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The eXtended – Finite Element Method (X – FEM) Through State of the Art Applications

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Abstract

Computational Fracture Mechanics (CFM) is key to several branches in science and engineering, such as Solid and Structural Mechanics, Geomechanics, Aerospace Engineering, Structural Health Monitoring and Damage Tolerant Design. Among the different numerical techniques that are being employed and developed in the framework of Computational Fracture Mechanics, the extended Finite Element Method (X-FEM) is one of the most powerful and versatile. By introducing enrichment functions along with standard Finite Element shape functions, the X-FEM enables very accurate simulation of fields with discontinuities and fields that feature singularities such as crack opening displacements and stresses at the vicinity of a crack tip respectively. This chapter aims to highlight the effectiveness of the X-FEM method for fracture simulation and structural integrity assessment through two state of the art applications namely; (1) Hydraulic Fracture Propagation in Naturally Fractured Porous Media, and (2) Coupling X-FEM with Peridynamics (PD) for dynamic fracture propagation in brittle materials. A brief introduction to Computational Fracture Mechanics with emphasis on the X-FEM method history, development and relevant literature is included.

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Highlights

- Fundamentals and recent developments of the Extended Finite Element Method (XFEM) are presented through two, state-of-the art, applications in fracture mechanics
- A brief and compact introduction, with literature review, to the history and basics of XFEM is included.
- Application of XFEM in hydraulic fracture propagation regarding naturally fractured porous media is presented first.
- Several numerical examples verify the effectiveness of XFEM in this challenging coupled-field problem
- Coupling of XFEM with Peridynamics for dynamic crack propagation simulation is the second application
- A dynamic relocation strategy of the Peridynamic grid at the crack tip vicinity with introduction of enrichment shape functions for crack opening displacements outside this grid is discussed.

Introduction to the eXtended Finite Element Method (XFEM)

Fracture Mechanics is the branch of solid mechanics that studies the formation and propagation of cracks in solids. Towards this aim, fracture mechanics employs analytical, experimental and numerical techniques. Early developments in numerical techniques for fracture simulation include, among other methodologies, Singular Integral Equations, the Boundary Element Method and the Finite Element Method.

Modeling crack propagation with the classical Finite Element method can be a cumbersome procedure since the evolving nature of the problem requires updating the domain topology to conform the FE mesh to the faces of the crack as it propagates. The extended finite element method eliminates the requirement of remeshing by incorporating additional discontinuous functions into the standard finite element approximation based on the partition of unity method (Melenk and Babuška, 1996). In this method, the discontinuity is modeled by introducing additional degrees of freedom, called as the enrichment degrees of freedom, to the nodal points whose supports are cut by the discontinuity (Belytschko and Black, 1999; Moës *et al.*, 1999). Hence, the geometry of the discontinuity can be updated independent of the FE mesh simply by employing an enrichment strategy (Sukumar *et al.*, 2001; Belytschko *et al.*, 2001). The X-FEM has been extensively applied in various fracture mechanics problems including, cohesive crack growth (Wells and Sluys, 2001; Moës and Belytschko, 2002), three-dimensional crack initiation and propagation (Areias and Belytschko, 2005), branched and the intersecting multiple crack growth (Zi *et al.*, 2004; Budyn *et al.*, 2004), and plastic fracture mechanics (Elguedj *et al.*, 2006; Broumand and Khoei, 2013). A comprehensive survey for various developments in the context of X-FEM was performed by Khoei (2015).

The extended finite element method (X-FEM) has gained a lot of attention in last decades for its advantages in replicating the discontinuity of the displacement field across the crack surface and the singularity at the crack-front without the need for remeshing. The extended finite element method enables the accurate approximation of fields that involve jumps, kinks, singularities, and other nonsmooth features within elements. This is achieved by adding additional terms, i.e., the enrichments, to classical finite element approximations. These terms enable the approximation to capture the nonsmooth features independently of the mesh. The X-FEM has shown its full potential for applications in fracture mechanics. Applications with cracks involve discontinuities across the crack surface and singularities, or general steep gradients, at the crack front. In the classical FEM, a suitable mesh that accounts for these features has to be provided and maintained; this is particularly difficult for crack propagation in three dimensions. The X-FEM, however, can treat these types of problems on fixed meshes and considers for crack propagation by a dynamic enrichment of the approximation.

The enrichment can be attributed to the degree of consistency of the approximation, or to the capability of approximation to reproduce a given complex field of interest. The principal of enrichment is basically equivalent to the principal of increasing the order of completeness that can be achieved *intrinsically* or *extrinsically*. However, the enrichment is aimed to increase the accuracy of the approximation by including information of the analytical solution. There are basically two ways of enriching an approximation space; enriching the basic vector known as the *intrinsic enrichment* and enriching the approximation known as the *extrinsic enrichment*. The approximation field on the basis of *extrinsic enrichment* is generally known as the extended finite element method, defined as

$$u(x) = \sum_{i=1}^{N} N_i(x)\overline{u}_i + \text{enrichment terms} \tag{1}$$

where N is the set of all nodal points and $N_i(x)$ denotes the standard shape functions. The partition of unity method is a concept for enriching the approximation *extrinsically* by adding the enrichment functions to the standard approximation. In the X-FEM method, two approaches are simultaneously applied to the elements located on discontinuity; the *partition of unity method* and the *enrichment of displacement field* (Khoei, 2015). The partition of unity method is used to enhance the approximation by adding the enrichment functions to the standard approximation. The enrichment of displacement field is applied to correct the standard displacement based approximation by incorporating discontinuous fields through a partition of unity method. It must be noted that the enrichment varies from node to node and many nodes require no enrichment. Consider the enriched approximation field (1), the enriched approximation for a single interface Γ_d can be written as