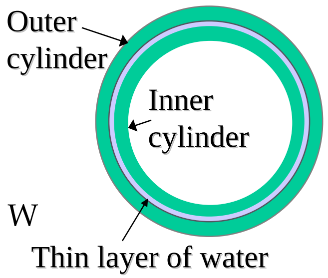
Viscosity Measurement: Solution

$$F = \mu \frac{AU}{t} \qquad U = \underline{\omega r} \qquad A = \underline{2\pi r h}$$

$$F = \mu \frac{2\pi \omega r^2 h}{t}$$

$$P = \underline{F\omega r}$$

r = 5 cm
t = 2 mm
h = 10 cm
P = 50 x 10^{-6} W
ω=10 rpm



$$P = \mu \frac{2\pi \omega^2 r^3 h}{t} \qquad \mu = \frac{Pt}{2\pi \omega^2 r^3 h}$$

ω=10 rpm
=10(2π)/60 1/s =1.047/s

$$\mu = \frac{(50\text{x}10^{-6}\,\text{W})\,(0.002\,\text{m})}{2\pi(1.047/\text{s})^2\,(0.05\,\text{m})^3\,(0.1\,\text{m})} = 1.16\text{x}10^{-3}\ \text{N}\cdot\text{s/m}^2$$

Role of Viscosity

- Statics
  - Fluids at rest have no relative motion between layers of fluid and thus du/dy = 0
  - Therefore the shear stress is <u>zero</u> and is independent of the fluid viscosity
- Flows
  - Fluid viscosity is very important when the fluid is moving

Dynamic and Kinematic Viscosity

- Kinematic viscosity (ν) is a fluid property obtained by dividing the dynamic viscosity (μ) by the fluid density

$$\nu = \frac{\mu}{\rho} \qquad \mu \Rightarrow \left[\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right] \qquad [\text{N}] = \left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right]$$

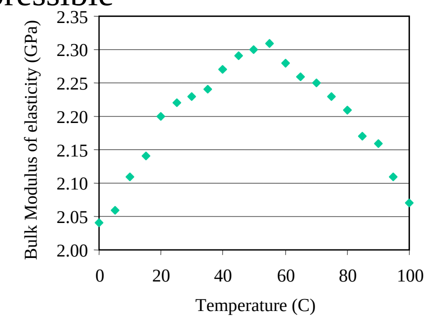
$$\nu = \frac{\left[\frac{\text{kg}}{\text{m}\cdot\text{s}}\right]}{\left[\frac{\text{kg}}{\text{m}^3}\right]} \qquad \underline{[\text{m}^2/\text{s}]}$$

Elasticity (Compressibility)

- Deformation per unit of pressure change

$$E_v = -\frac{dp}{dV/V} = \frac{dp}{d\rho/\rho}$$ •Bulk Modulus of Elasticity

- For water E_v = 2.2 GPa,
  1 MPa pressure change = 0.05% volume change
  Water is relatively incompressible



Example

- **Given:** Pressure of 2 MPa is applied to a mass of water that initially filled 1000-cm^3 volume.
- **Find:** Volume after the pressure is applied.
- **Solution:** E = 2.2x10^9 Pa (Table A.5)

$$E_v = -\frac{\Delta p}{\Delta V / V}$$

$$\Delta V = -\frac{\Delta p}{E_v} V$$

$$= -\frac{2x10^6\, Pa}{2.2x10^9\, Pa} 1000 cm^3$$

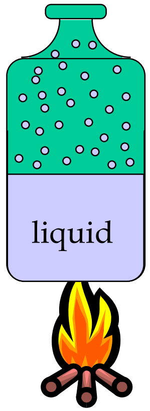
$$= -0.909\, cm^3$$

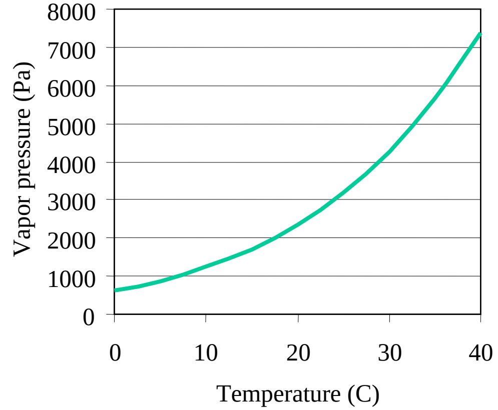
$$V_{final} = V + \Delta V$$

$$= 1000 - 0.909$$

$$\boxed{V_{final} = 999.01 cm^3}$$

Vapor Pressure

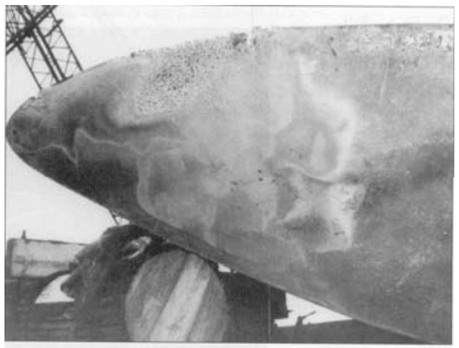




- Pressure at which a liquid will boil for given temp.
- Vapor pressure increases with temperature
- If you reduce the pressure in water at this temperature, boiling will occur (cavitation)

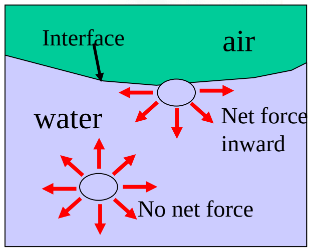
What is vapor pressure of water at 100°C? 101 kPa

Cavitation Damage



Surface Tension

- Below surface, forces act equally in all directions
- At surface, some forces are missing, pulls molecules down and together, like membrane exerting *tension* on the *surface*
- If interface is curved, higher pressure will exist on concave side
- Pressure increase is balanced by surface tension, σ
- σ = 0.073 N/m (@ 20°C)



Capillary Rise

- **Given:** Water @ 20°C, d = 1.6 mm
- **Find:** Height of water
- **Solution:** Sum forces in vertical
  Assume θ small, $\cos\theta \rightarrow 1$

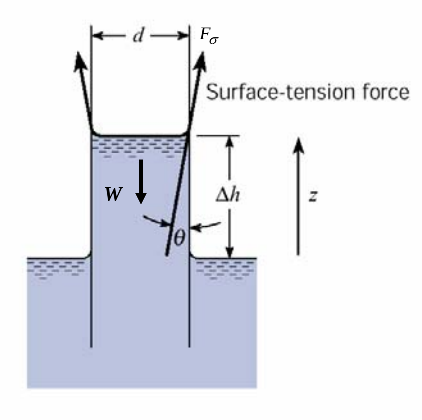
$$F_{\sigma,z} - W = 0$$

$$\sigma \pi d \cos\theta - \gamma(\Delta h)(\frac{\pi}{4} d^2) = 0$$

$$\Delta h = \frac{4\sigma}{\gamma d}$$

$$= \frac{4 * 0.073}{9790 * 1.6x10^{-3}}$$

$$\boxed{\Delta h = 18.6\, mm}$$



Example

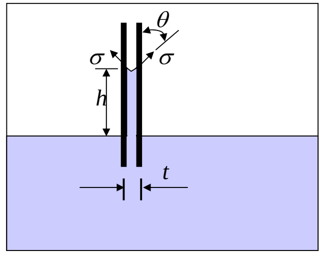
- **Find:** Capillary rise between two vertical glass plates 1 mm apart.
- σ = 7.3x10^{-2} N/m
  l is into the page
- **Solution:** $\sum F_{vertical} = 0$

$$2\sigma l - hlt\gamma = 0$$

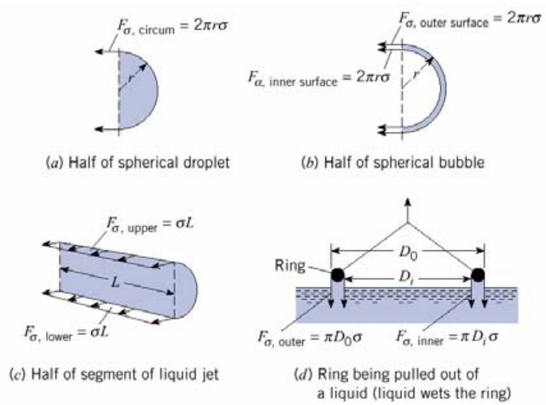
$$h = \frac{2\sigma}{t\gamma}$$

$$= \frac{2 * 7.3x10^{-2}}{0.001 * 9810}$$

$$\boxed{\begin{aligned} h &= 0.0149\, m \\ h &= 14.9\, mm \end{aligned}}$$



Examples of Surface Tension



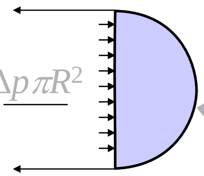
(a) Half of spherical droplet
(b) Half of spherical bubble
(c) Half of segment of liquid jet
(d) Ring being pulled out of a liquid (liquid wets the ring)

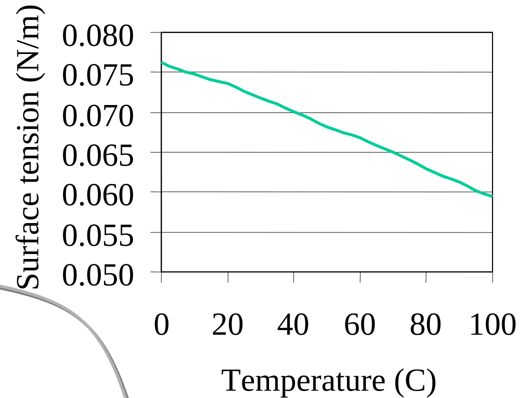
Surface Tension

- Pressure increase in a spherical droplet

$\Delta p \pi R^2$

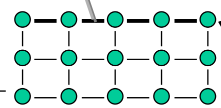
$2\pi R\sigma$





$$\Delta p \pi R^2 = 2\pi R\sigma$$

$$\Delta p = \frac{2\sigma}{R}$$



Surface molecules

Example: Surface Tension

- Estimate the difference in pressure (in Pa) between the inside and outside of a bubble of air in 20ºC water. The air bubble is 0.3 mm in diameter.

$p = \frac{2\sigma}{R}$ $\quad p = \frac{2(0.073\ \text{N/m})}{0.15 \times 10^{-3}\text{m}}$ $\quad R = 0.15 \times 10^{-3}$ m

$\sigma = 0.073$ N/m

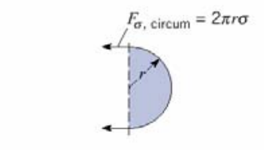
$p = 970$ Pa

Statics! $p = \gamma h$ $\quad h = \frac{p}{\gamma} = \frac{974 Pa}{9806 N/m^3} = 0.1$ m water

What is the difference between pressure in a water droplet and in an air bubble?

Example

- **Find:** The formula for the gage pressure within a sperical droplet of water?
- **Solution:** Surface tension force is reisited by the force due to pressure on the cut section of the drop

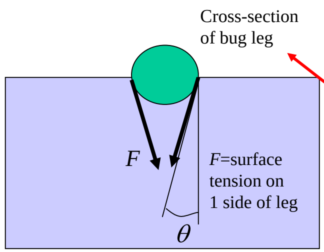


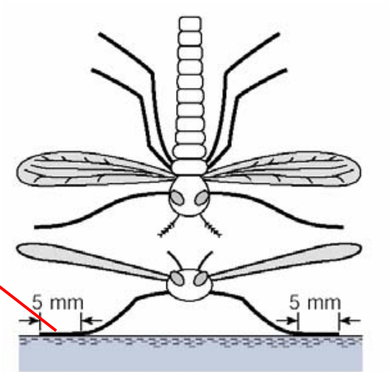
(a) Half of spherical droplet

$$p(\pi r^2) = 2\pi r\sigma$$

$$\boxed{p = \frac{2\sigma}{r}}$$

Bug Problem





Dimensions and Units

Quantity	Symbol	Dimensions
Density	ρ	ML^{-3}
Specific Weight	γ	$ML^{-2}T^{-2}$
Dynamic viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	L^2T^{-1}
Surface tension	σ	MT^{-2}
Bulk mod of elasticity	E	$ML^{-1}T^{-2}$

These are __fluid___ properties!

How many independent properties? _4___

Exercise:

1.10, 1.13, 1.54, 1.64, 1.97