Online Energy-Based Error Indicator for Assessment of Numerical and Experimental Errors in Hybrid Simulation

Authors:

Mehdi Ahmadizadeh, University at Buffalo, Buffalo, NY 14260, mehdia@buffalo.edu
Gilberto Mosqueda, University at Buffalo, Buffalo, NY 14260, mosqueda@ang.buffalo.edu

ABSTRACT

Hybrid simulation is an effective structural test technique that takes advantage of numerical simulation of substructures with well-identified behavior, and experimental testing of complex and nonlinear components. As a result of combining numerical and experimental simulations, hybrid simulation is prone to both numerical and experimental errors. In this study, the dominant sources of numerical and experimental errors of hybrid simulation are studied. It is shown that analytical stability and accuracy limits of the utilized test procedures may fail to adequately predict the outcome of hybrid simulation due to the experimental errors and nonlinearities. As a result, these criteria may not be suitable for the assessment of the accuracy and stability of hybrid simulations. An alternative approach using the energy balance of the system as an overall error indicator is proposed. First, an online error monitor for experimental errors is studied that evaluates the difference between the actual experimental energy dissipation and the energy dissipation apparent from the final states at each integration step. Next, this energy error is extended to evaluate the overall energy balance of the system in order to capture both numerical and experimental errors. The effectiveness of this energy error indicator in predicting unacceptable levels of error is demonstrated through numerical and experimental simulations.

INTRODUCTION

Hybrid simulation combines the physical testing of experimental substructures that may be difficult to model with the numerical analysis of the remaining parts of the structural model. As a result, both numerical and experimental errors are expected to contaminate the results of a hybrid simulation. Numerical errors can usually be reduced beyond the desired precision for engineering purposes, by following certain modeling and analysis guidelines. The errors in experimental substructures can also be reduced by proper tuning and calibration of test equipment and using high-performance instrumentation. However, it is virtually impossible to entirely eliminate all numerical and experimental errors, and compensation procedures are often necessary in order to reduce their effects.

In feedback systems like hybrid simulation, even small errors can accumulate during the experiment and significantly affect the simulation, yielding inaccurate or unstable results. This is due to the fact that in time-stepping integration algorithms, experimental measurements contaminated by errors are used to compute subsequent commands. These errors can accumulate through the simulation and hence, it is imperative to recognize the most important sources of error in hybrid simulation and develop error indices for quantification of the errors in order to assess the reliability of the results. The errors of individual hybrid simulation procedures have been widely studied, and their stability and accuracy limits are well established. Examples of these procedures are the integration methods [Shing and Mahin 1983; Shing and Mahin 1984] and delay compensation procedures [Horiuchi et al. 1999; Nakashima and Masaoka 1999]. However, the results from most of these studies are based on a linear single-degree-of-freedom (SDF) system, and often do not include experimental errors and nonlinearities of hybrid simulations. For this reason, most of the analytically-calculated stability and accuracy limits may not adequately predict the outcome of hybrid simulations. It should also be noted that repeating a hybrid simulation due to unacceptable accuracy can be very costly, as the experimental substructures may sustain damage during the simulation and need to be replaced. For this reason, rather than post-experiment error evaluation [Thewalt and Roman 1994], it may be beneficial to assess the accuracy and stability of the system during the simulation [Mosqueda *et al.* 2007a]. These measures can provide early detection of excessive errors in preliminary low-level simulations or during the main test, to stop the simulation and prevent damage to experimental substructure.

In this paper, an online error monitor for experimental errors is studied that evaluates the difference between the actual experimental energy dissipation and energy dissipation resulting from the final states at each integration step. Next, the error indicator is extended to capture the overall energy balance of the system. The overall energy balance error is used as a reliability measure that indicates the severity of both numerical and experimental errors in hybrid simulation results. The effectiveness of this error indicator is demonstrated through numerical and experimental simulations.

HYBRID SIMULATION

In a hybrid simulation, the equation of motion of the combined numerical and experimental structural model can be expressed as:

$$\mathbf{Ma} + \mathbf{Cv} + \mathbf{Kd} + \mathbf{r} = \mathbf{f} \tag{1}$$

in which $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping, and stiffness matrix of the numerical substructure, $\mathbf{f}$ is the external force vector, $\mathbf{d}$, $\mathbf{v}$, and $\mathbf{a}$ are displacement, velocity and acceleration vectors, respectively, and $\mathbf{r}$ is the restoring force measured in the experimental substructures.

In hybrid simulation, an integration procedure is employed to satisfy the above-mentioned equation of motion in each simulation step. In addition, the integration procedure should maintain proper kinematic relations among the states. For example, the finite difference kinematic relations in the Newmark's Beta integration procedure are:

$$\mathbf{d}_n = \mathbf{d}_{n-1} + \Delta t\,\mathbf{v}_n + \left(\frac{1}{2} - \beta\right)(\Delta t)^2\,\mathbf{a}_{n-1} + \beta(\Delta t)^2\,\mathbf{a}_n \tag{2}$$

$$\mathbf{v}_n = \mathbf{v}_{n-1} + (1-\gamma)\Delta t\,\mathbf{a}_{n-1} + \gamma\,\Delta t\,\mathbf{a}_n \tag{3}$$

where Δt is the integration time step, n is the integration step number, and β and γ are integration parameters that influence the stability and accuracy of the integration scheme. Note that the presence of the term $\mathbf{a}_n$ on the right-hand side of (2) when $\beta \neq 0$ (the implicit form) results in an iterative solution scheme in nonlinear problems, which is not desirable for hybrid simulation. For this reason, most hybrid simulations utilize simplified integration procedures that do not need iterations. Examples of these methods include explicit methods that eliminate iterations by choosing $\beta = 0$, and the operator-splitting [Nakashima et al. 1990] method that use the initial stiffness approximation to account for implicit terms of (2). These integration methods were used in the hybrid simulations presented in this paper, and their actual performance in experiments have been evaluated using the proposed energy error indicator.

SOURCES OF ERROR IN HYBRID SIMULATION

In a hybrid simulation, errors can occur from the structural model idealizations, the approximate numerical methods used to solve the equation of motion, and the experimental setup. Figure 1 shows the components of a displacement-controlled (pseudo-dynamic) hybrid simulation, along with the error sources that may exist in each component. As illustrated, the major components of hybrid simulation consist of the numerical integration module and the experimental setup. The experimental subsystem is shown to consist of mechanical loading controllers, servo-hydraulic actuators for application of forces and displacements, and the measurement instrumentation. The numerical integration in this figure is shown to have two substeps. The first substep updates the states using the measurements received from the experimental setup to complete the integration step. Then, the next integration step begins by calculating the desired displacement and sending to the experimental setup. Two compensation and correction blocks are also shown as parts of the numerical simulation subsystem in Figure 1, which tend to minimize the effects of experimental errors in the simulation [Ahmadizadeh et al. 2007]. In addition, a signal generation module is required to ensure the continuity of the simulation, and generate experimental command signals from numerical simulation results [Nakashima and Masaoka 1999], which normally have different sampling rates.

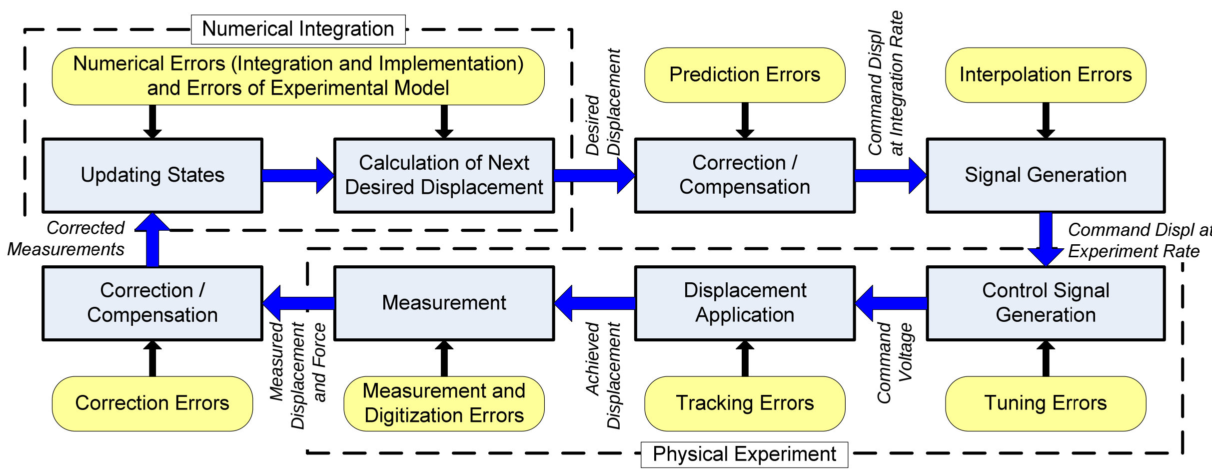


FIGURE 1 - ILLUSTRATION OF ERRORS IN HYBRID SIMULATION

ASSESSMENT OF ACCURACY AND STABILITY

It is often difficult to extend the numerical accuracy and stability limits of individual test procedures to hybrid simulations due to system nonlinearities and experimental errors that are unique to each simulation. As a result, simulation instability may occur well before reaching the stability limits that are calculated analytically for linear SDF systems. For example, when the negative damping effect resulting from actuator delay [Mosqueda *et al.* 2007b] in a linear simulation becomes greater than the specified structural damping, instability occurs. Instability may also occur in a linear hybrid simulation with smaller delays due to measurement noise (acting as a high-frequency excitation signal) and other errors. On the other hand, a nonlinear hybrid simulation with larger delay may remain stable (but inaccurate) as a result of additional hysteretic energy dissipation that overcomes the negative damping effect. These nonlinearities and experimental errors also affect the analytical stability limits of numerical integration procedures, and often shrink their stability ranges. For this reason, it is important to develop error indicators that account for nonlinearities and experimental errors, preferably without dependency on the numerical and experimental models.

Reliability Measures for Experimental Errors

One of the main goals of a hybrid simulation is to identify the structural properties of the experimental substructure, thus an effort should be made to ensure these properties are accurately captured in the simulation. One way of assessing the accuracy of the captured behavior of experimental substructure is through the observed hysteresis; the hysteretic behavior of the experimental substructure that is conceived in the numerical simulation, consisting of measured forces and desired displacements. This hysteretic behavior should be in agreement with the actual behavior of the experimental substructure. Note that in a hybrid simulation, the best data available for the actual hysteretic behavior of an experimental specimen is the measured hysteresis, obtained from the measured forces *versus* measured displacements. The difference between actual (measured) experimental behavior and that used in the numerical simulation can be evaluated by comparing the energy dissipated through these hysteretic loops [Mosqueda *et al.* 2007a]:

$$E_{\mathrm{E}}^{\mathrm{err}} = E_{\mathrm{E}}^{\mathrm{O}} - E_{\mathrm{E}} = \int \left(\mathbf{r}^{\mathrm{m}}\right)^{\mathrm{T}} \mathbf{du}^{\mathrm{d}} - \int \left(\mathbf{r}^{\mathrm{m}}\right)^{\mathrm{T}} \mathbf{du}^{\mathrm{m}} \tag{4}$$

in which $\mathbf{r}$ and $\mathbf{du}$ are experimental restoring force and incremental displacement vectors, and superscripts d and m denote the desired and measured values, respectively; E_{E} is the energy stored in, or dissipated by the experimental substructures, $E_{\mathrm{E}}^{\mathrm{O}}$ is that observed by the numerical analysis subsystem, and $E_{\mathrm{E}}^{\mathrm{err}}$ is the experimental energy dissipation error. This equation takes into account the difference between the desired and measured displacements, but does not account for the corrections made in the measured force vector, if any. Note that this energy error includes the effects of servo-hydraulic actuator delay, which is one the most important errors of hybrid simulation. The observed hysteretic behavior significantly differs from actual behavior when delay is not properly compensated [Ahmadizadeh *et al.* 2007; Mosqueda *et al.* 2007a]. This energy error term can be normalized by input energy to give a non-dimensional error indicator that is merely dependent on the experimental errors (hybrid simulation error monitor, HSEM) [Mosqueda *et al.* 2007a]:

$$\mathrm{HSEM} = \frac{E_{\mathrm{E}}^{\mathrm{err}}}{E_{\mathrm{I}} + E_{\mathrm{E}}^{\max}} \tag{5}$$

where:

$$E_{\mathrm{I}} = \int \mathbf{f}^{\mathrm{T}} \mathbf{du} \tag{6}$$

is the input energy (e.g. from earthquake excitation), and:

$$E_{\mathrm{E}}^{\max} = \frac{1}{2} \mathbf{u}_0^{\mathrm{T}} \mathbf{K}^{\mathrm{e}} \mathbf{u}_0 \tag{7}$$

is the maximum experimental strain energy. This constant energy term is used to prevent large values of error indicator in the beginning of simulation, when the input energy is very small. $\mathbf{K}^{\mathrm{e}}$ is the initial stiffness matrix of the test structure, and $\mathbf{u}_0$ is an experimental displacement vector, which can be roughly selected as the yield displacement of the experimental substructure. The choice of this displacement vector depends on the available information, and since it is used in the normalization of energy error, this selection should be considered in the selected limit for error indicator. Mosqueda *et al.* [2007b] showed that one can limit the displacement and force errors of a hybrid simulation by limiting the amount of the above-mentioned error indicator. Since the majority of errors in a hybrid simulation are likely from experimental sources, the above-mentioned HSEM can be a suitable choice for monitoring the simulation quality.

A number of error compensation procedures [Ahmadizadeh *et al.* 2007] and integration methods [Nakashima *et al.* 1990; Wu *et al.* 2006; Mosqueda and Ahmadizadeh 2007] may apply corrections on the measured force, or modify desired displacements and measured forces to improve the stability of the simulation. In order to consider the effects of those modifications on the accuracy of the captured experimental hysteresis, the energy error between the experimental and analytical subsystems can be defined as:

$$E_{\mathrm{EA}}^{\mathrm{err}} = E_{\mathrm{E}}^{\mathrm{C}} - E_{\mathrm{E}} = \int \mathbf{r}^{\mathrm{T}} \mathbf{du} - \int \left(\mathbf{r}^{\mathrm{m}}\right)^{\mathrm{T}} \mathbf{du}^{\mathrm{m}} \tag{8}$$

in which $\mathbf{r}$ and $\mathbf{du}$ (without superscripts) are the final force and displacement values used in numerical analysis (immediately after updating the states in the numerical integration subsystem in Figure 1). These values are found at the end of each integration step, possibly after some modifications or iterations, and are used in the next simulation step. For this reason, the hysteretic behavior using these values is called herein the corrected or converged hysteresis, and the corresponding energy dissipation is termed $E_{\mathrm{E}}^{\mathrm{C}}$.

In order to ensure proper identification of experimental substructure properties, the energy error $E_{\mathrm{EA}}^{\mathrm{err}}$ should be monitored rather than $E_{\mathrm{E}}^{\mathrm{err}}$, since the latter only considers the errors that may occur outside of numerical simulation module. Hence, it is important to note that not all errors in $E_{\mathrm{EA}}^{\mathrm{err}}$ have experimental sources, and part of them may be originating from numerical simulation module. Examples of these errors are the errors resulting from the use of approximate models of the experimental subsystem, piecewise linear approximations of nonlinear experimental hysteretic behavior, and inaccurate correction of forces for actuator tracking errors. This error can also be normalized by a relation similar to (5) for online monitoring of simulation errors.

Overall Energy Balance for Evaluation of Total Errors

In this section, an energy-based error measure is introduced for hybrid simulation that includes both numerical and experimental errors, and does not require a numerical model of the test system. Filiatrault *et al.* [1994] proposed the use of energy balance equation to estimate the extent of numerical errors in nonlinear seismic analyses. They showed that the energy balance is a better accuracy measure than a comparison among peak response parameters, such as displacements and accelerations. The error index introduced in this section also uses the energy balance for online assessment of simulation accuracy. In order to include both numerical and experimental errors in this index, the energy balance evaluation procedure is slightly modified as described below.

The energy balance equation of a simulation can be obtained by integrating the equation of motion (1) over displacement:

$$E_{\mathrm{K}} + E_{\mathrm{D}} + E_{\mathrm{S}} + E_{\mathrm{E}}^{\mathrm{C}} = E_{\mathrm{I}} \tag{9}$$

in which E_{K} is the kinetic energy of numerical mass, E_{D} is the energy dissipated through viscous damping in numerical substructure, E_{S} is the strain energy stored or dissipated in numerical substructure:

$$E_{\mathrm{K}} = \frac{1}{2} \mathbf{v}^{\mathrm{T}} \mathbf{M}\, \mathbf{v} \tag{10}$$

$$E_{\mathrm{D}} = \int \mathbf{v}^{\mathrm{T}} \mathbf{C}\, \mathbf{du} \tag{11}$$

$$E_{\mathrm{S}} = \int \mathbf{u}^{\mathrm{T}}\mathbf{K}\,\mathbf{du} \tag{12}$$

and $E_{\mathrm{E}}^{\mathrm{C}}$ is the energy stored or dissipated in the experimental substructure from an analytical standpoint, as discussed in the preceding section.

Both numerical and experimental errors affect how well the energy balance is maintained. For example, experimental errors make experimental energy E_{E} differ from $E_{\mathrm{E}}^{\mathrm{C}}$ used in numerical analysis to satisfy the equation of motion. On the other hand, numerical truncation errors or relaxed convergence tolerances may result in small differences between left- and right-hand sides of (9). Hence, an overall energy error can be defined as:

$$E^{\mathrm{err}} = E_{\mathrm{I}} - \left(E_{\mathrm{K}} + E_{\mathrm{D}} + E_{\mathrm{S}} + E_{\mathrm{E}}\right) \tag{13}$$

Within the engineering precision requirements, and if the convergence tolerance is sufficiently small, the energy error obtained from (13) will be very close to $E_{\mathrm{EA}}^{\mathrm{err}}$ from (8). That is, it essentially includes the difference between actual experimental and converged energies, when the experiment and the numerical simulation are in phase. Particularly, it cannot capture all of the errors of numerical integration procedure, since all integration methods satisfy the equation of motion and its integral form, (9). However, perfect satisfaction of equation of motion is not sufficient for an accurate and stable simulation; the numerical simulation procedure should also maintain proper kinematic relations between displacement, velocity and acceleration such as (2) and (3). To include the kinematic errors that may occur in the numerical simulation module of hybrid simulation, it is proposed to replace the velocity in (10) and (11) by the first derivative of displacement:

$$E_{\mathrm{K}} = \frac{1}{2}\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{M}\,\dot{\mathbf{u}} \tag{14}$$

$$E_{\mathrm{D}} = \int \dot{\mathbf{u}}^{\mathrm{T}}\mathbf{C}\,\mathbf{du} \tag{15}$$

With this modification, any error in the kinematic relation between displacement and velocity (and hence, between displacement and acceleration) will be reflected as a discrepancy of kinetic and damping energies from those satisfying (9). Similar to (5), an energy error indicator (EEI) can be calculated based on overall unbalanced energy:

$$\mathrm{EEI} = \frac{E^{\mathrm{err}}}{E_{\mathrm{I}} + E_{\mathrm{E}}^{\mathrm{max}}} \tag{16}$$

Since the error terms that constitute (16) can be calculated merely based on the experimental measurements and the states calculated in the simulation, the above energy error indicator can be calculated online. Note that this error index does not directly depend on the structural properties, such as natural period or the extent of nonlinearities. The behavior of this error indicator with different levels of hybrid simulation errors is studied through numerical and experimental simulations in the following sections.

NUMERICAL STUDIES

A series of numerical simulations have been carried out to study the effects of hybrid simulation errors on the simulation results and the energy error indicator given by (16). In this study, simplified hybrid simulation models including numerical and experimental error sources [Mosqueda *et al.* 2007b] have been used for numerical studies. In these models, a Bouc-Wen stiffness model is utilized for calculation of experimental restoring force. Actuator delay and measurement noise are also introduced in the signals to mimic the behavior of an actual hybrid simulation system. These errors have been calibrated to actual experimental data and laboratory equipment information.

The errors studied include actuator tracking errors, measurement noise, errors of the numerical integration method, and errors of the experimental substructure models. For this purpose, a SDF system has been considered with a natural period of 0.4 seconds. The damping is assumed to be 5% of critical, and is numerically modeled. The entire stiffness of the system is modeled in an essentially strain-dependent experimental substructure, using a Bouc-Wen model with parameters selected to produce a ductile nonlinear behavior and a yield displacement of 10 mm. The structural response subjected 1940 El Centro earthquake has been simulated using explicit Newmark and Operator-Splitting [Nakashima *et al.* 1990] integration methods. The exact simulation results are obtained for comparison purposes using a small-time-step implicit integration without considering the hybrid simulation errors. Note that such benchmark does not exist in an actual hybrid physical and numerical simulation, since in that case, the response of the experimental substructure is not exactly known.

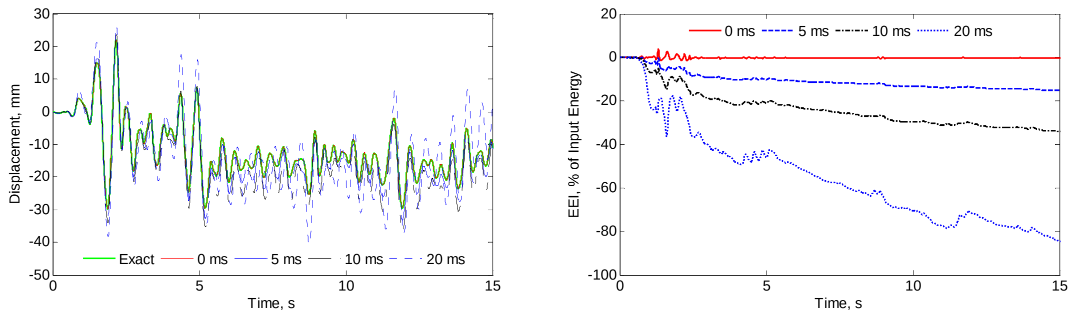


FIGURE 2 - EFFECT OF SERVO-HYDRAULIC ACTUATOR DELAY ON DISPLACEMENT HISTORY AND ERROR INDICATOR

One of the most important errors in hybrid simulation is the systematic servo-hydraulic actuator delay. This delay is known to introduce erroneous energy in the simulation through an apparent negative damping [Horiuchi *et al.* 1999; Mosqueda *et al.* 2007b]. In the first series of the numerical simulations, all artificial error sources are eliminated, except for the delay. Simulations have been carried out with different values of actuator delay up to 20 milliseconds, to observe the effects on the displacement history and the energy error indicator, as shown in Figure 2. It can be observed that delay normally increases the response parameters by adding energy to the system. The added energy can be clearly seen from the negative values of the energy error indicator, which show the actual input energy is smaller than the internal energy stored or dissipated in the system. The comparison of displacement histories shows that the displacement errors are negligible as long as the energy error remains within about 20% of input energy.

Random tracking errors also occur in hybrid simulation, and may alter the results. However, the effects are less significant than delay, as these tracking errors are normally very small. Of course, this is only true for a properly tuned experimental setup with minimal control and tracking errors. Considerable amounts of random tracking errors have been observed to result in positive values of energy error indicator in nonlinear experiments. This implies the occurrence of erroneous experimental energy dissipation through additional hysteretic cycles resulting from random tracking errors. A similar effect may occur in hybrid simulations using fully implicit integration algorithms with physical application of iterative displacements.

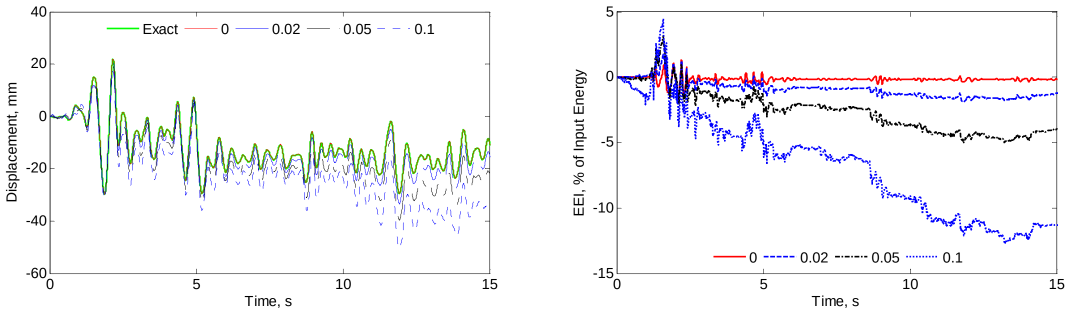


FIGURE 3 - EFFECT OF FORCE MEASUREMENT NOISE ON DISPLACEMENT HISTORY AND ERROR INDICATOR WITH DIFFERENT RATIOS OF NOISE STANDARD DEVIATION TO THE MAXIMUM RESTORING FORCE

Next, the effects of force measurement noise are studied by eliminating delay and tracking errors of the actuator and using different ratios of noise standard deviation to the maximum restoring force. The simulation results shown in Figure 3 demonstrate the structural response modification with noise. The displacement history shows that the peak response is mildly affected by the measurement noise at levels considered in these simulations. However, even small amounts of noise are observed to alter quantities such as nonlinear deformations and permanent drifts. The energy error index shows that measurement noise can also add energy to the system, with less severity compared to actuator delay. It is important to note that the measured experimental energy dissipation includes the effects of noise, and excessive measurement errors may lead to the alteration of simulation outcome through modification of measured experimental response. Hence, it is highly important to ensure the proper tuning and calibration of the test equipment and instrumentation before the simulation begins.

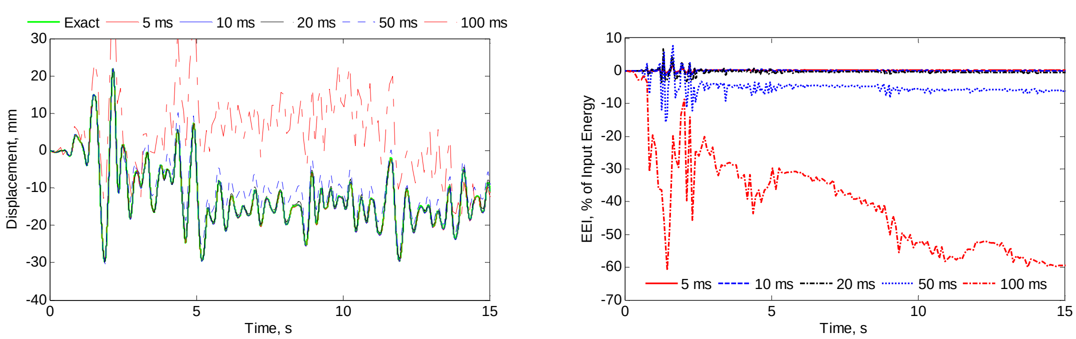


FIGURE 4 - EFFECT OF INTEGRATION TIME STEP ON DISPLACEMENT HISTORY AND ERROR INDICATOR (EXPLICIT NEWMARK METHOD)

In order to study the effects of the numerical errors on the energy error indicator proposed in this study, numerical simulations have been carried out with explicit Newmark and operator-splitting integration methods. In these simulations, the actuator tracking errors and measurement noise are calibrated to the actual experimental results [Mosqueda *et al.* 2007b]. A delay compensation procedure [Ahmadizadeh *et al.* 2007] is also used to compensate the delay in actuator response, similar to an actual hybrid simulation. Figure 4 and Figure 5 show the simulation results using these methods with different integration time steps, ranging from 0.005 to 0.100 seconds. As illustrated, the energy error increases with using larger integration time steps. In the simulations using explicit integration, the energy error tends to constantly increase throughout the simulation. However, the energy error in the simulations using operator-splitting remains mostly steady with abrupt increments near the earthquake peak. Overall, the operator-splitting method demonstrates smaller errors compared to the explicit method. It has been observed that displacement errors can be limited within reasonable ranges by restricting the energy balance error value to about 20%.

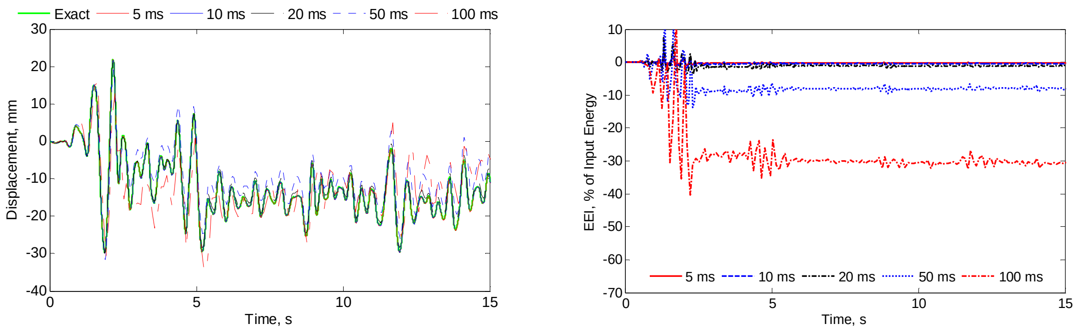
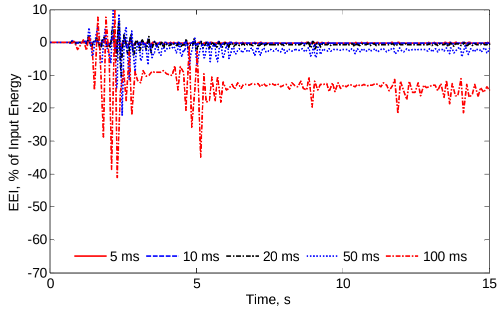


FIGURE 5 - EFFECT OF INTEGRATION TIME STEP ON DISPLACEMENT HISTORY AND ERROR INDICATOR (OPERATOR-SPLITTING METHOD)

In order to explain the above observations, it should be mentioned that the equation of motion is satisfied in both integration methods considered in this study. In these integration methods, the desired displacement is calculated using only the information available up to that step. This is achieved by using (2) with $\beta = 0$ in the explicit method, or only using the first three terms of this equation in the operator-splitting method. In the explicit approach, this displacement serves as the final displacement of the step. However, in the operator-splitting method, the elimination of the term $\beta(\Delta t)^2 \mathbf{a}_n$ is only temporary, and the displacement is later modified to satisfy (2) in its complete implicit form. This modification occurs in a corrector step (the first of two integration substeps shown in Figure 1), after the application of the desired displacement and measurement of the restoring force, to account for the change of the acceleration from the beginning to the end of the integration step. Since the explicit Newmark method does not consider this change of acceleration in the calculation of the displacement, the satisfaction of the kinematics equations will only be approximate, and its errors will be larger than those of operator-splitting approach throughout the simulation.

In the operator-splitting method, the experimental restoring force is also modified to account for the change in the displacement vector, $\beta(\Delta t)^2 \mathbf{a}_n$. In this method, the initial stiffness matrix of the experimental substructure is used due to the difficulties associated with online calculation of experimental tangent stiffness matrix. This approximation introduces additional errors in the restoring force of nonlinear experimental substructures. Further, this approximation is believed to result in abrupt increments of energy error indicator at integration steps with large nonlinear deformations, as shown in Figure 5. The SDF structure considered here shows a ductility of about 3 when subjected to El Centro earthquake. The initial stiffness approximation was observed to result in a converged hysteretic behavior that was notably different from the actual behavior. This error can therefore be categorized as an error of the experimental substructure model that takes place when the experimental substructure behaves nonlinearly. Note that the explicit approach does not apply any modification on the experimental displacement or restoring force, and does not have the potential to introduce this type of error in the simulation.

(a) 

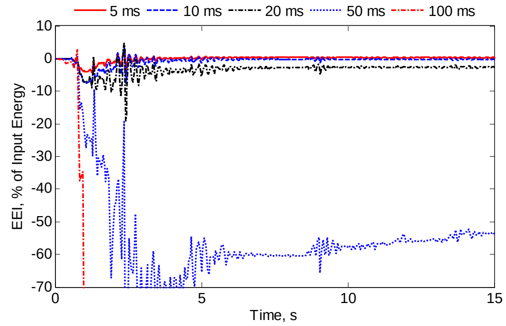
(b) 

FIGURE 6 - EFFECT OF THE INTEGRATION TIME STEP ON THE DISPLACEMENT HISTORY AND ERROR INDICATOR IN THE SIMULATION OF A STRUCTURE WITH LOW DUCTILITY (A) OPERATOR-SPLITTING METHOD, (B) EXPLICIT NEWMARK METHOD

To further study the effect of experimental substructure ductility on the numerical errors and errors of experimental substructure model, the above simulations have been repeated for the same SDF structure, with an increased yield displacement (50 mm) to result in a close-to-linear response. It should be mentioned that in these simulations, the same $E_{\mathrm{E}}^{\max}$ based on yield displacement of 10 mm is used for normalization of energy error to provide better references for comparison. The results of the operator-splitting method, shown in Figure 6(a), demonstrate reduced displacement and energy errors in all simulations. As mentioned above, this improvement can be explained by the increased accuracy of the correction step utilizing initial stiffness as a result of the reduced experimental ductility. However, it should be noted that errors in the estimation of initial stiffness matrix of the experimental substructure may also increase the energy error, regardless of the ductility level of the simulation.

On the other hand, the simulation of the considered SDF with reduced ductility using explicit Newmark approach results in considerably larger displacement and energy errors (Figure 6(b)). Particularly, the simulation with 0.1-second time step becomes unstable, although the ratio of the time step to the natural period is 0.25, which is less than the explicit integration stability limit of $1/\pi$. Note that this stability limit is calculated for linear undamped structures, and should increase for structures possessing damping or hysteretic energy dissipation capacity. The increased errors of the explicit integration can be attributed to the reduced energy dissipation capacity of the system, and the fact that the structure shows a more stiff behavior with the reduction of ductility. Stiff structures are more sensitive to errors, and require smaller time steps for accurate simulation using explicit integration.

Similar trends have been observed in simulations of other structural systems and earthquake excitations. These simulations demonstrate that analytically-calculated stability and accuracy limits of numerical integration procedures have limited application for the assessment of the reliability of hybrid simulations. It has been observed that in addition to the properties of the test structure, experimental errors, nonlinearities and properties of the excitation signal may affect the accuracy and stability of hybrid simulations. On the other hand, a comparison of the numerical and experimental simulation results is generally not suitable for actual hybrid simulations, due to the unavailability of exact models of the experimental substructure. Instead, the energy error indicator given by (16) can be used for this purpose, to assess the accuracy and stability of the hybrid simulation results.

EXPERIMENTAL VERIFICATIONS

In this section, the results of a few hybrid numerical and experimental simulations are presented to verify the results obtained from numerical simulations. In SDF experimental simulations, the cantilever column shown in Figure 7 was used as the experimental substructure. This column provides lateral resistance through replaceable coupons inserted in the clevis at the bottom. The number of coupons and the structural mass were selected to achieve a natural period of 0.5 s. Damping of 5% of critical was numerically modeled, and the response was simulated for the 1978 Tabas earthquake (a near-fault record with peak ground acceleration of 0.85g). The amplitude of this earthquake was scaled to small values, to avoid damage to the experimental setup in case of instabilities.

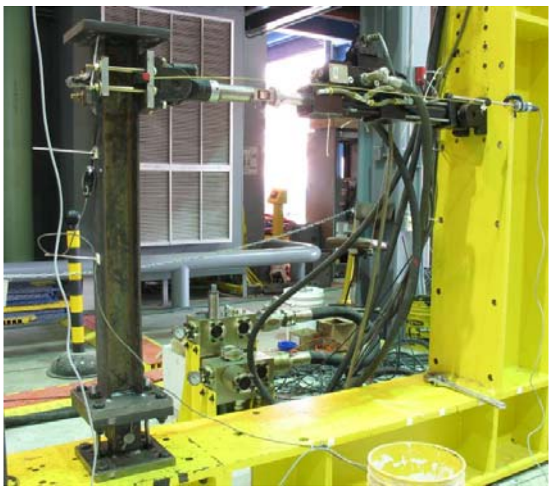


FIGURE 7 - SDF EXPERIMENTAL SETUP.

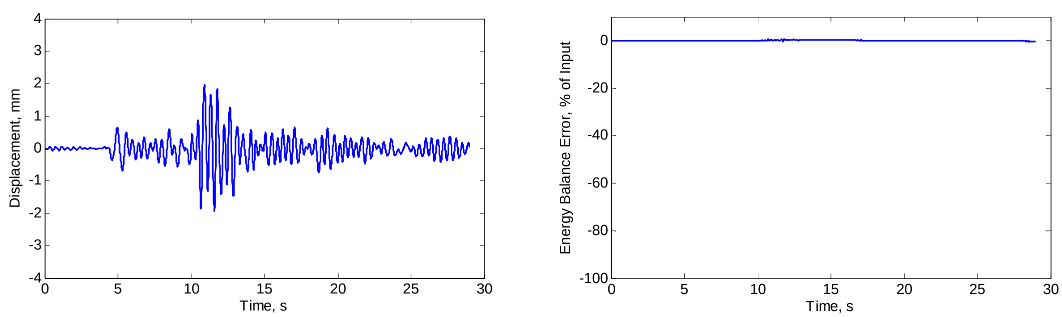


FIGURE 8 - HYBRID SIMULATION RESULTS OF A 0.5-SECOND PERIOD STRUCTURE SUBJECTED TO LOW-AMPLITUDE EXCITATION WITH DELAY COMPENSATION.

In the first series of the experimental studies, the effect of using a delay compensation procedure on the accuracy and stability of the simulation is studied. Delay was measured to be 15 milliseconds in the present experimental setup. Figure 8 shows the simulation results using a displacement extrapolation approach for delay compensation. As shown, the simulation remains stable, and according to the energy error indicator, with a good accuracy. When delay is not compensated, the simulation becomes unstable as shown in Figure 9. This figure demonstrates that the response constantly increases, even before the earthquake strong motion begins. The energy error indicator in this figure shows that a significant amount of erroneous energy is being added to the system. The simulation is stopped at about 6 seconds as a result of the detection of excessive errors due to system instability. It is shown that with an energy error limit of 15% of input, the simulation could have been stopped at 4 seconds, before the displacement exceeds the maximum expected displacement for this low-level simulation. In this simulation, the observed hysteretic behavior of the experimental substructure showed reverse (counterclockwise) loops resulting from the negative damping effect of uncompensated delay.

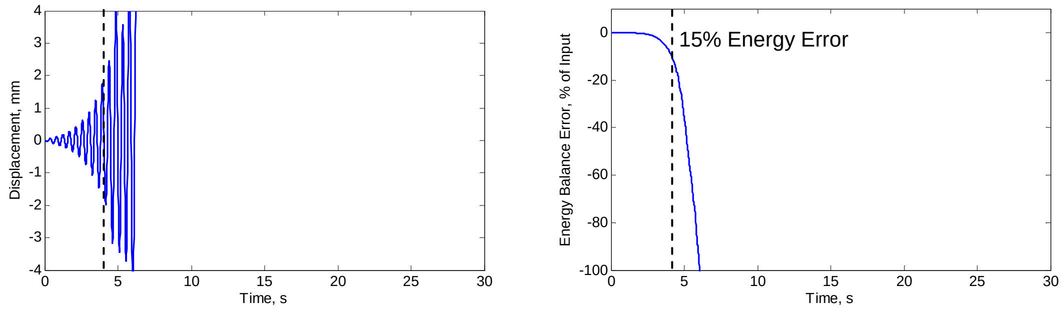


FIGURE 9 - HYBRID SIMULATION RESULTS OF A 0.5-SECOND PERIOD STRUCTURE SUBJECTED TO LOW-AMPLITUDE EXCITATION WITHOUT DELAY COMPENSATION.

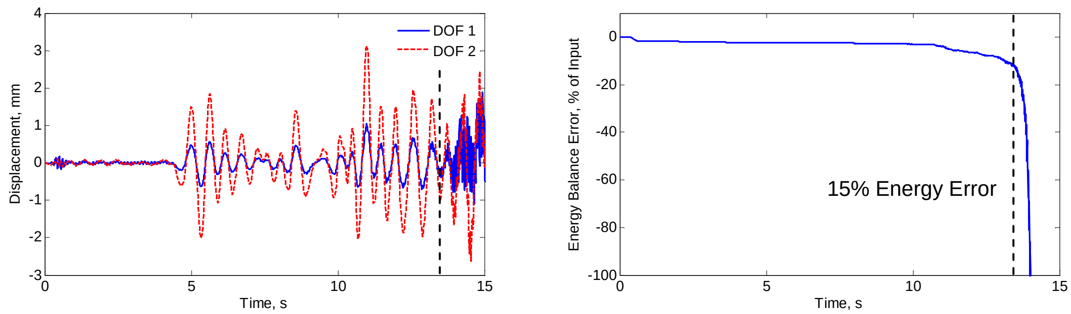


FIGURE 10 - HYBRID SIMULATION RESULTS OF A 0.5-SECOND PERIOD 2DF STRUCTURE SUBJECTED TO LOW-AMPLITUDE EXCITATION – EXPLICIT NEWMARK METHOD.

Next, a two-degree-of-freedom experimental setup was assembled by mounting two columns identical to that shown in Figure 7 on top of each other. The two degree of freedom system, with the same properties and fundamental period of 0.5 seconds, was simulated using operator-splitting and explicit Newmark integration methods. The simulation using operator-splitting integration approach was observed to be stable and accurate, with less than 0.5% energy error at the end of simulation. The explicit Newmark approach, however, failed to maintain the stability through the simulation, as shown in Figure 10. Up to about 13 seconds, the simulation shows an acceptable accuracy, which can be observed through the small values of the energy error indicator. The displacement history was also observed to be in agreement with that obtained from operator-splitting method up to this point. At this time, it appears that the accumulation of errors becomes significant, which eventually renders the simulation unstable with a large increase in the energy error. It should be noted that this simulation had a relatively large noise to signal ratio due to low excitation amplitude. This noise is particularly important in multi-degree-of-freedom systems, as it may result in the erroneous excitation of higher modes (the second mode in this simulation, with natural period of 0.13 s). Again, an energy error limit of 15% of input shows the instability problem shortly after 13 seconds, before the instability appears clearly in the displacement results. Hence, the energy error indicator proves to be useful as a reliable sign of excessive errors, which should be used to stop the simulation.

CONCLUSIONS

An energy-based error indicator is proposed for assessment of the accuracy and stability of hybrid simulations. This procedure expands the existing methods to monitor experimental errors by considering both experimental and numerical errors. For this purpose, the energy balance equation was modified to include the effects of: (i) the discrepancies between the actual experimental energy dissipation (hysteretic behavior) and that conceived by the numerical simulation module, (ii) the improper kinematic relations between displacement, velocity, and acceleration, and (iii) the errors in satisfying the governing equation of motion.

Through numerical and experimental simulations, it was shown that the energy error indicator effectively captures the effects of the most important errors of hybrid simulation. These include the errors originating from actuator delay, numerical errors of the integration algorithm and errors of the experimental substructure models. It was observed that limiting the energy error to about 20% of input energy can ensure the reliability of the results from hybrid simulation. It was also found that an energy error warning limit of about 15% of input can be used to effectively capture the excessive errors during hybrid simulations. Random tracking errors and measurement noise were also observed to affect the energy error indicator, but with less significance. It was shown that the analytically-calculated stability and accuracy limits may not adequately portray the outcome of hybrid simulations, mainly due to the experimental errors and nonlinearities. The energy error indicator was demonstrated to be an efficient test-specific alternative that includes the effects of both numerical and experimental errors, and does not require a numerical model of the test system.

REFERENCES

[1] Ahmadizadeh, M., G. Mosqueda and A. M. Reinhorn (2007). "Compensation of actuator delay and dynamics for real-time hybrid structural simulation." *Earthquake Engineering & Structural Dynamics*: in press.

[2] Filiatrault, A., P. Leger and R. Tinawi (1994). "On the computation of seismic energy in inelastic structures." *Engineering Structures* **16**(6): 425-436.

[3] Horiuchi, T., M. Inoue, T. Konno and Y. Namita (1999). "Real-time hybrid experimental system with actuator delay compensation and its application to a piping system with energy absorber." *Earthquake Engineering & Structural Dynamics* **28**(10): 1121-1141.

[4] Mosqueda, G. and M. Ahmadizadeh (2007). "Combined implicit or explicit integration steps for hybrid simulation." *Earthquake Engineering & Structural Dynamics* **36**(15): 2325-2343.

[5] Mosqueda, G., B. Stojadinovic and S. A. Mahin (2007a). "Real-time error monitoring for hybrid simulation. I: methodology and experimental verification." *Journal of Structural Engineering* **133**(8): 1100-1108.

[6] Mosqueda, G., B. Stojadinovic and S. A. Mahin (2007b). "Real-time error monitoring for hybrid simulation. II: structural response modification with error." *Journal of Structural Engineering* **133**(8): 1109-1117.

[7] Nakashima, M., T. Kaminoso, M. Ishida and A. Kazuhiro (1990). *Integration techniques for substructure online test*. 4th US National Conference of Earthquake Engineering, Palm Springs, CA, Earthquake Engineering Research Institute.

[8] Nakashima, M. and N. Masaoka (1999). "Real-time on-line test for MDOF systems." *Earthquake Engineering & Structural Dynamics* **28**(4): 393-420.

[9] Shing, P. B. and S. A. Mahin (1983). Experimental error propagation in pseudodynamic testing (UCB/EERC-83/12), Earthquake Engineering Research Center, University of California, Berkeley.

[10] Shing, P. B. and S. A. Mahin (1984). Pseudodynamic test method for seismic performance evaluation (UCB/EERC-84/01), Earthquake Engineering Research Center, University of California, Berkeley.

[11] Thewalt, C. R. and M. Roman (1994). "Performance parameters for pseudodynamic tests." *Journal of Structural Engineering* **--** *ASCE* **120**(9): 2768-2781.

[12] Wu, B., G. Xu, Q. Wang and M. S. Williams (2006). "Operator-splitting method for real-time substructure testing." *Earthquake Engineering & Structural Dynamics* **35**(3): 293-314.